

## Integrating Python Programming Within A Multiple Representation Framework To Enhance Student Mathematical Numeracy

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**Abstract:** By using digital technology, this study aims to develop a learning design that facilitates multiple representations and numeracy skills. The ASSURE model was used as the design approach for this development. The first stage was a needs analysis to ensure the development aligns with the needs of educators and students. A validation stage was also conducted before classroom use to ensure it is applicable and addresses teaching and learning challenges. The method used was a mixed methods approach. The development stage provided an overview of the process, followed by descriptive and inferential statistical analyses to assess the results of the learning design implementation. The results showed that Python-based learning led to a greater increase in numeracy skills than conventional learning. The experimental group increased its average score from 60.12 in the pretest to 83.72 in the posttest, with an N-gain of 0.60, while the control group achieved an N-gain of 0.36. Statistical tests showed that the increase in both groups was significant, but the magnitude of the increase in the experimental group was more dominant. Qualitative findings from observations and questionnaires showed that students became more active, helped their understanding of concepts through various representations, and were more confident in solving numeracy problems, despite initially experiencing difficulties using Python. In addition, expert validation results showed that the developed learning design was in the very feasible category for content, media, and learning. Students demonstrate various types of representations in learning, including graphical, symbolic, pictorial, tabular, and verbal representations. Therefore, it can be concluded that developing a learning environment that aligns with learning objectives and is tailored to each student's skills and conditions can significantly increase student engagement and achieve desired competencies.

**Keywords:** python-based learning design, multiple representation, numeracy skills, digital.

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### ■ INTRODUCTION

The current digital era demands that all areas of life evolve accordingly. This is also true in education, and especially in mathematics education. In today's mathematics education, educators must develop learning strategies to help students understand concepts effectively (Serin, 2023). One way to do this is to explore various types of representations and improve their numeracy skills. This is based on students' needs, where they should not only master procedures but also manage how to think conceptually. With

these technological advances, educators can utilize them to develop appropriate learning designs or media to facilitate various types of representations and numeracy skills.

Multiple representations in mathematics teaching combine various forms, such as numbers, symbols, graphs, and language, to illustrate mathematical concepts. This method is crucial for developing conceptual understanding and improving students' problem-solving abilities (Dreher et al., 2016; Ozyildirim et al., 2009; Sa'Dijah et al., 2018). Research shows that

students who learn with more than two representations perform better on problem-solving tasks than those who use fewer representations (Rexigel et al., 2024). Integrating appropriate technology and learning strategies can further support the use of multiple representations in mathematics learning and students' numeracy skills.

Numeracy is a crucial skill in mathematics education, encompassing the ability to apply mathematical knowledge in a variety of everyday situations. Numeracy encompasses not only mastery of the fundamentals of mathematics but also problem-solving, critical thinking, and understanding meaning in contexts beyond mathematics (Goos et al., 2020b; Adelia et al., 2024). Effectively teaching numeracy is a challenge. Educators often struggle to incorporate numeracy into their teaching methods and grasp the associated concepts (Liljedahl, 2015). Furthermore, learning activities that utilize games and specific contexts have demonstrated positive results in improving students' mathematical abilities (Maher & Smith, 2017; Pettigrew et al., 2020). The importance of numeracy spans many areas, from early childhood education and teacher training to critical thinking and everyday learning. Addressing these areas holistically can substantially improve numeracy skills and their use in various areas of life. However, mathematics learning practices are still dominated by procedural activities and do not provide students with the opportunity or space to fully explore how to represent their thinking. Research shows that students often excel at procedural tasks but struggle with conceptual questions. For example, research by Kurniadi et al. (2022) and Žakelj & Klanèar (2024) showed several differences: students' procedural problem-solving success rate was 60%, whereas conceptual knowledge was only 40%. Furthermore, in the research by Kurniadi et al., it was stated that although students were often able to carry out mathematical

procedures to obtain solutions to differential equations, they often had difficulty understanding the meaning and behavior of the solutions, as well as interpreting the model. Therefore, most students could follow the steps to solve the problem and obtain the answer, but they had difficulty understanding it more deeply. Learning activities are still dominated by lecture-based methods that direct students to solve problems using prescribed steps; educators do not emphasize deeper understanding (Woods & Weber, 2020). Therefore, learning activities that can explore students' representation and numeracy skills are needed to optimize learning outcomes. One way to do this is by utilizing technology as a learning medium.

Technology integration can be a bridge to enrich representation and support students' numeracy skills, namely through visualization, simulation, computation, and data exploration. To improve representation in mathematics, various technologies and pedagogical approaches have been explored to make mathematics education more inclusive and engaging (Archer et al., 2024; Eppard et al., 2021). Digital tools can substantially enhance student understanding and interaction in mathematics lessons. These technologies assist in exploring and analyzing complex mathematical tasks, thus creating a collaborative learning environment that allows students to delve deeper into the content (Orrill & Polly, 2013; Pedersen et al., 2021). The integration of Google Colab, Python, Matplotlib, GeoGebra, and spreadsheets in education significantly improves learning outcomes by making abstract concepts more tangible and engaging (Arvind et al., 2023; Dahal et al., 2022; Mishra et al., 2023; Rasid et al., 2020). These devices support interactive and visual learning, which is crucial for subjects like mathematics. This research uses Python as the programming language.

Previous research has shown that the rapid development of digital technology creates

significant opportunities in education, particularly for learning. This technology allows students to utilize and facilitate various forms of mathematical representation. It also supports the strengthening of students' numeracy skills. The opportunity to demonstrate various types of representation emphasizes the importance of visualizations, graphs, symbols, and other forms in building a deeper understanding of concepts (Capraro et al., 2014; Li et al., 2022; Ozyildirim et al., 2009). Furthermore, research on numeracy is currently receiving special attention, for example, in mathematics education, specifically in the context of mathematical literacy and applications in everyday life (Nuraini & Humaidi, 2020). However, most previous research still places representation and numeracy as separate elements. In fact, in some studies, the integration of the two has not yet been a primary focus in the development of effective learning designs. This statement is not intended to diminish the existing literature, but rather to highlight methods that clearly integrate various representations with numeracy systems in a holistic pedagogical context that are still relatively rare.

This gap was believed to present a new opportunity for research, particularly in technology-based learning activities. There is a significant opportunity to integrate the two. This is because technology provides more interactive visual images or representations and can enrich various areas of thinking. However, using technology alone in learning is not sufficient to provide the right solution, as it often relies on visual aids. Therefore, appropriate learning designs are needed when integrating technology into learning. This will accommodate various representations and students' numeracy skills.

As part of the researchers' response to this gap, this study aims to design a learning approach that integrates technology to facilitate various representations and strengthen students' numeracy. Furthermore, it can support students'

exploration in real-life applications. In practice, learning activities are conducted through various approaches that integrate various aspects. To clarify the research direction, the research problem is formulated as follows: "How can learning designs facilitate various representations and students' numeracy skills?"

This research is expected to contribute ideas and insights to preparing and managing learning in schools. Theoretically, it will be achieved by formulating learning designs that integrate various representations using technology and that accommodate numeracy skills. The practical contribution will be the provision of guidelines that can serve as a reference for classroom implementa

## ■ **METHOD**

### **Participants**

This research was conducted at a private university in Indonesia, specifically in the Mathematics Education department. The sample consisted of third- and fifth-semester students, with third-semester students serving as the control group and fifth-semester students as the experimental group receiving an integrated learning intervention. A total of 10 teachers who already understood programming concepts were selected for the limited trial, along with 22 third-semester and 28 fifth-semester students. Subjects were selected using purposive sampling, namely those who had taken courses in algebra and programming languages, thus making them relevant to the research objectives.

To control for differences in students' initial abilities across semesters, the data were analyzed using Analysis of Covariance (ANCOVA). In this analysis, posttest scores were used as the dependent variable, learning group as the independent variable, and pretest scores as the covariate. The purpose of ANCOVA was to control for initial differences between groups, thereby providing a more accurate estimate of the learning intervention's impact.

