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The Efficacy of The Open-Ended Approach in Fostering Students' Mathematical Conceptual Understanding in Terms of Initial Ability

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Abstract: The low mathematical ability of vocational high school (SMK) students is a strategic problem because mathematics is the foundation of technical competence in the era of the Industrial Revolution 4.0. Various pedagogical innovations have been proposed to overcome this problem, but previous studies have often encountered obstacles in effectively improving the conceptual understanding of students with low Initial Mathematical Ability (KAM). This quasi-experimental study was designed to test the effectiveness of the open-ended approach as an alternative learning method and to investigate the role of KAM as a moderating variable, particularly regarding aspects that have not received attention in vocational education research. The research sample consisted of 72 students in grade XI majoring in Computer and Network Engineering (TKJ), divided into a non-equivalent control group design, with the experimental group receiving open-ended approach learning (N=36) and the control group receiving conventional learning (N=36). Data collection was conducted using a mathematics concept comprehension test instrument and an observation sheet for the implementation of learning. Statistical analysis was conducted using the Kruskal-Wallis H test on the normalized N-Gain scores. The results show a significant difference in conceptual understanding between the two groups, with the experimental group's N-Gain score (0.72; high category) statistically higher than the control group's (0.62; medium category). These findings contribute to the literature by demonstrating that the open approach not only improves average academic achievement but also reduces the learning gap through differential mechanisms responsive to heterogeneity in initial abilities. Thus, this study recommends the Open Approach as a pedagogical framework with the potential to transform vocational mathematics learning practices toward a more equitable and adaptive system.

Keywords: initial mathematical ability, open-ended approach, conceptual understanding ability.

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■ INTRODUCTION

Vocational education (SMK) plays a strategic role in preparing a skilled and competitive workforce to face the era of the 4.0 industrial revolution, which is characterized by the digitization and automation of work systems (Rais, 2018). The need for graduates who are not only technically proficient but also adaptable to technological changes makes the quality of learning in SMK the main foundation of national competitiveness. In this context, mastery of fundamental mathematics often determines the success of vocational competencies themselves. Recent research confirms that mathematical ability is not merely a tool for calculation but a critical foundation for understanding the concepts of engineering, technology, and logic that form the backbone of vocational competence (Lee et al., 2023; Rifai et al., 2025; Acheampong & Awuah, 2025). For example, to understand electrical circuits, read technical drawings, or analyze production data, a vocational school student needs a deep conceptual understanding of functions, geometry, or statistics.

However, the gap between industry needs and graduate competencies remains a serious challenge. Data show that vocational school graduates consistently have the highest open unemployment rate (TPT), at around 9–11%, surpassing those with other levels of education. One of the main causes is the weak mastery of applied mathematics, which is essential in industry (Soelistiyono & Feijuan, 2021). The weakness of vocational school students' ability to solve non-routine problems is evident from their unfamiliarity with contextual questions. This leads to difficulties in designing mathematical models and connecting abstract concepts with real-world situations (Mailisman et al., 2020). This is in line with the findings (Adelabu, 2022) that one of the main obstacles in vocational education is the challenge of teaching relevant mathematics topics and connecting mathematical ideas to practical situations.

This low achievement is often rooted in students' weak initial mathematical abilities (Heriyana et al., 2025), which include mastery of prerequisite material such as sequences and series, indicating that their basic understanding is not yet solid (Nurhasanah & Anita, 2025). A study conducted by Vale & Barbosa (2023) found that technical-vocational students often experience a disconnect between the embodied and symbolic aspects of mathematical concepts, such as measurement. This means that even though they can memorize formulas, their connection to real-world applications, such as calculating volume or flow rate in a technical context, is not well internalized. This condition becomes a serious obstacle for vocational high school students when they encounter complex, applied vocational concepts. Procedural and mechanistic understanding without a solid foundation can lead to misconceptions and an inability to apply this knowledge to diverse real-world problems (Al-Mutawah et al., 2019; Braithwaite & Sprague, 2021).

The significance of prior mathematical knowledge in constructing new understanding cannot be overstated. Research on how humans learn confirms three main principles, one of which is the importance of engaging students' prior understanding (Dawson & Venville, 2020). Students come to class with a set of beliefs and knowledge, both formal and informal, about how mathematics works. If these prior understandings are not identified and actively engaged, students risk failing to internalize the new concepts being taught, or simply memorizing them for the test and then reverting to their prior misconceptions outside of class (Dawson & Venville, 2020). In the context of advanced mathematics education, these prior abilities provide a deep factual foundation that enables students not only to understand facts and procedures separately but also to organize them into a coherent conceptual framework, thereby facilitating the retrieval and application of knowledge in new situations (Dawson & Venville, 2020).

