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The Limits of Cognitive Preferences: How Gender and Conceptual Understanding Shape Mathematical Representations

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Abstract: This study aims to describe the profile of students' mathematical representations across visual, symbolic, and verbal aspects, and to examine visualist and verbalist cognitive styles from a gender perspective. A descriptive qualitative approach was used. Four students were purposively selected, consisting of male and female students with visualist and verbalist cognitive styles from a public high school. The instrument used was the Object-Spatial and Verbal Imagery Questionnaire (OSIVQ) to classify students into visualist and verbalist cognitive styles. Representation data were collected through a mathematical representation test focusing on function composition, and were supplemented by semi-structured interviews. Data analysis was carried out through data reduction, data presentation, and conclusion drawing. The results showed that visual representations by male and female visualist and verbalist students in solving mathematical problems were presented as images. In problems without image instructions, both male and female visualist students continued to use visual representations before switching to symbolic representations. There was no variation in the visual representations used by the subjects; the images used tended to be copied from their teachers. Male students, both visualists and verbalists, used symbolic representations less systematically than female students. In symbolic representation, visual students made errors at the algebraic operations stage, resulting in symbol errors, and vice versa. All subjects used verbal representation, but to a very small extent. Verbal representation was not used to explain the steps for completing the worksheet; it was only used to write the names of the concepts. Visual students, both male and female, tended to point to pictures when explaining the work process, whereas verbal students tended to point to symbols. Female visual and verbal students were more systematic in their explanations, while male visual and verbal students were less coherent.

Keywords: mathematical representation, cognitive style, gender.

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INTRODUCTION

Mathematics has abstract objects derived from the abstraction process of concrete objects. Therefore, learning mathematics requires symbols, images, and expressions as forms of mental representation. Representational skills play a crucial role in mathematics learning, alongside reasoning, communication, and problem-solving skills (NCTM, 2000). Representations are necessary for us to express the mathematical ideas in our minds so that they can be understood by others. In mathematical learning, concepts are not only expressed verbally but also through symbols, graphs, specific expressions, diagrams, figures, tables, and other forms. To communicate mathematics, both between students and teachers, representations are needed that facilitate understanding of concepts and their use in solving real-world problems (Arcavi, 2003; Stylianou, 2010).

Representations may be internal (i.e., mental structures/cognitive constructions within students' minds) vs. external (observed symbols, graphs, tables, diagrams, written explanations). The two kinds of representations mutually explain mathematics when students struggle with it. Students use these internal representations to generate multiple external descriptions; they offer explanatory clarity and serve to consolidate meaning (Björklund & Palmér, 2022; Schoenherr & Schukajlow, 2024). In the mathematics education research literature, representations have been recognized as vital for connecting mathematical ideas to the reasoning processes students engage in and for enabling effective learning in mathematics (Duval, 2006; Goldin, 2002). Those difficulties in creating relevant representations occur when the task may inhibit not only the understanding or elaboration of problem knowledge (or a path/plan for doing so, even while solving a problem) but also the expression of reasoning.

Understanding mathematical concepts requires the ability to connect and interpret the meaning of various forms of mathematical representation. Using concepts to solve real-world problems begins with representing the information in the problem. For example, if the problem is presented in written or verbal form, we can often convert it into another form, such as symbolic, graphic, or pictorial. Changing the form of this representation allows us to understand the information in the problem more quickly and thus facilitate solving it (Birgin & Eryýlmaz, 2025; Nurrahmawati et al., 2021). In the classroom learning process, each student has a variety of representational forms. As teachers, we should provide ample opportunities to accommodate various representations to facilitate all students' understanding of concepts according to their preferred type of representation (Post & Prediger, 2024; Rau & Matthews, 2017). Providing a variety of possible representations in mathematics learning on a particular topic is a challenge for teachers, as they themselves have preferences for preferred types of representation. For example, the concept of fractions can be represented in various forms, including visual geometric, numerical, contextual, verbal, and number line representations. The wide variety of representations used to understand a particular concept poses a challenge for teachers in maximizing the teaching and introduction of these representations (Ainsworth, 2006; Duval, 2006). Various representations need to be introduced so that students can easily understand and adapt to the preferred form of representation, thereby avoiding misconceptions and supporting higher-level understanding (Ainsworth, 2006; Lesh et al., 1987). Several studies have found that using multiple representations in learning can improve reasoning, creativity, in-depth understanding, and problem-solving skills (Bicer, 2021). Other research has also found that various representations used in mathematics learning allow mathematics to be taught at different ages according to students' mental ages; for example, at the concrete operational age, representations in the form of concrete objects are needed to make certain concepts easier to understand (Schoenherr & Schukajlow, 2024).