### **Research Design**

This research adopted a Research and Development (R&D) approach based on the ASSURE instructional design framework. The ASSURE framework provides a set of structured strategies for incorporating technology into the educational process, encompassing six main steps. The first stage begins with analyzing learners. This stage is used to understand learners' characteristics in their learning experiences, their readiness to use technology, and the needs of learning activities. This stage is expected to form the basis for developing the stages of the learning design, including content requirements and the support needed during learning activities. The next stage is to state learning objectives, which aim to define specific learning goals. With these objectives, the design development becomes clearer and more focused in its step-by-step development, which is then translated into a sequence of activities within a learning activity.

The third stage is selecting methods, media, and materials. This third stage is crucial in the development process because it forms the framework for learning activities. For example, selecting the learning strategies and technology to be used, and the materials that form the basis for exploring mathematical problem-solving that accommodates various representations and numeracy skills. The fourth stage is utilizing media and materials. At this stage, the learning flow or syntax is established, namely, how Python and the materials are used directly in learning. The next stage requires learner participation. This stage requires active student participation in each step of learning syntax. This is achieved through contextual problem exploration, discussion activities, and the use of Python in learning. The final stage is evaluation and revision, which aims to evaluate and reflect on what has been developed to determine an appropriate learning design that accommodates various representations and students' numeracy abilities.

The ASSURE approach is highly advantageous because it is planned and adaptable, making it easy to adapt teaching techniques to student needs while encouraging the meaningful use of digital technology in mathematics learning. In this project, the primary tool used was Python to create technology-based learning modules that are part of the framework with multiple representations.

This study focused on the development and implementation of a mathematics learning model integrating multiple representations and digital resources to improve numerical skills. Python provides a variety of interactive learning tools, such as dynamic visualizations, live graphs, computational simulations, and more abstract conceptual models. This provides students with a more comprehensive and integrated approach to learning mathematics.

### **Research Procedure**

The development process followed the six stages of the ASSURE model. This research began with an analysis of student needs and characteristics through classroom surveys, document studies on numeracy skills, and interviews with lecturers and students regarding obstacles and needs in understanding numeracy and using digital media. Then, Determining Learning Objectives, learning objectives were established based on the results of the needs analysis and the curriculum. The objectives formulated included the ability to understand numeracy concepts through multiple representations and to use digital media to solve contextual mathematical problems. And then the selection of Methods, Materials, and Resources. The learning method used was a multiple representation-based approach that integrates various forms of representation (visual, verbal, symbolic, and contextual) in mathematics learning. A digital project-based learning module was developed that included interactive simulations,

data-based problem solving, and visualization tools for mathematical concepts. The next procedure is for Media and Materials. The media and materials used were developed. Expert validation of the learning tools was conducted by lecturers with expertise in mathematics and learning technology. Then, the media and materials were revised based on input from the validators.

Implementation is carried out through the following learning activities: pre-tests; material boosters with videos or interactive quizzes; concept exploration through visual representations in Python; and attention to symbolic (formulas), verbal (explanations), and contextual (story problems) representations. Numeracy assignments are given using digital tools; students solve numeracy problems and present their results using digital applications. The final stage is a post-test. Evaluation is conducted in two stages: 1) Process evaluation: Using observations of student engagement, the effectiveness of digital media, and lecturer reflections. 2) Outcome evaluation: Measuring students' numeracy skills before and after the

implementation of the learning design. The instruments are a numeracy test and a digital project assessment rubric.

### Research Instruments

There are three main instruments used in this study: 1) a needs questionnaire and a student response questionnaire regarding the learning design, an observation sheet, and a test of various representational and numeracy skills. The needs questionnaire is necessary to analyze the needs for developing the learning design, so it is administered prior to development. This questionnaire contains three main categories: learning experience, the use of Python in learning, learning styles, and learning needs. The questionnaire uses a 1-5 Likert scale (strongly disagree to strongly agree). This questionnaire was developed based on a literature review conducted by the researcher, drawing on various relevant sources in the fields of mathematics learning, learning technology, and mathematical representation (Bosman & Achulze, 2018; Exploring, 2020; Rais & Xuezhi, 2024).

**Table 1.** Components and aspects measured in the needs analysis

No	Component	Measure aspect
1	Academic Experience in Learning	a. Student enjoyment in learning mathematics b. Understanding of mathematical concepts c. Experience of difficulty interpreting numerical data in a learning context
2	Experience Using Technology and Python	a. Student interest in using Python in learning b. Perception of Python's potential for data visualization c. Need for support or training in learning
3	Learning Styles and Instructional Needs	a. Student preferences for the use of various representations (visual, symbolic, and multiple representations) b. Contextual examples c. Visual aids d. Digital learning media

Before being used in the research, the needs analysis questionnaire was first tested for validity and reliability. This questionnaire consists of three components: academic experience in learning, experience using technology and Python, and

learning styles and learning needs. The validity test results showed that all items had item-total correlations greater than the r-table value (0.30). Correlations ranged from 0.37 to 0.49 for the academic experience component, 0.61 to 0.68

for experience with technology and Python, and 0.53 to 0.64 for learning styles and learning needs. These results indicate that all questionnaire items were valid. Reliability testing using Cronbach's alpha also showed good results, with component reliabilities of 0.81, 0.74, and 0.73, respectively, and an overall reliability of 0.76, which falls within the reliable range. Thus, the needs analysis questionnaire was deemed suitable for use in this research.

In addition, the reliability of the numeracy test instruments used in the pretest and posttest stages was also tested. The results showed that the pretest instrument had a Cronbach's Alpha value of 0.78 and the posttest instrument a Cronbach's Alpha of 0.76. These values indicate that both instruments have good internal consistency, making them reliable for measuring students' numeracy abilities before and after the implementation of the learning.

A response questionnaire was administered after the implementation of the developed learning design to assess its use. The needs and response questionnaire used a Likert scale. Observation sheets were used during the limited trial and implementation of the learning-to-observe approach to student activities, including engagement in problem-based learning, use of Python as a tool for numerical representation, collaboration with other students, and responses to instructor guidance. Ability tests were administered during the pre-test and post-test. Each consisted of four contextual mathematical problem-solving questions, each designed to facilitate various types of representations and numeracy skills. The questions addressed linear programming or optimization problems. The pre-test and post-test questions share similar characteristics and cognitive levels, but are designed in different contexts.

To ensure that the pre-test and post-test questions can be used effectively and meet the development objectives, instrument validity and

reliability testing were conducted. This was conducted concurrently with the validity testing of the learning design. The content was validated by mathematics content experts. Based on the validation results, the pre-test and post-test questions were deemed suitable for use, with several points noted, including more contextual wording and questions that begin with interrogative words.

### **Data Analysis**

This research used a mixed-methods approach, focusing primarily on developing instructional design and on quantitatively analyzing students' numeracy learning outcomes. Qualitative data were obtained through observations, questionnaires, and interviews, which, in this article, serve as supporting data for understanding the learning implementation process. In-depth qualitative analysis of these data is not the primary focus of this article and will be discussed in more detail in a separate publication.

The needs analysis data and questionnaire responses were analyzed using descriptive statistics and thematic analysis. The questionnaire data were analyzed using percentages and averages, then categorized by trend. This analysis provided insights into the needs for developing instructional designs, as well as students' perceptions and learning experiences after their implementation.

The feasibility assessment data were analyzed quantitatively by converting the average score into feasibility categories based on the score range proposed by Adiguna et al. (2025). An average score between 1.00 and 1.75 is categorized as not feasible, a score of 1.76 and 2.50 as moderately feasible, a score of 2.51 and 3.25 as feasible, and a score of 3.26 and 4.00 as highly feasible.