More structurally, students' prior knowledge can be classified into interconnected elements, including abstraction, cognition, context, metacognition, and perception (Chumdari et al., 2018). These elements form students' long-term memory, which serves as the foundation for every new mathematical task. As a concrete example, middle school students typically have mastered the four basic operations (addition, subtraction, multiplication, division) and general mathematical notation (Chumdari et al., 2018). Even understanding opposing binary concepts, such as "big-small" or "up-down," is valuable foundational knowledge for grasping more complex concepts, such as the conservation of size or gradients in Cartesian coordinates. In vocational schools, this initial ability serves as an essential foundation for solving technical problems. Students who can apply their knowledge of basic operations will realize that even complex mathematical procedures are essentially a series of simple steps they are already familiar with (Chumdari et al., 2018). Thus, diagnosing and reinforcing initial abilities are non-negotiable pedagogical steps before exploring deeper concepts, as the open-ended approach offers.

The positive correlation between the level of initial mathematical ability and the achievement of conceptual understanding has been empirically proven many times. A study at the college level confirms that when students' initial mathematical abilities are strong, their understanding of mathematical concepts is likely to be strong as well (Asri et al., 2020). The descriptive-qualitative research shows that students with high initial abilities can restate concepts, choose appropriate procedures, and apply concepts in problem solving. Conversely, students with low initial abilities have difficulty communicating ideas, selecting appropriate procedures, and applying concepts (Asri et al., 2020). Similar findings are also observed at the secondary education level. Analysis of high school students' conceptual understanding confirms that students with high initial abilities dominate in mastery of conceptual understanding indicators compared to students with medium or low initial abilities (Fererde et al., 2024). Furthermore, research at the elementary school level shows the same ability gradient: students with high initial abilities can demonstrate almost all indicators of conceptual understanding, whereas students with low initial abilities can master only one indicator, such as verbal explanation (Murtianto et al., 2019). The consistency of these findings across various educational levels indicates a strong structural relationship between initial ability and an individual's capacity to construct new understanding.

The learning conditions in vocational schools are further exacerbated by mathematics teaching methods that are still often dominated by conventional approaches (Vimbelo & Bayaga, 2023), oriented toward solving single problems with definite answers (Nguyen et al., 2020), thereby failing to train students' reasoning, creativity, and flexibility in thinking (Rahmatin et al., 2025). Pedagogical practices like this are not in line with the demands of the workforce, which requires innovative problem-solving. Research conducted by Manginsay et al. (2025) shows the need for teaching strategies specifically designed to strengthen the mathematical foundations of vocational students, often through intensive remediation programs outside regular study hours to address their basic weaknesses. This underscores that learning interventions cannot be uniform; they must take into account students' cognitive readiness.

To address this issue, a learning approach is needed that not only transfers knowledge but also builds conceptual understanding and higher-order thinking skills. One of the proposed alternatives is the open-ended approach. This holistic approach emphasizes the explicit integration of various mathematical representations, in which students' understanding is formed by connecting new information to subjective experiences and by moving between representations in the context of situational problems (Podkhodova et al., 2025). In the context of vocational schools, this approach is highly coherent, as it reflects the nature of work in the industrial world, which often requires problem-solving using a range of strategies and innovations (Faroh et al., 2022).

The open-ended approach has proven to be not only theoretically relevant but also empirically effective in the context of vocational education. Research on the development of open-ended mathematics modules for vocational high school students in the fields of arts, crafts, and tourism shows that the modules meet the criteria of validity, practicality, and effectiveness in improving student learning outcomes (Arrasyid et al., 2021). This effectiveness is measured by the achievement of the Minimum Completeness Criteria (KKM) and significant improvement in learning outcomes (N-gain), proving that open-ended problems contextualized to students' fields of expertise can encourage deeper understanding (Arrasyid et al., 2021). A similar development study on exponential equations and functions in private vocational high schools also confirms these findings. The open-ended module developed is not only highly valid and practical but also effective, with student conceptual understanding increasing to 79% (Isnani et al., 2020).