Three domains of mathematically represented students are identified in the present study as students represent themselves with simple examples from mathematics: visual representation, symbolic representation, and verbal representation. This includes diagrams showing a graph (e.g., a plot, picture, table, or similar) that represents the relationships between the math ideas. Mathematical expressions, equations, or notations that encode/codify ideas constitute what we might think of as symbolic representation. Verbal representation is how a student expresses reasoning and describes the reasoning process used, in both written and oral form, to either support or detract from a task. They are significant because they indicate how students understand data, apply knowledge, and communicate meaning in mathematics (Castellanos et al., 2009). Unfortunately, research and classroom observations indicate that students may lack at least one representational aspect. For example, students do better at creating symbolic or visual representations than at providing coherent, clear explanations of their reasoning. They can therefore contribute to shaping the quality and understanding of mathematical language (Schleppegrell, 2007). This is an even larger challenge: the discourse needs of mathematics, which require students to properly convey their ideas and find their way to the logic of mathematics (Moschkovich, 2015).

During the learning process, students tend to rely on their own natural ways of organizing and structuring information, which ultimately influences their effectiveness in constructing mathematical representations. Cognitive style has long been recognized as an important factor underlying individual differences in mathematical thinking and problem solving. Previous research has primarily examined students' thinking styles in relation to conceptual understanding, knowledge construction, communication, and overall mathematical performance (Fauziyah & Hakim, 2025). Cognitive style refers to an individual's relatively stable tendencies in receiving, encoding, and using information during the learning process (Riding & Rainer, 2014). In mathematics learning, these preferences can influence how students interpret problems, choose strategies, and visualize relationships between mathematical concepts.

However, despite growing research on cognitive styles, little research specifically examines how they contribute to the formation of mathematical representation profiles that include visual, symbolic, and verbal representations. On the other hand, research on gender differences in mathematics generally focuses on aspects of learning achievement, anxiety, accuracy, persistence, or participation in mathematics, so there is little research on how gender intersects with cognitive styles in shaping students' representational processes. As a result, the integrated role of cognitive style and gender in constructing mathematical representations remains relatively unexplored. Furthermore, most previous research uses a comparative quantitative approach that has not been able to describe the characteristics of students' representational profiles in depth. Therefore, this study seeks to fill this gap through an in-depth qualitative analysis of students' mathematical representation profiles by cognitive style and gender, which is expected to provide a more integrative perspective on how individual differences influence the representational process in mathematical problem solving.

As representation is also about creating, as well as generating meaning from, the latter, the relevance of students' cognitive style may outweigh the discrepancy in students' representations. One of these stylistic varieties for representational work is the Visualizer-Verbalizer. Visualizers process information through images and spatial images, while verbalizers process it through words/sentences and textual explanations (Koæ-Januchta et al., 2017). Research in multimedia learning suggests that learners of all cognitive styles approach content with text visualisation (and use textual visuals content) in different ways (Mayer, 2021; Schnotz & Bannert, 2003). Koæ-Januchta et al. (2017) found that Visualizers and verbalizers are presumed to experience attentional dissociation between visual and verbal stimuli, which can significantly influence learning processes and outcomes. As a result, the latter distinction is particularly important in mathematics learning environments, as students have been asked multiple times to provide verbal explanations of their responses, based on the visually generated models and their symbolic representations, to justify how they presented their recommendations.

Therefore, differences in students' cognitive modality can affect their levels of (1) dependence on certain information, and (2) coordination of that information between them. Beyond cognitive styles, we see across numerous literatures that gender differences lead to poorer mathematics performance and differences in problem-solving. Other research has documented how such gender differences may manifest in a student's coping style when facing more challenging mathematical problems, either through strategizing or flexibility (Gallagher et al., 2000). However, a much broader analysis of international evidence, however general, also notes discrepancies in performance around gender bias, and gender bias towards socio-cultural, educational, and political forces (Else-Quest et al., 2010). There were also gender differences in students' perceptions and learning properties related to mathematical construction, factors that would have a secondhand impact on how students created and articulated mathematical conceptions (Ganley & Vasilyeva, 2014). Even though the gender differences in effects depend on context: the education climate and the activities, we also need to see the representation profile in gender to better understand how students are learning to make and communicate mathematical constructions, as the support of instructional teachers seems or feels more or less the same.

Representation is especially important in cases involving functional topics, such as function composition, and subsequently, when many representations need to be integrated. To represent functions symbolically, students must interpret function notation and provide an expression in symbolic form; they need to conceptualise the relationship of the function. This refers to tasks in which students must balance (symbolic) manipulatory behaviour and (conceptual) thinking; they must think in terms of both and articulate the solution procedures they perform (Martins et al., 2023; Post & Prediger, 2024). Without representational competence in the realm of verbal explanation, students will not reason, even when they describe sound, symbolic responses to the questions they ask, and are willing to talk through their steps along with the output; as a result, they will not understand the idea behind the outcome.