Data obtained from the pre- and post-tests were analyzed quantitatively to assess the

development of numeracy skills between the two groups. Before proceeding with further analysis, normality and homogeneity tests were conducted to ensure that the data met the requirements for parametric statistical analysis. After these assumptions were verified, an independent-samples t-test was conducted to assess the statistical significance of the observed learning gains between the groups. In addition to evaluating the magnitude of improvement in raw scores, this study also analyzed normalized gain (N-gain) as an indicator of teaching effectiveness, using the approach developed by Nissen et al. (2018). N-gain was calculated to indicate the extent of progress students made.

Students' numeracy scores represent the aggregated performance across the four numeracy indicators. These indicators are studied from various references, including: symbolic (ability to formulate mathematical models an symbol correctly) (Ainsworth, 2006), visual (ability to interpret graphs and solution region) (Lesh et al., 1987), numerical/computational (Ability to calculate values) (Goos et al., 2020a) and contextual (ability to interpret mathematics result) (OECD, 2019). The four indicators do not represent separate constructs but rather complementary dimensions of numeracy,

reflecting students' representational competence in mathematical problem solving. To make it easier to interpret the results and to see the distribution of students' numeracy abilities in more detail, numeracy scores are categorized into five ability levels. These categories include very low (0–20%), low (21–40%), moderate (41–60%), high (61–80%), and very high (81–100%). This categorization is used to analyze the percentage distribution of students across ability levels at both the pretest and posttest stages, so that changes in students' numeracy ability profiles are more clearly observed.

## ■ RESULT AND DISCUSSION

### Student Analysis (A)

The ASSURE approach begins with an analysis of students to identify their previous learning experiences, their use of media in learning, and their readiness to engage in learning that utilizes technology, such as Python. The questionnaire comprises three main components, as described in the research methods section. Average scores for each indicator are presented in Table 2.

In the Academic and Learning Experience aspect, the average score was 3.88. This indicates a positive student perspective on learning

**Table 2.** Average scores by student category in the analysis

No	Category	Related Items	Average Score
1	Academic Experience and Learning	Enjoying mathematics, understanding basic concepts, and having difficulty with real-life data interpretation	3.88
2	Technology and Python Experience	Interest in Python, its potential for visualizing data, and the need for introductory training	3.93
3	Learning Styles and Instructional Needs	Multiple representations, contextual examples, visual aids, and digital media preferences	4.34

mathematics. The indicators included feelings about learning mathematics, understanding of mathematical content such as graphs, percentages, and ratios, and difficulties in interpreting data or graphs in real-life contexts.

These results indicate that although students have a basic understanding, there are gaps when they are asked to apply concepts in practical situations. This indicates the need for teaching strategies that connect mathematical concepts to real-life

applications, highlighting the relevance and usefulness of numeracy in everyday decision-making.

The average score in Technology and Python Experience was 3.93, indicating increasing student interest in adopting technology particularly Python in their mathematics learning process. Several factors contributed to this achievement: students demonstrated enthusiasm for learning Python and recognized its benefits in supporting tasks related to graphics and data. While students generally appeared enthusiastic about using digital tools, their responses indicated that all agreed on the need for training before utilizing these technologies. This indicates that students are open to using tools like Python in their learning, but they are also aware of technical issues that could hinder their effective use. Therefore, Python should be gradually integrated into the instructional design, with adequate support and structure ensure a smooth transition.

With an average score of 4.34, the section Learning Styles and Instructional Needs received the highest score. High-scoring elements include ways to demonstrate arithmetic ideas, to understand through contextualized examples, and to express symbols. Students also reported a preference for visual forms such as graphs and diagrams, as well as interactive media such as animations, illustrations, and other visual aids. This indicates that they strongly prefer learning through hands-on experiences. These findings demonstrate the importance of creating instructional designs that utilize a variety of representational methods visual, symbolic, contextual, and interactive to enhance students' understanding and engagement in numeracy.

Based on this description, it can be concluded that students are ready to engage in learning activities that use digital media. Therefore, developing instructional designs to facilitate multiple representations and numeracy skills is feasible.

### **States Objectives (S)**

This stage refers to the needs analysis conducted. The needs analysis indicates that the use of multiple representations (graphs, symbolic notation, tables, and contextual problems) can significantly support students' understanding of mathematical concepts. Based on the research questions, the objectives are to develop and implement a mathematics learning design that uses Python-based programming to facilitate various forms of representation and develop students' numeracy skills.

### **Selecting Methods, Media, and Materials (S)**

This stage involves selecting the methods, media, and materials that will form the basis of the learning design. The results of the needs analysis in the first stage served as a reference in its development. The analysis showed that students tend to prefer contextual learning experiences that include visualization and interactivity. Students are also open to learning using Python. The selection of methods, media, and materials is as follows, in 3 stages.

First stages is material selection. The material used is linear programming. This material was chosen because it is applicable and relevant to real-world decision-making problems. The subtopics studied include formulating mathematical models of contextual problems, such as objective functions and constraint functions, representing them using linear inequalities, identifying and interpreting optimal values, and connecting mathematical solutions to real-world contexts.

Second stages is method Selection. One of the main objectives of this research is to provide students with the opportunity to explore multiple representations. The use of various representations can help students build connections between mathematical models and their graphical interpretations. The

representations used in linear programming materials include, like a symbolic: The objective function and system of linear inequalities in the problem to be solved, then graphical: The solution set and objective function, numerical: A table showing the values of the objective function, and the last is contextual: A narrative case study that resembles a real-world problem.

The third stage is media selection. The media were developed using Python. Students may choose to use Python or Google Colab, depending on their devices' compatibility. This is

because Google Colab is a cloud-based service that lets you write, run, and share Python code directly in your browser, with no installation required. In developing this design, several features of the Python Library are used, such as SymPy (to define symbolic mathematical expressions and inequalities), Matplotlib (to visualize the solution set), NumPy (to handle computational problems), and so on.

After selecting these three aspects, the learning modules were developed. The learning modules were designed in the following phases

**MODUL PEMBELAJARAN**

**PENGUNAAN PHYTON/VISUALISASI GRAFIK UNTUK PENGEMBANGAN KEMAMPUAN NUMERASI MAHASISWA**

**BAB I PENDAHULUAN**

**1.1 Latar Belakang**

Kemampuan numerasi merupakan bagian penting dari literasi matematika yang harus dimiliki oleh setiap mahasiswa. Khususnya dalam menghadapi tantangan abad 21. Dalam konteks pendidikan tinggi, kemampuan ini tidak hanya melibatkan keterampilan berhitung, tetapi juga mencakup pemahaman terhadap masalah nyata, pemodelan matematis, pendataan, dan interpretasi hasil. Salah satu materi yang dapat digunakan untuk melatih kemampuan numerasi adalah program linear, yang mengharuskan mahasiswa memahami situasi nyata, menyusun model matematis, dan menafsirkan solusi dalam konteks permasalahan.

Di sisi lain, perkembangan teknologi memberikan peluang besar dalam mendukung pembelajaran matematika, termasuk penguatan numerasi. Salah satu platform yang sangat potensial digunakan adalah Google Colab, yaitu lingkungan pemrograman berbasis cloud yang memungkinkan mahasiswa menjalankan kode Python secara langsung melalui browser, tanpa perlu instalasi. Google Colab kompatibel dengan format Jupyter Notebook, dan memungkinkan integrasi teks, matematika (LaTeX), visualisasi data, serta algoritma komputasi secara bersamaan. Oleh karena itu, Google Colab sangat mendukung pendekatan representasi multiple dalam pembelajaran matematika.

Modul ini dikembangkan sebagai panduan belajar mahasiswa untuk memahami dan menguasai materi program linear berbasis representasi multiple dengan dukungan Google Colab. Melalui kegiatan eksploratif, kontekstual, dan komputasional, diharapkan mahasiswa dapat membangun pemahaman yang mendalam dan meningkatkan kemampuan numerasi mereka secara utuh.