Furthermore, two recent experimental studies provide strong quantitative evidence. The research by Rifai et al. (2025) demonstrates that the open-ended approach is significantly more effective than the direct approach in improving the mathematical communication skills of vocational high school students. The greater improvement in this experimental group indicates that the exploration process in open-ended approaches trains students to articulate their mathematical ideas, a crucial skill in the workplace. The same study asserts that students taught with the open-ended approach achieved better mathematical understanding than those who received direct instruction (Rifai et al., 2025). These findings strengthen the argument that the open-ended approach is effective in promoting mathematical conceptual understanding through independent exploration.

However, the implementation of the open-ended approach cannot be separated from the context of student readiness. Boelens et al. (2018), in their study on adult learners with vocational education backgrounds, emphasize that structured support, such as worked examples, is crucial in open-ended tasks. This support not only helps produce high-quality artifacts but also provides a sense of security by giving an overview of the task (Boelens et al., 2018). This implies that in implementing an open-ended approach, needs diagnosis, including the reinforcement of initial abilities, is an absolute prerequisite. The needs analysis reinforces this, showing that from the students' perspective, the open-ended approach is essential. However, its success depends heavily on effective learning management and meticulous lesson planning, which implicitly include diagnostic assessments to determine students' starting points (Gunawan et al., 2023). Research on measurement learning among vocational students also emphasizes the importance of understanding students' initial comprehension as a foundation for designing appropriate interventions; neglecting this can lead to persistent misconceptions (Vale & Barbosa, 2023).

The synergy between strengthening initial mathematical abilities and applying open-ended approaches is believed to create a multiplicative effect that enhances students' conceptual understanding. Adequate initial abilities will serve as cognitive tools that enable students to engage productively with open-ended problems (Jonsson et al., 2020). Conversely, the experience of solving open-ended problems can deepen and broaden their understanding of these fundamental concepts, as they are forced to view them from various perspectives and in different contexts (Nurkaeti et al., 2020). Without adequate prior knowledge, students may feel frustrated and unable to grasp the core of the given open-ended problems (Nurpratiwi & Setianingsih, 2021).

The issue of vocational high school students' readiness to face digital transformation and an increasingly complex work environment has become very relevant (Nurjanah et al., 2022). The *Merdeka Belajar* program from the Ministry of Education, Culture, Research, and Technology emphasizes student-centered learning and fosters

21st-century skills, including creativity and critical thinking (Zainudin et al., 2025). Although the open-ended approach aligns with this vision, its implementation must be adaptable to students' diverse initial abilities. The open-ended approach often becomes ineffective if not preceded by diagnosis and intervention to address the gaps in students' basic abilities (Chernikova et al., 2019). Failure to bridge this gap can widen the disparity in understanding between students with high and low initial abilities.

Therefore, research that specifically examines the influence of early mathematical ability on vocational high school students' conceptual understanding, using an open-ended approach, is highly relevant and urgent. Although the effectiveness of the open-ended approach has been widely studied, a significant research gap remains, particularly in vocational education in Indonesia. Previous studies have tended to focus on the direct effects of the open-ended approach on dependent variables such as mathematical understanding, without investigating in depth how students' initial abilities moderate or mediate its effectiveness. Based on the empirical evidence presented, the focus of research has shifted from simply proving the effectiveness of the open-ended approach to understanding how this approach can work optimally and for which groups of students it is most effective. This shift opens up space to explore initial mathematical ability as a moderator variable in learning with an open-ended approach.

The novelty of this research lies in its effort to empirically test the moderating or mediating effect of initial mathematical ability on the relationship between the use of the open-ended approach and improvements in conceptual understanding among vocational high school students. This research is expected to provide empirical data on the extent to which initial ability influences the effectiveness of the open-ended approach. In the context of the proposed research, previous findings provide strong evidence that the effectiveness of the open-ended approach in vocational high schools will also depend heavily on students' initial mathematical abilities. Groups of students with high initial abilities are expected to use open-ended problems to deepen their understanding. In contrast, groups with low initial abilities may require more intensive support to keep up.

The findings of this study are expected to serve as a guide for vocational school teachers and curriculum developers in designing effective mathematics learning. The learning referred to can reach all levels of student ability, both by strengthening their basic understanding and by being solution-oriented. More specifically, this study aims to answer the crucial question: "Is the improvement in mathematical comprehension of students who receive open-ended learning better than that of students who receive conventional learning in terms of high, medium, and low initial mathematical ability (KAM)?" Based on this research question, the following research hypothesis is proposed: "The improvement in mathematical comprehension skills of students who receive open-ended learning is better than that of students who receive conventional learning based on high, medium, and low initial mathematical ability (KAM) categories."