Comparative analysis of students' representation profiles, constructed using function composition, demonstrates differentiation across the board. In reality, representation, in either a typical and standardized form or a strong single factor, may be a central part of maths learning. Indeed, few studies have been conducted with senior high school students on the representation of mathematics across visual, symbolic (e.g., sign), and verbal domains, while considering multiple visualizer-verbalizer cognitive styles and gender distinctions among students. Hence, this study aims to describe a descriptive profiling study design that explores students' representational performance as a function of their cognitive style (visualizer versus verbalizer) and gender, in relation to their performance on functional composition tasks. The implications of this study will theoretically inform individual differences in the construction of representational processes and the practices that teachers can implement to support students' representational competence, i.e., individual representational process construction, to enhance students' representational competence in how they use language and written expressions of mathematics in their mathematical problem solving.

Based on the research objectives above, this study poses the following research question: What is the profile of student representation in solving mathematical problems, with respect to differences in cognitive style and gender? The representations used in this study are visual, symbolic, and verbal representations. Meanwhile, cognitive styles are divided into two types: visual and verbal.

METHOD

Research Design

This research employed a qualitative design with an exploratory multiple-case study approach to examine in-depth the mathematical representation profiles of four students selected based on their cognitive style and gender characteristics. This research did not aim to make statistical generalizations, but rather to generate rich contextual descriptions of each case to understand how representation patterns are formed within specific cognitive style and gender configurations.

Participants

Participants were selected using a purposive sampling method from students in grades 11-6 at a public high school. Subject selection was conducted purposively based on a combination of cognitive style and gender, not for statistical generalization. Each category was represented by a single subject chosen as an information-rich case, allowing for in-depth analysis of visual, symbolic, and verbal representations in mathematical problem solving. Based on the questionnaire results, four students were selected as study subjects: one female visualist, one male visualist, one female verbalist, and one male verbalist. The student with the highest score in each cognitive style group was selected to ensure a clear classification and a strong representation of the visualist and verbalist profiles. While providing depth of analysis, this study uses a single subject for each category, which is a limitation. The findings are not intended to generalize fixed patterns across gender groups or cognitive styles, but rather serve as contextual descriptions of the cases studied. Therefore, further research is recommended that involves more diverse participants or uses mixed-method designs to test the consistency of the findings.

Instruments

The instrument for classifying cognitive styles used the Object-Spatial and Verbal Imagery Questionnaire (OSIVQ) (Kozhevnikov et al., 2002). The OSIVQ is used to classify students into visualist and verbalist cognitive styles. The results of the questionnaire serve as the basis for selecting participants and identifying cognitive style profiles (Kozhevnikov et al., 2002). This instrument has undergone construct and reliability validation in various previous studies (Blazhenkova & Kozhevnikov, 2009; Kozhevnikov et al., 2005), making it suitable for identifying students' cognitive style profiles. The OSIVQ instrument was translated and adapted to the research context through an expert-judgment process to ensure content validity before being administered to research subjects. The OSIVQ instrument was based on three indicators: object imagery, spatial imagery, and verbal style. Object imagery indicators included vividness and detail of visual images; spatial imagery included spatial manipulation and mental rotation abilities; and verbal style related to language-based information-processing preferences. The object and spatial imagery dimensions were combined as visualizers, while the verbal style dimension was categorized as verbalizers. Example of an object imagery statement: Looking at pictures helps me understand the content of a reading. Example of a spatial imagery statement: Diagrams or schematics help me understand the relationships between parts of an object. Example of a verbal style statement: I find it easier to understand written explanations than pictures.

The second instrument was a mathematical representation test designed to assess students' ability to coordinate and translate across various forms of representation when solving mathematical problems. The representation test focused on the composition of functions. The test items were designed based on a representation framework that emphasizes the role of various representations in supporting mathematical reasoning and understanding (Rau & Matthews, 2017; Schoenherr & Schukajlow, 2024). Students' mathematical representation abilities were evaluated using three indicators: (1) visual representation, (2) symbolic representation, and (3) verbal representation. The indicators of mathematical representation ability used in this study are presented in Table 1. The question development process begins with creating a question grid based on the learning objectives, with each question tailored to them. Next, student answer predictions were made for each question based on the mathematical representation indicators. To ensure alignment between the learning objectives, the instrument, and the representation indicators, a correspondence matrix was created. The following is an example of a question from the mathematical representation test instrument used.

Given , $f(x) = \frac{4x+7}{3x-5}; x \neq \frac{5}{3}$ determine the formula f^{1} *(x)* and the value of the function f^{1} *(x)*, for x = {1,2,3,4}. The third instrument used in this study was a semi-structured interview guide designed to explore and clarify students' written responses to the written representation test. Interviews were conducted to gain a more in-depth understanding of how students construct representations during problem-solving. The second and third research instruments were reviewed by mathematics education experts to establish content validity and clarity prior to data collection.