**1.2 Tujuan Modul**

Modul ini bertujuan untuk:

1. Memberikan pemahaman konsep dasar program linear melalui pendekatan kontekstual.
2. Membimbing mahasiswa dalam menyusun model matematis dari permasalahan nyata.
3. Mengembangkan kemampuan mahasiswa dalam menggunakan Google Colab untuk menyelesaikan program linear.
4. Meningkatkan kemampuan numerasi mahasiswa melalui representasi multiple.

secara bersamaan. Titik-titik di dalam daerah ini adalah solusi yang memungkinkan, dan kita akan mencari titik yang memberikan nilai maksimum dari fungsi objektif.

**4.4 Implementasi di Google Colab**

Google Colab memungkinkan kita memalukan dan menjalankan kode Python secara langsung tanpa instalasi. Untuk memodelkan dan menyelesaikan kasus ini, ikuti langkah-langkah berikut:

**1. Impor pustaka yang diperlukan**

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import linprog
```

**Penjelasan:**

- `numpy` digunakan untuk operasi numerik dan array.
- `matplotlib.pyplot` digunakan untuk visualisasi grafik.
- `scipy.optimize.linprog` adalah fungsi dari pustaka SciPy untuk menyelesaikan masalah program linear.

**2. Visualisasi kendala dan daerah solusi**

```
x = np.linspace(0, 60, 400) # Membuat 400 titik antara 0 hingga 60
y = np.linspace(0, 120, 400) # Membuat 400 titik antara 0 hingga 120

# Gambarkan kendala pertama
plt.plot(x, 1.5*x, label='2x + 3y = 120')

# Gambarkan kendala kedua
plt.plot(x, 1.5*x, label='2x + 3y = 120')

# Batas dan label
plt.xlim(0, 60)
plt.ylim(0, 40)
plt.xlabel('x (kg)')
plt.ylabel('y (kg)')
plt.grid(True)
plt.legend()
```

**3. Nilai domain yang memenuhi semua kendala**

```
obj = lambda x, y: 20*x + 40*y
res = linprog(-obj, A_ub=[[-2, -3], [-1, -1.5]], b_ub=[-120, -40],
             bounds=[(0, 60), (0, 40)])
```

**Penjelasan:**

- Kita menggunakan dua garis batas dari kendala.

- `plt.legend()` digunakan untuk menampilkan daerah layak sebagai representasi visual dari solusi yang memungkinkan.

**3. Menyelesaikan Program Linear (Optimasi)**

```
# Fungsi objektif (maksimum untuk maksimum)
c = [-4000, -5000] # Nilai negatif karena linprog meminimalkan maksimum

# Matriks kendala
A = [[2, 3], [1, 1.5]]
b = [120, 40]

# Batasan variabel (x, y >= 0)
bounds = [(0, 60), (0, 40)]

# Optimasi menggunakan metode 'linprog'
result = linprog(c, A_ub=A, b_ub=b, bounds=bounds, method='linprog')
```

**4. Tampilkan hasil**

```
print("Nilai maksimum: %d" % -result.fun)
print("Nilai x optimal: %d" % result.x[0])
print("Nilai y optimal: %d" % result.x[1])
```

**Penjelasan:**

- `linprog` secara default melakukan minimisasi, jadi kita ubah tanda koefisien fungsi objektif menjadi negatif.
- Matriks `A` dan `b` masing-masing mewakili koefisien dan batasan dari kendala.
- `bounds` mendefinisikan bahwa solusi tidak boleh negatif.

**4.5 Interpretasi Solusi**

Solusi optimal memberikan nilai terbaik dari fungsi objektif dalam daerah yang memenuhi semua kendala. Sebagai contoh, jika hasil eksekusi kode adalah:

```
• x = 20
• y = 40
• Z = 400.000
```

Artinya: UNM sebaiknya memproduksi 20 unit Fish Jaha dan 40 unit Sari Kumri untuk memperoleh nilai keuntungan sebesar Rp400.000, tanpa melanggar batasan waktu dan bahan baku yang tersedia.

**4.6 Aktivitas Mahasiswa**

1. Uraikan total bahan baku menjadi 140 unit. Apa dampaknya terhadap solusi?
2. Jika keuntungan Sari Kumri naik menjadi Rp7.000, bagaimana perubahan model dan solusinya?
3. Buat grafik baru dan tunjukkan titik optimum berdasarkan hasil laju.

Figure 1. (a) Learning module introduction, (b) Python/Google colabs scripts in the module

contextual learning: Introducing a contextual problem close to students' lives, Problem Detailing: Identifying decision variables and constraints, Python-Based Modeling: Guiding students through modeling, coding, and visualization, Solution Construction: Encouraging interpretation and reasoning based on the output and Reflection and Extension: Encouraging students to evaluate their strategies, explore alternatives, and relate them to other problems. Each module contains: Realistic problem scenarios; Structured, directed coding that allows students to simulate and analyze the model;

Dynamic visualization of constraint regions and optimal points; Numerical analysis to evaluate the best solution; and Reflection prompts to support critical thinking and conceptual understanding.

Given that many students are beginners in Python, the module is structured in stages to ensure gradual mastery: It begins with well-annotated, well-explained Python code examples and progresses to semi-guided exercises in which students modify the provided code. Then it leads to independent assignments in which students build their own models and visualizations (Figure 2).

BAB VI  
TUGAS AKHIR KONTEKSTUAL

6.1 Tugas Terbuka Berbasis Konteks Nyata  
Judul: Optimalisasi Penyaluran Paket Bantuan Mahasiswa Rantau

**Latar Belakang:** Sebagai bentuk kepedulian terhadap mahasiswa perantauan, sebuah koperasi kampus berinisiatif menyalurkan dua jenis paket bantuan logistik: *Paket Sehat* dan *Paket Hemat*. Kedua paket memiliki kandungan gizi, biaya, serta kebutuhan distribusi yang berbeda. Sumber daya koperasi terbatas baik dari segi anggaran maupun tenaga distribusi. Permasalahan yang dihadapi adalah bagaimana merancang strategi penyaluran paket yang efisien, adil, dan tetap memperhatikan keterjangkauan serta kecukupan gizi mahasiswa penerima.

**Tugas Anda:**

1. Kembangkan asumsi sendiri berdasarkan kreativitas dan rasionalitas Anda. Beberapa contoh yang perlu Anda tentukan:
  - o Komposisi isi masing-masing paket (jumlah dan jenis item)
  - o Biaya tiap paket dan tenaga distribusi yang dibutuhkan
  - o Sumber daya yang tersedia: dana maksimum, jumlah relawan distribusi, dll
  - o Tujuan optimasi: misalnya memaksimalkan jumlah penerima, meminimalkan biaya, atau mengkombinasikan aspek efisiensi dan pemerataan
2. Susun model program linier berdasarkan asumsi tersebut, minimal memuat:
  - o Definisi variabel
  - o Fungsi objektif
  - o Kendala sumber daya
  - o Syarat non-negatif
3. Implementasikan penyelesaian model tersebut menggunakan Google Colab. Anda perlu:
  - o Menuliskan narasi dan latar belakang masalah dalam Markdown
  - o Menuliskan kode Python dengan struktur yang rapi dan terdokumentasi
  - o Menampilkan grafik visualisasi kendala dan daerah layak
  - o Menyelesaikan model dengan `scipy.optimize.linprog`
  - o Menafsirkan solusi: apa maknanya, bagaimana pengaruh perubahan asumsi terhadap hasil
4. Buat refleksi terhadap:

Figure 2. Sample assignments in the module

### Utilizing Media and Materials (U)

This stage is a limited trial of the media and materials. This stage involves evaluating and identifying potential areas for improvement. The trial was conducted with 10 educators with no prior programming knowledge. Therefore, the limited trial focused on the material's clarity, the

module's ease of use, and the learning flow. Participants engaged in learning by following the steps outlined in the interactive module.

The results of the limited trial were that the module was quite clear and usable, although some students experienced difficulty understanding Python syntax. Therefore, the module needed to

be reorganized to be more gradual and step-by-step. Furthermore, command explanations were simplified yet operational. Material explanations were also simplified, and visual elements were strengthened in the module. Revisions were made before the implementation stage with students directly.

This limited trial consisted of several steps like orientation (A brief presentation on how to use the module), guided individual exploration (participants worked independently using coding, following a series of activities in the module), group discussion (Participants observed real-life situations and understood mathematical models in small groups) and reflection (participants answered reflective questions and provided feedback on how clear, accessible, and useful the program was). This trial was conducted to determine whether the module could be used effectively and to assess its implementation.

### **Requiring Student Participation (R)**

This stage emphasizes active student involvement. Researchers began implementing and developing learning strategies for students by distributing the modules they had developed. The main student activities in this implementation are shown in three activities. In the first activity, Python Investigation for Interactive Visualization, students used VS Code to execute mathematical solutions to linear inequalities. By changing settings, they saw the solution's movement and ideal location in real time, encouraging intuitive, concept-based learning. The next activity is identifying multiple representations. Students working in small groups analyzed results across multiple representations, exploring the relationship between graphs and systems of inequalities, the impact of corner points on objective values, and the contextual significance of variables and solutions. Furthermore, the next activity is about how exploration contextualizes numeracy problem-solving. Students solved linear programming

problems to address real-world challenges, such as production planning, by mathematically modeling scenarios, using coding for exploration, and assessing results in context.

The final stage of module development focused on evaluating its impact and refining the content using actual classroom data. The evaluation addressed three main areas: student learning outcomes, the quality of the learning process, and the robustness of the learning design.

### **Student Learning Outcomes (Numeracy)**

Students' numeracy skills were measured by comparing pre- and post-test scores. The test tasks were based on real-world situations and designed to encompass four types of representation: symbolic, visual, numerical, and contextual.

The analysis showed that the majority of students demonstrated significant improvement in their numeracy performance. Students became more proficient in interpreting graphs, constructing mathematical models of contextual problems, and selecting efficient and relevant solution strategies. Descriptive statistics of students' pre- and post-test scores are presented in Table 3. The analysis of numeracy skills, based on the indicators in this study, focused on the experimental group. This focus was carried out to obtain a more in-depth picture of how the Python-based learning design affects each aspect of numeracy, specifically symbolic, visual, numerical, and contextual representation. Meanwhile, the control group was analyzed at the aggregate score level to ensure the overall comparability of learning outcomes between the two groups. This approach was chosen to ensure the analysis remains aligned with the main objective of the study, namely, the development of a learning design developed to strengthen student numeracy.

Table 3 shows that the average pre-test and post-test scores showed improvement. Before the intervention, students in the experimental class

had an average pre-test score of 60.12 (SD = 8.59). After going through the designed learning process, their average post-test score increased to 83.72 (SD = 5.60). In comparison, students in the control class had an average pre-test score of 58.60 (SD = 6.80), which increased to 73.36

**Table 3.** Descriptive statistics of pre- and post-test numeracy scores

Group	N	Pretest Mean	Pretest SD	Posttest Mean	Posttest SD
Experimental	31	60.12	8.59	83.72	5.60
Symbolic		61.4	8.2	84.9	5.4
Visual		59.6	8.8	86.1	5.1
Numerical / Computational		58.9	9.1	82.7	5.9
Contextual		60.6	8.5	81.2	6.0
Total Exp.		60.12	8.59	83.72	5.60
Control	31	58.60	6.80	73.36	6.79

(SD = 6.79) on the post-test. Both groups made progress, but the experimental group showed more substantial improvement. To provide a more detailed understanding of the intervention’s impact, students’ numeracy performance was analyzed separately across four indicators: symbolic, visual, numerical/computational, and contextual numeracy. The results indicate that improvements occurred across all indicators; however, the most substantial gains were

observed in visual and symbolic numeracy. These indicators showed higher posttest means and normalized gain values, suggesting that the use of Python-based visualizations and symbolic modeling played a key role in supporting students’ learning. Although numerical and contextual numeracy also improved meaningfully, their relatively lower N-gain values indicate that applying mathematical reasoning within real-world contexts may require more sustained practice.