METHOD

Participants

The population in this study was all 11th grade students at a vocational school in Pandeglang Regency, Banten Province, consisting of five classes with a total of 148 students, including 11th grade Light Vehicle Engineering (21 students), 11th grade Office Automation and Management (25 students), 11th grade Computer and Network Engineering 1 (36 students), XI Computer and Network Engineering 2 (36 students), and XI Motorcycle Engineering and Business (30 students). Sampling was carried out based on the existing class structure, learning schedule, and external validity, resulting in two classes: XI TKJ 1 as the experimental group and XI TKJ 2 as the control group, with a total sample of 72 students.

The students' initial mathematical ability was determined based on their Grade 10 Final Assessment scores for the 2024/2025 academic year. The descriptive analysis results showed that the average KAM for the experimental group was 35.64, and for the control group, 36.47, out of an ideal score of 100. The two groups had relatively close average scores, but statistical testing was still carried out to ensure their equivalence. The Shapiro-Wilk normality test at a 5% significance level showed that the KAM data for the experimental group had a p-value of 0.097, and the control group had a p-value of 0.229. Because both values exceeded 0.05, the null hypothesis was accepted, and the KAM data for both groups were declared normally distributed.

Next, Levene's test of homogeneity of variance yielded a significance value of 0.949. This value exceeds 0.05, so the null hypothesis is accepted, indicating that the variances of the KAM data across the two groups are homogeneous. With the assumptions of normality and homogeneity fulfilled, a test of mean difference was performed using the Independent Sample t-test. The test results showed a significance value (2-tailed) of 0.823, which is greater than the significance level of 0.05. Thus, the null hypothesis was accepted, indicating that there was no significant difference in the mean mathematical ability between the experimental and control groups. Based on the entire series of tests, it can be concluded that both groups had equivalent initial abilities before being given treatment.

Research Design

This study is a quasi-experimental study with a nonequivalent control group design (Creswell, 2012). The research subjects were not selected randomly but were drawn from existing classes to avoid disrupting the learning schedule and structure (Ary, Jacobs, & Sorensen, 2010). The design used is a pretest-posttest control group design (Fraenkel et al., 2019; Ruseffendi, 2005), in which both groups received pretests and posttests to measure changes in ability before and after treatment.

This study involved three types of variables. The independent variable was the learning approach, namely the open-ended approach in the experimental group and the direct approach in the control group. The dependent variables included mathematical comprehension skills. The control variable is students' initial mathematical ability, which is classified into high, medium, and low categories based on the average score from the previous semester (Ary, Jacobs, & Sorensen, 2010). Controlling this variable aims to ensure that the research results truly reflect the influence of the learning approach, rather than differences in students' initial abilities.

The research subjects comprised two classes, each designated as the experimental or control group. Both groups underwent a pretest to measure their initial abilities. The experimental group then received instruction using an open-ended approach that encouraged the exploration of creative problem-solving strategies. At the same time, the control group received instruction using a structured, teacher-centered, direct approach. After the treatment was completed, both groups underwent a posttest to measure the improvement in their abilities. Data analysis was conducted both overall and by category of students' initial mathematical abilities to determine the effectiveness of the open-ended approach at each ability level (Creswell, 2012). The research design can be described as follows:

O (pretest)	X1 (open-ended)	O (posttest)
O (pretest)	X2 (conventional)	O (posttest)

Procedures

This research was conducted in three interrelated stages: preparation, implementation, and completion. In the preparation stage, the researchers systematically compiled an instrument grid based on the indicators of the variables under study. The grid contained detailed descriptions of the material covering all learning content, the aspects of ability to be measured, and the achievement indicators used as benchmarks for success. In addition, the grid also determined the number of questions for each variable with a balanced proportion to ensure that all dimensions of the research were measured representatively. The preparation of this well-developed grid is an important foundation for creating a research instrument that accurately and comprehensively captures data in accordance with the research objectives.