Table 1. Mathematical representation indicators

Aspect of Mathematical Representation	Indicators
Visual representation (pictures, diagrams, graphs, tables)	Students visualize the problem, representing the information as an arrow diagram, table, or graph that shows how the elements are related.
Symbolic representation (mathematical expression, equations, notation)	Students record what they recognize about the problem and apply the solution by showing the answer in written form, adding what is known about the set, the symbol for a function from one set to another, and the composition of functions.
Verbal representation (written or oral explanations)	Students write the problem, provide detailed explanations of the given information, explain the question asked, and solve the problem.

Data Analysis

The research data were analyzed using qualitative research procedures, including data reduction, data presentation, and conclusions/verification (Miles et al., 2016). In the data reduction stage, researchers transcribed interview results and collected student answer sheets. They then conducted initial coding based on mathematical representation indicators, including visual, symbolic, and verbal representation. Each student's response was identified and coded according to its representational characteristics. Information from the interviews that was irrelevant to the research objectives was reduced. This process aimed to simplify the raw data into meaningful units relevant to the research objectives.

Next, the coded data were presented as a profile matrix and a descriptive narrative to compare the representational characteristics of each combination of cognitive style and gender. The conclusion verification stage involved doublechecking the consistency among written data, interview results, and researchers' interpretations, as well as discussions among researchers to minimize subjective bias. Conclusions were drawn gradually by tracing the representational patterns that emerged in each subject, ensuring that the resulting interpretations remained grounded in empirical research evidence.

To ensure the validity of the analyzed data, triangulation was used as a data collection technique, comparing written test results and interview data to ensure consistency in students' interpretations of mathematical representations. For example, for one subject with a visualizer cognitive style, the answer sheet showed a predominant use of images but a lack of verbal explanations. This finding was then clarified through interviews, where the student explained that he found it easier to understand problems through images than through verbal explanations. This process helped the researcher confirm that the student had a stronger preference for visual representations. Furthermore, member checking was conducted by showing each participant a summary of the interpretation results following the initial analysis. This was possible because high school students are much more articulate in communicating. Participants were asked to respond to the researcher's description of the representation profile at specific points requiring confirmation. In several cases, participants provided additional clarification regarding the rationale for their chosen strategy or the sequence of steps, prompting the researcher to revise previously inaccurate interpretations. This process helped increase the credibility of the findings because the conclusions were based on the congruence between the researcher's interpretation and the participants' lived experiences.

Participants were informed of the study's purpose and that their participation was voluntary. Their identities were anonymized to protect confidentiality, and all data were used solely for research purposes. Teachers and the principal granted permission for this research process.

RESULT AND DISCUSSION

Based on the results of the Object–Spatial Imagery and Verbal Questionnaire (OSIVQ), four participants were purposively selected for further investigation through a mathematical representation test and in-depth interviews. The selection was guided by two criteria: (1) a clear cognitive-style classification and (2) balanced gender representation. The OSIVQ scoring scheme ranged from 0 to 100 for each dimension. Students were classified as visualizers when the visualizer score was at least 10 points higher than the verbalizer score, and as verbalizers when the verbalizer score was at least 10 points higher than the visualizer score. Students whose score difference was less than 10 points were categorized as negligible. The complete OSIVQ results and classifications are presented in Table 2.

Table 2. OSIVQ results and participant classification

Student ID	Gender	Visualizer Score	Verbalizer Score	Difference (V-VB)	Category	Research Code
S01	M	**82**	65	+17	Visualizer	MV
S02	M	68	**81**	−13	Verbalizer	MVB
S03	M	72	69	+3	Negligible	-
S04	M	70	73	−3	Negligible	-
S05	M	69	67	+2	Negligible	-
S06	F	**85**	62	+23	Visualizer	FV
S07	F	64	**88**	−24	Verbalizer	FVB

S08	F	71	69	+2	Negligible	-
S09	F	70	72	−2	Negligible	-
S10	F	68	71	−3	Negligible	-
S11	F	69	66	+3	Negligible	-
S12	F	67	70	−3	Negligible	-
S13	F	72	70	+2	Negligible	-
S14	F	69	68	+1	Negligible	-
S15	F	70	69	+1	Negligible	-
S16	F	68	66	+2	Negligible	-
S17	F	71	69	+2	Negligible	-
S18	F	73	71	+2	Negligible	-
S19	F	69	70	−1	Negligible	-
S20	F	70	68	+2	Negligible	-
S21	F	71	72	−1	Negligible	-
S22	F	69	67	+2	Negligible	-
S23	F	68	69	−1	Negligible	-
S24	F	70	71	−1	Negligible	-
S25	F	69	68	+1	Negligible	-

Notes. V = Visualizer (Visualizer Score e" Verbalizer Score + 10); VB = Verbalizer (Verbalizer Score e" Visualizer Score + 10); N = Negligible (Difference < 10). Research codes: FV = Female Visualizer; MV = Male Visualizer; FVB = Female Verbalizer; MVB = Male Verbalizer.