**Table 4.** Descriptive statistics of gain and normalized gain (n-gain) scores

Group	Gain Mean	Gain SD	N-Gain Mean	N-Gain SD
Experimental	23.60	5.82	0.60	0.12
Control	14.76	5.44	0.36	0.14

To further verify the statistical significance of the observed improvement, a paired sample *t*-test was conducted. The result shown in Table 5.

To control for differences in students’ initial abilities, an Analysis of Covariance (ANCOVA) was conducted with posttest scores as the

**Table 5.** Paired sample *t*-test result

Group	t-value	p-value
Experimental	21.56	< 0.001
Control	15.13	< 0.001

**Table 6.** Analysis of covariance (ANCOVA)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	317.251a	3	105.750	72.358	.000	.825
Intercept	16.673	1	16.673	11.408	.001	.199

Group	.527	1	.527	.361	.551	.008
PreTest	81.550	1	81.550	55.799	.000	.548
Group * PreTest	1.211	1	1.211	.828	.368	.018
Error	67.229	46	1.461			
Total	300852.000	50				
Corrected Total	384.480	49				
a. R Squared = .825 (Adjusted R Squared = .814)						

dependent variable, learning groups as the independent variable, and pretest scores as the covariate (Table 6). The results of the analysis showed that the ANCOVA assumptions were met, as indicated by an insignificant homogeneity-of-variance test ( $p = 0.541$ ) and no significant interaction between groups and pretest scores ( $p = 0.368$ ). Pretest scores significantly influenced posttest scores ( $F = 55.799$ ;  $p < 0.001$ ), indicating that students' initial abilities contributed to learning outcomes. However, after controlling for the influence of initial abilities, the results of the corrected mean comparison showed that the experimental group had a significantly higher posttest score than the control group, with a mean difference of 4,403 points ( $p < 0.001$ ). The partial eta squared value of 0.780 indicates that the learning intervention had a significant influence on improving student learning outcomes.

To better understand this progress, the improvement scores were analyzed separately, as shown in Table 4. The experimental group showed an average improvement of 23.60 (SD = 5.82), whereas the control group gained only 14.76 points (SD = 5.44). When calculated using normalized gain (N-Gain), the experimental group achieved an average of 0.60 (SD = 0.12), which is considered moderate to high. The control group's N-Gain was 0.36 (SD = 0.14), which is considered low to moderate. These results indicate that the learning model used in the experimental group had a stronger impact on student improvement. A paired-sample t-test also confirmed the significance of this improvement,

as shown in Table 5. Both groups showed a statistically significant difference between pretest and posttest scores. The statistical analysis revealed a t-value of 21.56 ( $p < 0.001$ ) for the experimental group and 15.13 ( $p < 0.001$ ) for the control group. This means that the learning improvement was statistically significant and not simply a coincidence. These results reinforce the claim that the learning methodology, rooted in diverse digital representations and instruments, successfully facilitated student learning. The greater learning improvement in the experimental group suggests that integrating this model fostered a deeper, more practical understanding of numeracy. This also indicates that students are more confident and skilled in using numerical reasoning in real-life situations. Taken together, these results provide a strong rationale for further iterations and broader implementation of the instructional design.

In addition to presenting data in tabular form, scatter plots were used to provide a clearer picture of the distribution of students' pretest and posttest scores. Figure 3 shows a scatter plot of the pretest and posttest scores for the experimental and control classes. The red line indicates the diagonal line. From the figure, it can be observed that for the experimental class, the data points are located in the upper part of the graph and form a dense, upward-sloping pattern, indicating that nearly all subjects demonstrated consistent improvement. In contrast, for the control class, the data points are positioned below those of the experimental class, though they remain above the diagonal line. The spread also

appears lower than in the experimental class, which may indicate an improvement, albeit not as significant. Quantitatively, it is evident that the experimental class's posttest scores fall within the 85–90 range, whereas the control class's scores are in the 75–80 range; this suggests that the

intervention applied to the experimental class was more effective. This visualization reinforces the results of the previous statistical analysis by presenting the increasing trend in numeracy scores in a more intuitive, understandable way.

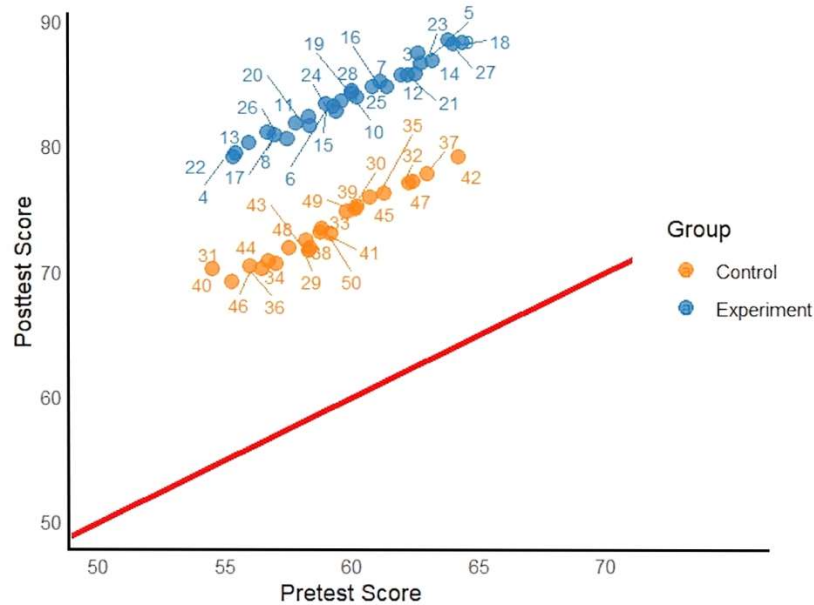


Figure 3. Scatter plot of pretest and posttest scores of students' numeracy ability

Figure 4 presents the distribution of students' numeracy levels in the pretest and posttest phases using a 100% stacked bar chart across four numeracy indicators: symbolic, visual,

numerical/computational, contextual numeracy. This visualization allows a detailed examination of how students' numeracy profiles changed following the implementation of the learning design.

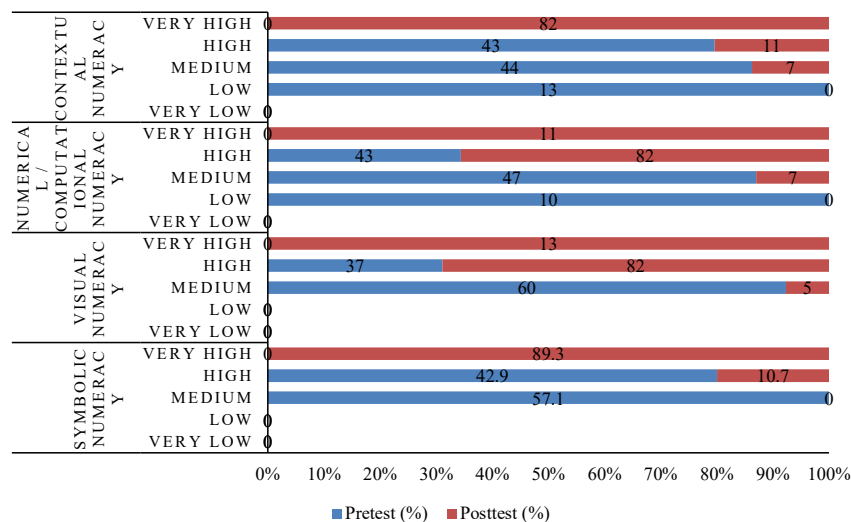


Figure 4. Percentage distribution of students' numeracy levels in pretest and posttest

Prior to the intervention, students' numeracy abilities across all indicators were predominantly concentrated in the medium and high categories. In the symbolic numeracy indicator, for example, 57.1% of students were classified at the medium level and 42.9% at the high level, with no students reaching the very high category. A similar pattern was observed across the visual, numerical/computational, and contextual indicators, with most students demonstrating moderate proficiency. At the same time, the lower categories (very low and low) were minimally represented or absent. These patterns suggest that although students possessed a foundational understanding of numeracy concepts, their skills had not yet reached an advanced level prior to the learning intervention.

After the implementation of the learning model, a substantial shift in the distribution of numeracy levels was evident across all indicators. The posttest results show that the majority of students progressed to the very high category, ranging from 82% to 89.3% across the four indicators. In contrast, the remaining students were largely classified within the high category. Notably, no students remained in the medium, low, or very low categories at the posttest stage. This consistent upward shift across symbolic, visual, numerical, and contextual numeracy indicates a marked improvement in students' overall numeracy competence.

The observed changes demonstrate that the implemented learning intervention was effective in elevating students' numeracy abilities, not only in procedural or symbolic aspects but also in their capacity to interpret visual information, perform numerical reasoning, and apply mathematical concepts within contextual problem situations. The convergence of students' performance toward the higher ability categories across all indicators highlights the positive impact of the instructional design in supporting comprehensive numeracy development.

### **Learning Process Assessment**

During module implementation, classroom observations, student questionnaires, and interviews were used to directly assess students' experiences of the learning process. Observations showed that students were more engaged when learning involved Python-assisted visualization and exploration, although some students initially struggled with basic commands. Interviews and questionnaire responses indicated that the use of visuals and contextual examples helped them understand numeracy concepts more clearly and increased their confidence in solving problems, especially after becoming accustomed to the learning flow.

Step Feedback from Subject Matter Experts aimed to assess the module's depth, its ability to invoke a variety of mathematical representations, and its consistency of use. Validators considered the information conceptually sound and appropriate in its context. The combination of symbolic, visual, and quantitative representations was considered logical and well-organized. However, validators provided feedback to improve the visual content, better demonstrating how different types of representations work and emphasizing mathematical reasoning.

Validators observed how easy it was to use the digital components, how well the code and visuals were organized, and how clear the instructions were. Participants liked the module because it was easy to use and had features that helped them learn. Some ideas for improvement include adding short instructional videos, a search engine menu, and useful external links to support student self-study. c) Feedback from educators working with students. The validators stated that the content was well developed and included exercises that could help students explore and collaborate. They also suggested adding annotations to the code used. The results of the validation process confirmed the module's

suitability for integration into mathematics education. Systematic revisions were conducted in response to evaluator feedback, with specific improvements aimed at providing a clearer framework for beginners and enhancing guidance during the module's initial exploration phase.