The implementation stage begins with the selection of an experimental group and a control group based on the students' initial abilities to minimize potential bias. After the two groups are formed, a pretest is administered to measure students' conceptual understanding before treatment. The pretest data is analyzed first to ensure that there are no significant differences in ability between the two groups. Next, the experimental group receives open-ended learning, while the control group receives conventional learning. To maintain consistency in the treatment's implementation, the researcher serves as a teacher in both groups.

After all learning interventions are complete, the completion stage is carried out by analyzing all collected data. Quantitative data were analyzed using statistical tests, including tests of normality, homogeneity, and comparison. Based on the results of this analysis, the researchers drew conclusions systematically in accordance with the study's objectives and hypotheses. The findings of this study were then used as a basis for recommendations to educational practitioners, policymakers, and future researchers.

Instruments

This study focuses on sequences and series as the main topic. This material was chosen because it has not been taught to 11th-grade students at a vocational high school in Pandeglang Regency, and because of its relevance to everyday life and its moderate difficulty for students. The research instruments used consisted of three questions to measure students' instrumental understanding and relational understanding. Instrumental understanding was measured using indicators of students' ability to identify the first term and the ratio of a geometric sequence. In contrast, relational understanding was measured using indicators of students' ability to use the concepts of geometric sequences and arithmetic series appropriately to solve contextual problems.

The questions used are as follows: 1) Given a geometric sequence with the first term 2 and ratio 3, determine the 7th term of the geometric sequence; 2) An online store sells cotton plain shirts as its flagship product. In the first week, 5 shirts were sold. In the second week, sales increased to 10 shirts. In the third week, 20 shirts were sold, and the same pattern continued. How many plain cotton shirts were sold in the 10th week? 3) A math teacher gave practice questions to students. In the first meeting, students had to do 3 practice questions. In each subsequent meeting, the number of practice questions students had to complete increased by 2 from the previous meeting. How many practice questions must the students complete in the 12th meeting? Each question has a score weight of 4, so the maximum total score for the three questions is 14. In addition to the test instruments, the researchers also used the previous year's final exam scores as data on the students' initial abilities. These scores were used to group students into high, medium, and low categories.

The instrument used was an essay test, so that students' steps and thought processes in solving problems could be described clearly and thoroughly. The essay format allowed researchers to comprehensively evaluate students' thought processes, including the accuracy of their answers, the systematic structure of their responses, and the accuracy of their problem-solving steps. All instruments were tested in advance through validity tests using product-moment correlations and reliability tests using Cronbach's Alpha formula. The validity test was conducted to assess the accuracy of the test instruments in measuring their intended constructs. In contrast, the reliability test assessed the instruments' ability to provide consistent results across measurements.

Data Analysis

This study applied quantitative data analysis sourced from pretest and posttest results. To measure the improvement in students' mathematical comprehension skills, N-Gain analysis was conducted by comparing pretest and posttest scores as proposed by Meltzer (2002). The N-Gain calculation results were then categorized into three categories: high improvement (g e 0.7), moderate improvement (0.3 d g < 0.7), and low improvement (g < 0.3). All statistical analyses in this study were conducted in SPSS version 30, with a significance level of á = 0.05.

Prior to hypothesis testing, inferential statistical analysis was conducted, beginning with prerequisite tests, including tests of normality and homogeneity of variance. The normality test was performed using the Shapiro-Wilk method to ensure that the pretest, posttest, and N-Gain data in both the experimental and control groups were normally distributed. The hypothesis tested is that H€ is accepted if the significance value is greater than 0.05, indicating that the data is normally distributed; H€ is rejected if the significance value is less than 0.05, indicating that the data is not normally distributed (Santoso, 2015). After that, a homogeneity of variance test was conducted using Levene's Test to assess whether the variances between groups were similar. The testing criteria were that H€ was accepted if the significance value was greater than 0.05, indicating homogeneous variance, and H€ was rejected if the significance value was less than 0.05, indicating non-homogeneous (Santoso, 2015).

After all prerequisite tests were met, hypothesis testing was conducted through three stages of interrelated analysis. The first stage was an analysis of the differences in pretest, posttest, and N-Gain means between the experimental and control groups. This analysis used an ANOVA test to examine the significance of the differences in mathematical comprehension ability between the two groups. The hypothesis formulated is that H€ is accepted if the significance value is greater than 0.05, which means that there is no significant difference in the mean between the groups, and H€ is rejected if the significance value is less than 0.05, which means that there is a significant difference in the mean between the groups (Trihendradi, 2009).