As seen in Table 2, only four students fell into the visualist and verbalist categories, two in each category, and the remainder fell clearly into the negligible category. The OSIVQ classification results indicate that the majority of students fall into the negligible category. This condition needs to be understood in relation to the classification criteria used, namely, a minimum difference of 10 points between visualizer and verbalizer scores. This threshold is conservative and designed to identify truly distinct cognitive style tendencies, thereby automatically increasing the number of students categorized as negligible. Therefore, the dominance of the negligible category does not merely indicate low instrument sensitivity, but also reflects the stringent classification criteria applied in this study. The scatter plot of the OSIVQ scores for 25 students is shown in Figure 1.

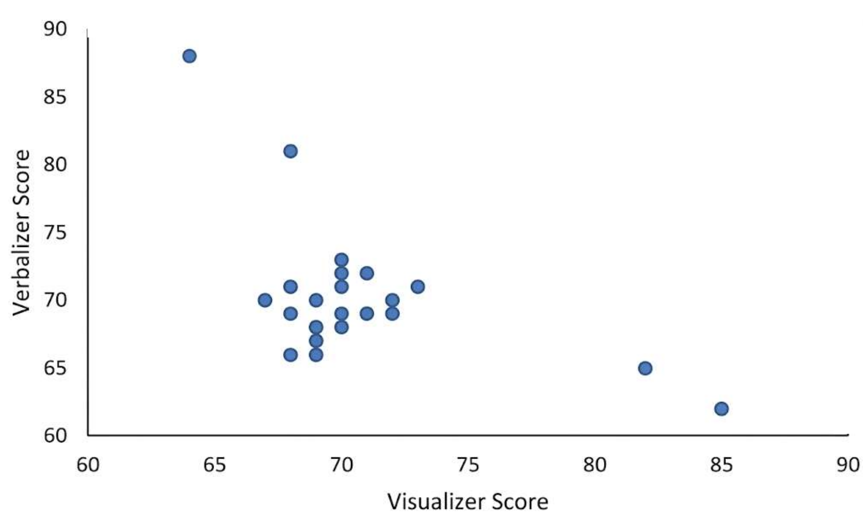

Figure 1. Scatter plot of the OSIVQ scores (25 students)

In the context of a qualitative study focused on in-depth profile mapping, the four participants were purposively selected based on scores that best demonstrated contrasting cognitive style tendencies and gender balance. This approach aimed to obtain information-rich cases for the exploration of mathematical representations, not to statistically describe population distributions. However, the use of strict classification thresholds may limit variation in identified cognitive styles, a research limitation that must be considered when interpreting the results.

Table 3. Category n mean visualizer score mean verbalizer score

Category	n	Mean Visualizer Score	Mean Verbalizer Score
Visualizer	2	83.50	63.50
Verbalizer	2	66.00	84.50
Negligible	21	70.00	69.50

The data in Table 3 show that the mean scores for visualizers and verbalizers in the negligible group were relatively balanced (70.00 and 69.50, respectively). This pattern indicates that most students tend toward moderate cognitive styles, but do not reach the minimum 10-point difference established as the classification threshold. Therefore, the dominance of the negligible category is likely not due to an insensitive instrument but rather to the use of sufficiently stringent classification criteria, which identified only profiles with contrasting scores as visualizers or verbalizers. This finding reinforces the rationale for purposive subject selection within the most distinct categories and underscores the importance of interpreting the research results with caution.

From Table 4, four individuals were selected as study subjects (FV, S06; MV, S01; FVB, S07; and MVB, S02). These students were chosen because they obtained the highest dominant scores in their respective cognitive-style categories and provided balanced gender representation. These four subjects were then analyzed to identify their profiles of mathematical representation (visual, symbolic, and verbal) when solving function-composition tasks.

Table 4. Selected research subjects

Subject Code	Student ID	Gender	Cognitive Style	Dominant Score
FV	S06	Female	Visualizer	85
MV	S01	Male	Visualizer	82
FVB	S07	Female	Verbalizer	88
MVB	S02	Male	Verbalizer	81

The following section presents research findings based on three types of mathematical representation: visual, symbolic, and verbal. For each type of representation, the analysis focuses on profile differences arising from cognitive style and gender to provide a more comprehensive picture of students' representation characteristics in mathematical problem solving.