The learning module demonstrated strong performance across all the evaluation criteria. The module effectively fostered students' numeracy development, provided a quality digital learning experience, and aligned with academic expectations and pedagogical standards. These results support broader implementation of the module, particularly for mathematics topics that utilize multiple representations and computational thinking.

### **Learning Module Quality Review**

Validation results showed that the developed product received very positive reviews from validators with diverse expertise. Assessments by subject-matter experts for conceptual accuracy, completeness of material, and coherence of representation yielded an average score of 3.80. This achievement indicates that the scientific content presented conforms to conceptual principles and is structured comprehensively and logically interconnected.

Assessments by media experts, covering aspects of visual design, integration between text and code, ease of navigation, and level of interactivity, yielded an average score of 3.75. These results indicate that the learning media is not only visually appropriate but also effectively supports readability and user learning experiences. Meanwhile, assessments by learning practitioners, focusing on the Problem-Based Learning (PBL) structure, student roles in learning, code comprehension, and scaffolding support, yielded an average score of 3.70. This score indicates that the product is relevant to classroom learning practices and supports active student engagement in the learning process.

Overall, the average validation score from all validators was 3.75. Referring to the feasibility categories proposed by Adiguna et al. (2025), the value falls within 3.26–4.00, indicating that the developed product is highly feasible. Therefore, this product is considered to have met the feasibility standards for content, media, and learning implementation and can be used in learning activities with minor revisions based on validator input.

Validators from three distinct but related fields, such as mathematics education, instructional technology, and classroom pedagogy, conducted a comprehensive review of the module. Each group had a distinct perspective and used its own criteria to assess the module within its area of expertise.

### **Adaptive Learning Syntax for Representation-Based and Technology-Based Mathematics Learning**

Based on the analysis of the development and classroom implementation process, an iterative learning sequence emerged. This sequence reflects a learning path that effectively supports conceptual understanding, facilitates multiple representations, and students' numeracy and problem-solving skills in technology-based mathematics learning. The resulting learning design consists of five phases and is presented Figure 5.

The instructional design model shown in Figure 3 was developed from the results of the analysis, development, and implementation phases of the ASSURE framework. The requirements and challenges encountered throughout the procedure are reflected in each stage of the model. The need to reinforce the context of the issue to increase student participation was highlighted by initial observations that led to the formulation of the Context Activation phase. In response to the discovery that students need a technology-based

environment for exploration and collaboration, the Collaborative Digital Exploration Phase was created. Conversely, the Strategic Analysis and Argumentation phase is grounded in expert

validation and observational learning, highlighting the importance of students' ability to analyze outcomes, develop arguments, and consider the solutions they propose.

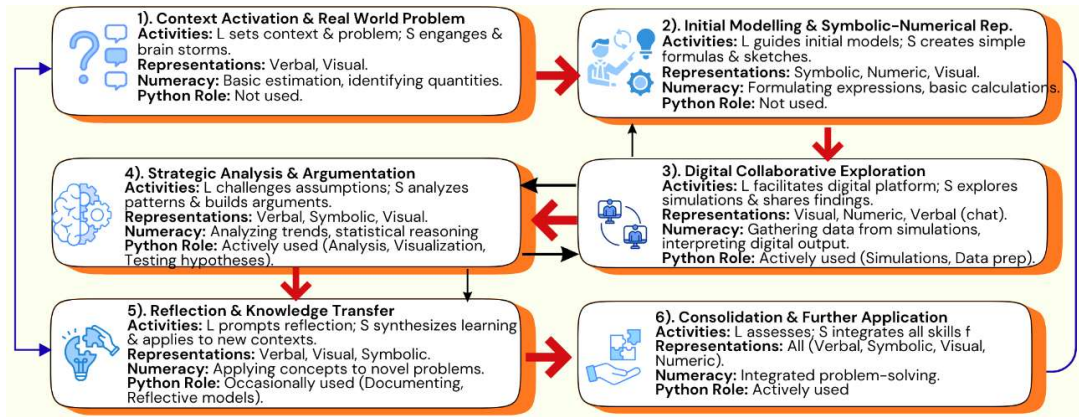


Figure 5. Learning design model

Figures 6 and 7 show the student results of the task. Figure 6 shows an example of Student A's work solving a linear programming problem

using Python, using a visual-symbolic approach. In the initial stage, the student formulated the problem as an objective function with constraints,

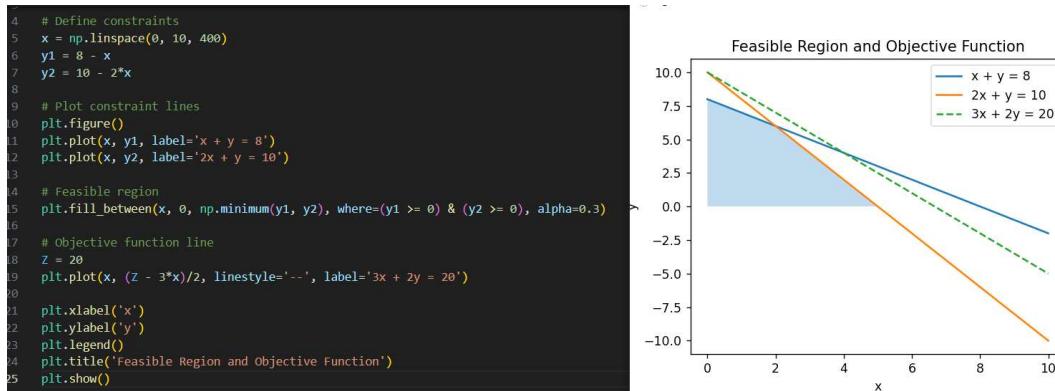


Figure 6. Students' A result

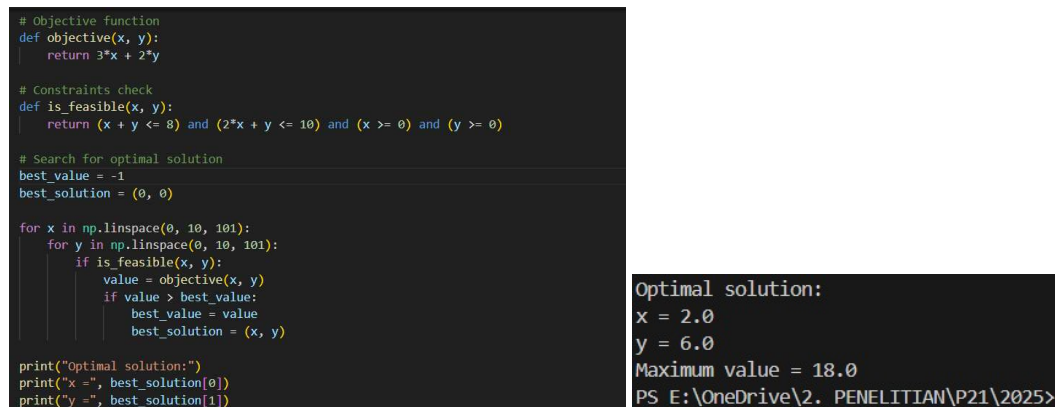


Figure 7. Student's B result

expressed in linear equations and inequalities. This process reflects the student's ability to construct a formal mathematical model from a given contextual situation.

Next, the student used Python to visualize the constraints and the feasible region graphically. The graph served as the primary tool for understanding the relationships between the constraints and identifying the corner points of the solution region. Through visual inspection of the graph, the student determined the optimal solution by interpreting the objective function's position relative to the feasible region. This approach demonstrates that the student emphasized conceptual understanding through visual representation over direct numerical calculations.

Although numerical output remained available, interpreting the graph played a primary role in decision-making. This demonstrates that Student A used the visual representation to build intuition and strengthen his understanding of optimization concepts. Thus, Script 1 predominantly facilitated the development of symbolic and visual numeracy, with contextual numeracy emerging through the interpretation of solution results in relation to real-world problems.

In contrast to Student A, Figure 7 displays the work of Student B, who used Script 2, which emphasized numerical and computational approaches. In this approach, the student defined the objective function and constraints in symbolic form, then used Python to systematically evaluate possible values of the variables that satisfy the constraints.

Student B determined the optimal solution based on the numerical calculations generated by the program. The maximum value of the objective function and the corresponding pairs of variable values were used as the primary basis for decision-making. The student's focus was on calculation accuracy and the quantitative comparison of alternatives, rather than on visual interpretation of the graph.

Although visual representations were not explicitly used in Script 2, the student still demonstrated symbolic numeracy skills through mathematical modeling and contextual numeracy by interpreting calculation results within the given problem. This approach demonstrates that Student B relied on computational processes to systematically and rationally validate solutions.

A comparison between Figures 6 and 7 shows that the developed learning design enables students to employ different problem-solving strategies through diverse mathematical representations. Student A emphasizes the use of visual and symbolic representations to build conceptual understanding and geometric intuition. In contrast, Student B emphasizes numerical and computational representations to determine optimal solutions quantitatively.

Although the focus of the representations differs, both students demonstrate a connection among them. Symbolic representations serve as the basis for visualization and numerical calculations, while visual and numerical results are used to interpret the solution within the context of the problem. These findings indicate that students do not use representations in isolation but rather complement one another according to their individual thinking strategies.

Analysis of the two scripts shows differences in emphasis on numeracy indicators that develop during the learning process. Script 1 strongly emphasizes visual and symbolic numeracy, while Script 2 predominantly emphasizes numerical and computational numeracy. However, both scripts still involve symbolic and contextual numeration as supporting elements in problem solving.

Table 7 confirms that different approaches to using Python contribute to the development of different aspects of numeracy. Script 1 is more effective in strengthening conceptual understanding through visualization, while Script 2 is more effective in training quantitative reasoning and calculation-based decision-making.

The combination of these two approaches demonstrates that the developed learning design can facilitate the comprehensive development of students' numeracy skills through multiple representations.

**Table 7.** Emphasis on numeracy indicators in script 1 and script 2

<b>Indikator Numerasi</b>	<b>Script 1</b>	<b>Script 2</b>
Symbolic	V	V
Visual	V	X
Numerical / Computational	X	V
Contextual	V	V

The results of the study indicate that improvements in students' numeracy skills were not uniform across all indicators but depended on the type of representation predominantly used in learning activities. This finding reinforces the idea that integrating Python into mathematics learning serves not only as a technical aid but also as a means of facilitating the transition and interconnection between mathematical representations. Students who interacted more with graphical visualizations demonstrated stronger visual and symbolic numeracy, while students who emphasized computational processes demonstrated stronger numerical and contextual numeracy.

This finding aligns with the view that deep mathematical understanding develops when students can flexibly coordinate multiple representations, rather than simply mastering a single representational form in isolation. In this context, the developed learning design provides students with the freedom to choose and combine representational strategies according to their way of thinking. This is an important contribution of this study, as it demonstrates that technology-based learning can support numeracy development through diverse, yet focused and meaningful pathways.