Next, the third stage is a further analysis to test differences in improvement across levels of initial mathematical ability (KAM). This analysis uses a two-tailed test with the following testing criteria: H€ is accepted if the significance value is greater than or equal to 0.05, which means that there is no significant difference in improvement between KAM levels, and H€ is rejected if the significance value is less than 0.05, which means that there are at least two KAM levels that have a significant difference in improvement (Trihendradi, 2009). This series of systematic statistical analyses was designed to test the hypothesis that the improvement in students' mathematical comprehension who received the open-ended approach was greater than that of students who received conventional instruction, across high, medium, and low initial mathematical ability categories.

■ RESULT AND DISCUSSION

Result

The N-Gain scores obtained by comparing pretest and posttest results were grouped into three levels of improvement (low, medium, high) and analyzed in SPSS version 30. The analysis produced descriptive data in the form of mean and standard deviation values that show the level of effectiveness of each approach in improving students' mathematical understanding at each KAM level, as explained below.

Table 1. Description of N-Gain data on students' mathematical comprehension ability

Learning	KAM	Mean	Std. Deviation	N
Open-Ended Approach	High	0.9500	0.06245	7
	Moderate	07138	0.11020	21
	Low	0.5250	003780	8
	Total	0.7178	0.16487	36
	High	0.8967	0.12275	6
Conventional Learning	Moderate	0.5854	0.12632	24
	Low	0.4517	0.06014	6
	Total	0.6150	0.17869	36

Based on the table of students' conceptual understanding abilities, it can be seen that the learning approach has a different effect on student improvement at each level of initial mathematical ability. In general, the open-ended approach yields a higher average conceptual understanding than conventional learning across all ability categories, whether high, medium, or low. This shows that the open-ended approach is more effective at supporting the overall improvement of students' conceptual understanding.

A consistent pattern was also observed: the higher the students' initial mathematical ability, the higher their average conceptual understanding, in both the open-ended and conventional approaches. This confirms that students' initial abilities are an important factor in determining learning success, regardless of the method used. However, the open-ended approach appears to be more effective at bridging the achievement gap among students in the moderate- and low-ability groups, as their average conceptual understanding is higher than that of their counterparts in the conventional learning group.

In terms of result consistency, the open-ended approach shows a more stable distribution of scores, especially in the high- and low-ability groups, indicating that it can create a more equitable learning experience. In contrast, conventional learning shows greater variability, particularly in the medium ability group, suggesting that students' learning outcomes in this group are more diverse and less uniform. Meanwhile, the largest distribution of students is in the medium ability group, which is an important concern because this group contributes significantly to the overall average.

Thus, these findings confirm that the open-ended approach not only excels at improving the average level of conceptual understanding but also at creating more consistent and inclusive results across different levels of students' initial abilities. This approach has the potential to be a more equitable and effective learning strategy, especially for reaching students with moderate-to-low abilities.

The ratio of the increase in mathematical comprehension of the experimental group through an open approach to that of the control group through conventional learning was further analyzed. This difference is illustrated in Figure 1, which compares the average N-Gain scores of the 2 groups. The results show that the open-ended approach produced greater improvements, particularly among students with moderate and low KAM scores. In contrast, the control group showed only limited progress across all KAM levels. This visualization further confirms the findings that the open approach is more effective at maximizing students' mathematical understanding than conventional learning.