Visual Representation Profiles

The first finding in general, both subjects with visualizer and verbalizer cognitive styles use arrow diagrams to represent the composition of functions $(f \circ g)(x)$, where the functions f and g are in the same set, namely $x = \{a, b, c, d, e\}$. The process of drawing an arrow diagram begins by drawing 3 ellipses. Each ellipse drawn is used to accommodate members of the set x. Next, draw arrows to determine the mapping of functions f and g according to the ordered pairs known in the problem, namely *(f) = {(a,c), (b,d), (c,a), (d,e), (e,b)}* and *g (x) = {(a,b), (b,c), (c,d), (d,e), (e,a)}*. To represent that the function composition is the function continued by the function , not the other way around, they drew a curved arrow labeled above it, followed by a curved arrow labeled . This was done by both visualizers and verbalizers. This can be seen in Figure 2.

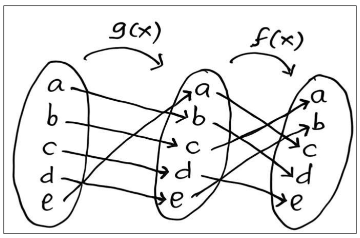

Figure 2. Visual representation of the composition of functions in the form of arrow diagrams drawn by visualizer and verbalizer subjects.

It appears that the differences in cognitive styles related to the representation of function composition are not significant. However, there are significant differences in gender. For both visualists and verbalists, male subjects begin by drawing function composition using arrow diagrams for each function, and . This is in line with previous research that male students tend to perform better on visual tasks, which may influence visual representation strategies in mathematical problem solving (Ramírez-Uclés & Ramírez-Uclés, 2020; Wei et al., 2016). Evidence that male subjects drew arrow diagrams for functions and g before drawing the function composition is shown in Figure 3.

Male subjects drew arrow diagrams for both functions, f and g, to simplify and assist in determining the order of arrows in the function composition. Meanwhile, female subjects, both visual and verbal, did not do this because they

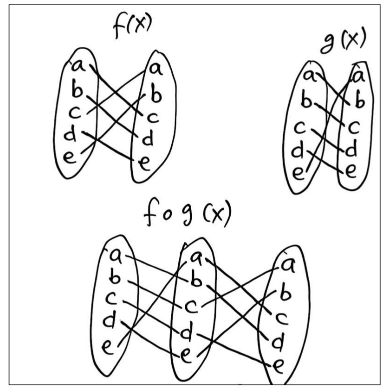

Figure 3. Arrow diagram of functions f and g by a male subject visualizer and verbalizer

could already directly see functions f and g from the known set of ordered pairs. This was conveyed by the male subjects as seen in the following interview transcript:

R (Researcher): What function is this arrow diagram for? (pointing to the arrow diagrams of functions f and g).
S (subject): Functions f and g.
R: Why did you draw arrow diagrams for functions f and g?
S: To help me illustrate the composition of functions.
R: Where did you get the mapping rule for functions f and g?
S: From what is known (pointing to functions f and g written as ordered pairs from what is known in the problem).
R: Why didn't you just look at the ordered pairs, without having to draw the arrow diagrams for functions f and g one by one?
S: I'm confused about the ordered pairs.
R: Oh, I see.
S: Yes, because when drawn in an arrow diagram, it's clear that the members of one set are mapped to the members of another set.

The following is an excerpt from an interview between a researcher and a female subject regarding their reasons for drawing function compositions without first drawing the functions individually.

R: Why did you draw the function composition straight away? How do you know the mapping rule for functions f and g?

S: Because we know the functions f and g (while pointing to the functions f and g in the ordered pair form from the problem).

R: Aren't you confused about reading the mapping from one set to another if the function is expressed in the form of a set of ordered pairs?

S: No.

R: Why?

S: Because each member of the ordered pair set represents a mapping of the function.

Therefore, there was a one-step difference in problem-solving. Male subjects started with two functions, represented as ordered pairs, drawn in an arrow diagram, and then drew their composition. Meanwhile, female subjects drew the composition of the functions directly for two functions in the form of a set of ordered pairs. Thus, there was a difference in the composition of the images; male subjects had more compositional images than female subjects, both visualizers and verbalizers. The following shows the difference in the order of drawing the composition of functions for male and female subjects; both visualizers and verbalizers are shown in Figure 4.