The results of this study align with previous research emphasizing the importance of well-planned and technology-supported instructional design, particularly in fostering deep mathematical understanding through multiple representations (Mierlu<sup>o</sup>-Mazilu & Yilmaz, 2024; Serpe & Frassia, 2020). Learning that encourages multiple representations is most effective because they are complementary, integrated, and follow a clear sequence. To support students in building a sound conceptual framework, learning must focus on developing deep conceptual understanding (Pink et al., 2016). On the other hand, research such as that conducted by Wu et al. (2025), which states that digital technology has significantly transformed educational patterns, offers numerous benefits that enhance the learning experience for students and educators. Therefore, this research contributes to understanding how to integrate various representations into learning environments in a collaborative manner.

Although the results of this study indicate an increase in students' numeracy skills in the experimental group, these findings are not entirely consistent with several previous studies that reported that the use of digital representations or technology does not always result in significant improvements in conceptual understanding. Gupta & Zheng (2020) showed that mathematical problem-solving performance is strongly influenced by the interaction among prior knowledge, learning strategies, and task difficulty, in which high extraneous cognitive load can decrease learning outcomes. In contrast, germane cognitive load is positively related to learning motivation. Nurjanah & Retnowati (2018) found that poorly structured presentations of mathematical material, such as split attention and minimal signaling, can increase cognitive load and hinder students' conceptual understanding. Conversely, Rexigel et al. (2024) showed that using more than two representations can improve students' cognitive performance and efficiency

when supported by appropriate learning design and adequate scaffolding. The findings in this study indicate that the use of technology in numeracy learning cannot be viewed solely as a determining factor for success; rather, it depends heavily on the learning design, the level of support provided, and students' readiness to manage various representations.

Furthermore, contextual learning modules also have a positive impact, engaging students throughout the learning process and encouraging them to analyze, reflect, and test. Research conducted by Hidayati et al. (2024) found that a Predict-Observe-Explain (POE) approach based on digital modules with computer-assisted feedback was effective in improving students' conceptual understanding of physics, demonstrating significant improvements in pre- and post-test scores. The integration of Python programming in various educational contexts has demonstrated positive results. For example, the use of Python in teaching STEM calculus led to better conceptual understanding and problem-solving skills than traditional methods (Zhang & Somasundram, 2025). Furthermore, the use of Python in teaching concepts through simulations and tutoring systems significantly improved students' understanding of complex theories (Zhao et al., 2025). These research findings reinforce the view that digital media platforms built with Python can serve not only as technical tools but also as spaces for collective reasoning and concept development.

These findings also suggest that the use of digital media in learning can help students explore various representations and improve their numeracy skills. Digital media have demonstrated significant potential to improve numeracy skills in mathematics education through various innovative approaches. The use of multimedia in numeracy learning has been shown to improve students' numeracy skills and elicit positive responses, indicating increased student engagement and interest (Rohendi et al., 2016). Research by

Rahardjo et al. (2025) on culturally nuanced electronic comics, designed to integrate local cultural elements, found that these comics made learning more relevant and engaging for students. These comics helped bridge abstract mathematical concepts with real-life contexts, thereby improving numeracy skills. These findings suggest that students can engage in learning in a challenging and personally relevant way, shifting from rote memorization to more meaningful, cognitively rich tasks through various interactive learning media.

Integrating contextual learning with digital media through learning modules can facilitate students' exploration of various representations and numeracy skills. This integration can be achieved through the support of relevant theoretical frameworks, effective design strategies, and evaluation tools for measuring student engagement. This research illustrates that learning designed systematically by paying attention to flow, involving students, and providing opportunities for exploration and the use of digital media can yield good results.

This study concentrates on the creation and preliminary assessment of a Python-based instructional design that incorporates various forms of representation into numeracy instruction. Consequently, this article focuses on the overall efficacy of instructional design in improving student learning outcomes. Although students utilize a variety of representations throughout the learning process, including mathematical symbols, graphs, and code-based representations, this study has not conducted a thorough examination of the quality and patterns of their use. In a future publication, a more thorough examination of students' ability to employ and switch between representations is planned.

## ■ CONCLUSION

Based on the research description, the results showed that the learning design was developed from needs analysis to reflection. The

learning stages include building context, presenting contextual problems, and then transforming those problems into various forms of representation. The third stage is using digital technology for exploration, with activities carried out collaboratively. Next, strategic analysis and communication are carried out to foster critical thinking. The final stage is reflection and knowledge transfer, where open-ended problems are presented to reconnect what has been learned to other situations. The developed learning design is categorized as highly feasible across content, media, pedagogical aspects, and learning structure. The design has also been proven to facilitate various forms of student representation and improve their numeracy skills. These representations include symbolic, verbal, contextual, numeric, and visual representations. Furthermore, the average test score of students in the experimental group increased from 60.12 in the pretest to 83.72 in the posttest. This improvement is also reflected in the normalized gain (N-Gain), which increased from 0.36 to 0.60, indicating greater learning progress.

The results of the paired-sample t-test showed a t-value of 21.56 and  $p < 0.001$ , further demonstrating that the improvement was not due to chance but rather resulted from several stages of the learning process. This strengthens the evidence that systematically designed technology-integrated learning activities positively impact students' numeracy skills. Indicator-based analysis shows that learning that integrates symbolic, visual, numerical, and contextual representations enables students to develop numeracy through distinct but complementary pathways. Qualitative findings from student work and learning observations provide empirical evidence that students actively use and connect various representations, both through graphical visualizations and computational processes.

Overall, this study shows that technology-supported learning can enhance students'

numeracy skills, but the findings remain limited by the use of intact classes and a relatively small sample size. Future research should therefore apply a more rigorous quasi-experimental or experimental design, involving larger and more diverse samples, and be complemented with qualitative methods to better understand how students engage with Python-based representations during learning.

This study has limitations in reporting qualitative data, which is not presented in depth. Although observation and interview data were collected, thematic analysis and presentation of participant quotes were not discussed in detail in this article. Therefore, further research or follow-up publications are expected to explore these qualitative aspects to provide a more comprehensive understanding of the learning process.

#### ■ **DECLARATION OF GENERATIVE AI USAGE IN THE WRITING PROCESS**

In writing this manuscript, the researcher used a limited generative AI tool for language editing, paraphrasing, and readability checks. All research data, analysis, interpretation of results, and conclusion drawing were handled entirely by the researcher. The researcher has reviewed and re-edited all output from the tool and takes full responsibility for the authenticity, accuracy, and integrity of the published article.

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