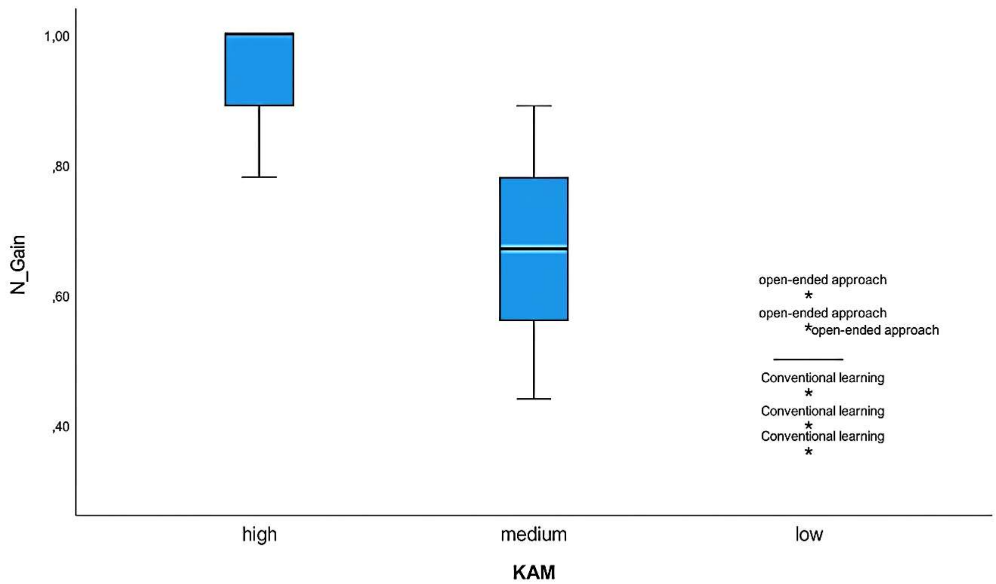

Figure 1. Improvement in mathematical comprehension skills

This line chart shows the distribution of exam scores, grouped by KAM, into three categories: high, medium, and low. In the high category, student scores range from 78 to 99, with a median of 98. This shows that, even though the questions were difficult, most students achieved very high, consistent scores, with half of the student population scoring above 98. The data distribution in this category is very narrow, indicating no significant variation in scores.

Meanwhile, in the medium category, the score range is wider, from 52 to 88. The median is 80, which means that 50% of students scored above 80. The first quartile (Q1) is 72, and the third quartile (Q3) is 85, indicating that the other 50% of students are spread across this range. This distribution appears to be more even than in the high category, with some students achieving high scores and others with fairly low scores.

Finally, in the low category, student scores ranged from 45 to 66. With a median of 58, half of the students scored below 58. The first quartile is at 45, and the third quartile is at 62 confirm that the distribution of scores tends to be low and clustered at the bottom of the scale. Overall, this graph shows a contrasting trend: the greater the difficulty of the questions, the higher and more uniform the students' scores, whereas questions with low difficulty show more varied results, with a tendency toward lower scores.

To gauge the effect of learning on mathematical comprehension of sequences and series, the next step is to assess the significance of the difference in N-Gain averages between the open-ended and conventional techniques. The initial stage is to test the normality of all dependent variables using SPSS version 30. The results of the prerequisite tests listed in Table 2 indicate that the assumption of normality ($p > á = 0.05$) between groups has not been met; however, the analysis of mean differences will continue. This comprehensive testing phase ensures the validity of the discoveries regarding the effectiveness of the two learning approaches.

Table 2. Normality of mathematical comprehension data

Learning	KAM	Shapiro-Wilk		
		Statistic	df	Sig.
Open-Ended	High	0.830	7	0.107
	Medium	0.836	21	0.068
	Low	0.211	8	0.200

Conventional	High	0.280	6	0.153
	Medium	0.166	24	0.084
	Low	0.251	6	0.200

Table 2 presents the results of the Shapiro-Wilk normality test. The significance values for all data groups in open and conventional learning, both in the high, medium, and low ability categories, are above 0.05. This indicates that all data are normally distributed, so the normality assumption is fulfilled.

With the normality assumption met, the analysis can proceed with the ANOVA test. This test aims to examine differences in students' average mathematical understanding across learning approaches (open and conventional) and KAM levels (high, medium, low). The results of the ANOVA are presented in detail in Table 3 to support conclusions about differences in students' mathematical understanding.

Table 3. According to the KAM variable analysis, there is a substantial difference in the

Table 3. Results of analysis of variance of mathematical comprehension ability data

Source	Dependent Variable	Type III Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	N_Gain_KPM	1.522[a]	5	0.304	27.277	0.000
Intercept	N_Gain_KPM	24.614	1	24.614	2205.128	0.000
Learning	N_Gain_KPM	0.094	1	0.094	8.441	0.005
KAM	N_Gain_KPM	1.300	2	0.650	58.248	0.000
Error	N_Gain_KPM	0.737	66	0.011		
Total	N_Gain_KPM	34.232	72			
Corrected Total	N_Gain_KPM	2.259	71			

increase in mathematical comprehension skills among students with medium, high, and low initial skills, with a significance value of 0.000 ($p < 0.05$) for the n-gain score. This condition is apparent: compared with students who received traditional instruction, students who received the open approach treatment showed a significant increase in their arithmetic comprehension skills. In other words, the open approach yields more optimal outcomes than conventional learning across all KAM levels (high, medium, low) in developing mathematical comprehension abilities.