The second finding is that, when solving problems that don't require drawing, both male

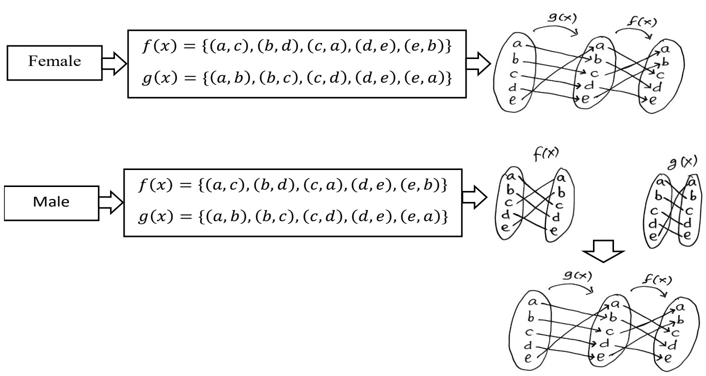

Figure 4. Differences in the composition of images and the order in drawing function compositions for male and female subjects, both visualizers and verbalizers

and female visualizer subjects tend first to draw concepts that will help them solve the problem. For example, in the problem of determining the domain of (f= g) (x) for x = {1,2,3,4}, given the functions f and g. The subject is completed by first finding and drawing the function f as an arrow diagram. This is consistent with previous research indicating that visualizers tend to use mental imagery when processing information and are visually based in their understanding of mathematical problems (Hasan, 2019; Kozhevnikov et al., 2005; Pitta-Pantazi & Christou, 2010). The following artifacts of their work are shown in Figure 5.

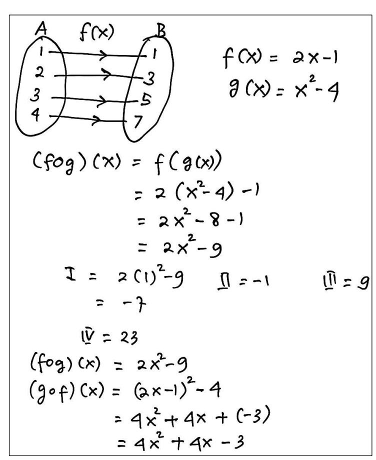

Figure 5. The visualizer subjects represented the function in the form of an arrow diagram

Meanwhile, verbalizer subjects tended to use symbols directly, prior visual representation, when working on problems that did not require drawing. This can be seen in Figure 6 below:

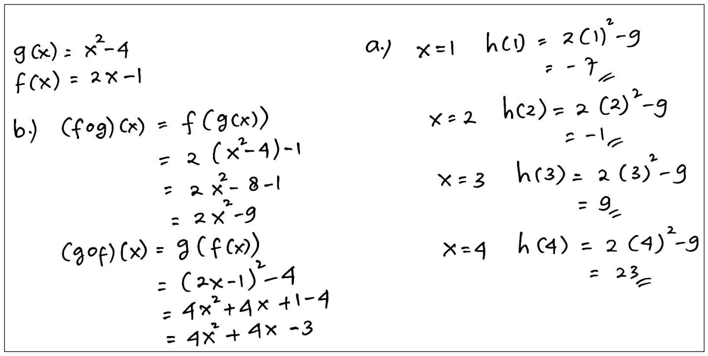

Figure 6. The verbalizer subjects work without any visual representation when solving problems that do not require drawing

Third finding: Although the second finding revealed a difference in the proportion or intensity of use between visualizers and verbalizers, visualizers used visual representations more frequently than verbalizers. This is because visualizers used visual representations first before moving on to symbolic representations, while verbalizers did not. However, the representational forms used by all subjects tended to consist only of the forms or models exemplified by the teacher. No elaboration or development of new visual representations outside of the given learning pattern was found. For example, when drawing the placeholders for set elements in a function arrow diagram, all subjects drew ellipses; there was no variation in other shapes, such as rectangles. Furthermore, all functions were depicted in arrow diagrams; no subjects depicted them graphically. This occurred because students imitated the teacher's representations, even though the teacher had taught them how to graph functions. However, during the discussion on function composition, the teacher never provided an example as a graphical visualization.

To clarify this finding, the researcher posed questions to the subjects, as excerpts from the interviews follow.

R: Why do you draw arrow diagrams in the shape of ellipses?
S: The teacher gave the example.
R: Weren't you taught to draw arrow diagrams in the shape of rectangles?
S: No.
R: Why do you draw functions in the shape of arrow diagrams?
S: Because that's the example.
R: Can't functions be graphed?
S: Oh yes, but graphs are more complicated.
R: What's the complicated part?
S: Draw its Cartesian coordinates.
R: So, which is easier to draw in an arrow diagram or graph form?
S: Arrow diagram.

This third finding is consistent with previous research indicating that students tend to follow, imitate, or model teachers' strategies when working on math problems (Çakmak Gürel, 2023; Ozturk, 2025; Wang et al., 2024). Another finding is also consistent with this study's results: students tend to follow the form of representation provided by the teacher rather than creating their own representation (Lundvin & Palmér, 2025). This finding aligns with research showing that the use of visual representations is strongly influenced by how teachers position the representations in learning. Teachers who tend to use a specific model encourage students to follow the same representational pattern, thus limiting the variety of representations, choosing familiar visual representations, and rarely exploring new variations in mathematical problem-solving (Rif'at et al., 2024; Weingarden et al., 2026). This indicates that teaching practices act as a structural framework that limits the variety of representational forms, while cognitive styles influence how and how often these representations are used. Thus, the expression of cognitive preferences is not entirely independent but interacts with the pedagogical context in shaping students' representational profiles.