Statistical analysis shows that open approaches are much more effective than conventional teaching in increasing students' mathematical understanding at each early mathematics ability level (KAM). Further examination showed substantial variations in mathematics comprehension gains among students with high, medium, and low KAM. Students taught through open-ended methods performed significantly better than those taught conventionally across all four levels of mathematics. "These findings demonstrate that the open approach effectively enhances students' prior mathematical understanding, facilitating a more tailored learning experience at each level of mathematics."

The study findings demonstrate that the open-ended method substantially enhances students' mathematical understanding skills across high, medium, and low KAM levels compared with traditional instructional approaches. The characteristic of open-ended problems is that they have many methods of solution and more than one correct answer, so students are encouraged to provide a variety of methods and answers (Hafidzah et al., 2021). The investigative process in open-ended learning helps students develop a more flexible mathematical mindset. Thus, this approach is effective for all levels of student KAM.

Improved mathematical understanding across all KAM levels can be attributed to cognitive load-optimized instruction, which minimizes extrinsic load and supports the acquisition of mathematical schemata (Maj, 2022). The open-ended approach provides gradual assistance through open-ended questions that guide students in finding solutions, thereby reducing the extraneous cognitive load that often arises from rigid procedural instructions. Students do not feel pressured because there are no rigid procedures to follow, which helps optimize working memory capacity for germane cognitive load, the mental effort dedicated to schema construction and automation. This contrasts with conventional learning, which emphasizes memorizing the steps to a solution, potentially increasing extraneous load that interferes with meaningful learning. As a result, students with understanding at all KAM levels find it easier to develop through an open-ended approach, as their cognitive resources can be focused on genuine understanding rather than rote memorization.

Open-ended learning increases student motivation across all KAM levels, as students feel more involved in problem-solving. Research by Silver & Cai (1996) shows that open-ended problems stimulate curiosity and reduce mathematical anxiety. Students become more active in exploring various possible solutions. A direct, one-way approach tends to make students passive and less motivated. Thus, an open-ended approach is more effective in building student engagement.

The open-ended approach encourages conceptual understanding, not just procedural understanding, making it suitable for students at all KAM levels. Shimada and Becker (1997) state that this approach helps students see mathematics as a tool for thinking. Students do not just memorize formulas, but also understand the reasoning behind each step of the solution. Contextual learning in open-ended approaches makes mathematics more meaningful. This contrasts with direct approaches, which tend to produce superficial understanding.

The analytical findings reveal that students across all KAM levels demonstrated substantial gains in mathematical comprehension through the open-ended method compared to traditional instruction. These outcomes suggest that the open-ended approach is beneficial at every KAM level, yielding superior conceptual understanding compared with the conventional cohort. The implication is that this instructional strategy has demonstrated efficacy in enhancing mathematical comprehension skills among vocational high school learners. Consequently, the open-ended approach represents a viable pedagogical option for students, considering their individual KAM characteristics.

CONCLUSION

A comprehensive analysis of N-Gain data reveals that the open-ended learning approach has proven superior to conventional learning in improving students' mathematical comprehension of sequences and series. This superiority is not only reflected in the higher overall N-Gain average score (0.72 compared to 0.62), but also seen in every stratum of Initial Mathematics Ability (KAM). In the high KAM group, although both approaches achieved a high improvement category, the open-ended approach produced near-perfect results (0.95). The most significant impact was seen in students with medium and low initial abilities, where the open-ended approach raised their improvement to the high (0.71) and moderate (0.52) categories. In contrast, conventional learning produced improvement only in the moderate category (0.59 and 0.45) at both levels. These findings are reinforced by ANOVA tests, which show significant differences between learning approaches and KAM levels. Thus, descriptively, it can be concluded that the exploratory and conceptual nature of the open approach successfully created a more optimal learning environment to accelerate students' mathematical understanding. The effectiveness of this approach was evident at all KAM levels, especially in closing the understanding gap between groups.

DECLARATION OF GENERATIVE AI USAGE IN THE WRITING PROCESS

This article was written with the assistance of AI tools. We, Rusdian Rifai, Trisnawati, and Heryani, used DeepSeek to refine the sentence structure and DeepL Translate for Indonesian-to-English translation. We have reviewed and edited the output from these tools and take full responsibility for the content of this published article.

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