Symbolic Representation Patterns

This section examines the symbolic representation patterns exhibited by subjects when solving mathematical problems, highlighting how each subject constructed symbolic notation and manipulated it. The analysis focused on the consistency of symbol use, the accuracy of algebraic steps, and the relationships between symbolic representation, cognitive style, and gender.

Based on the data analysis, both male and female visualizers represented known information as images and then switched to symbols during the solution process. This can be seen in Figure 5. Pictorial representation was used to facilitate the subjects' understanding of the information in the problem or their understanding of what was known in the problem. This finding aligns with previous research showing that visualizers prefer visual representations to represent known information in problems (Setyawan et al., 2020).

A second finding for female subjects, both visualizers and verbalizers, was that they worked on problems using highly systematic symbols that were easy to read and understand the flow or sequence. Meanwhile, male students, both visualizers and verbalizers, worked on problems less systematically, using a flow that was difficult to understand. This is shown in Figure 7, which demonstrates the differences in their systematic problem-solving methods. This finding has not been found in previous research, but when it comes to the ability to explain steps systematically, female students are stronger (Nurhajarurahmah & Arsyad, 2023).

The third finding was that both male and female visualizers made errors when performing algebraic operations, resulting in errors in writing

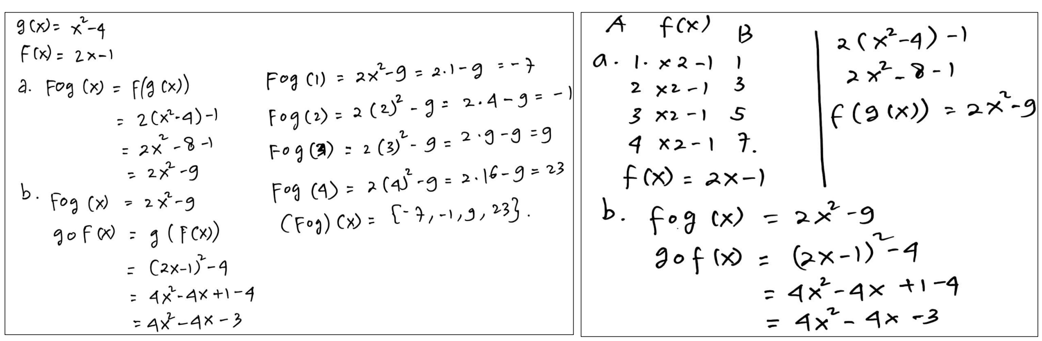

Figure 7. Differences in the systematic problem-solving methods of female and male visualizers

symbols. Meanwhile, neither male nor female verbalizers made errors in performing algebraic operations or writing symbols. The following is evidence consistently experienced by both male and female subjects with a visualizer cognitive style. For example, when squaring an algebraic expression, as shown in Figure 8, the correct answer should be $4x^2 - 4x - 3$, not $4x^2 + 4x - 3$.

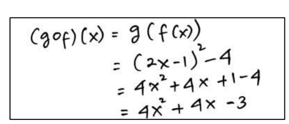

Figure 8. Algebraic operation errors in male and female visualizers that resulted in symbol errors

The third finding of this study is that visualizer students make errors when operating with algebraic forms due to a lack of care in symbolic operations (Munawwir et al., 2025).

Verbal Representation Challenges

The first research findings indicate that verbal representations account for a very small proportion compared to visual and symbolic representations. All subjects, with both visualizer and verbalizer cognitive styles and from both gender categories, more often convey problem solving through pictures and symbolic manipulation rather than through descriptive written explanations. Verbal sentences are generally used only to mention or label concepts, such as domain, codomain, and range, and are not followed by in-depth elaboration of reasons or conceptual relationships. This pattern indicates that verbal representations do not yet serve as a means of developing mathematical arguments, but rather as a terminological complement to the more dominant visual and symbolic processes. These findings indicate a general tendency for subjects to focus more on visual illustrations to show concepts. At the same time, the working procedures tend to use symbolic representations without any verbal explanations at each step. Verbal representation is a challenge for students (Castro et al., 2022; Nasrun et al., 2023).

The second set of research findings indicates that verbal representations occur in a very limited proportion compared to visual and symbolic representations. All subjects, with both visualizer and verbalizer cognitive styles, and from both genders, more often conveyed problem solutions using drawings and symbolic manipulation rather than descriptive written explanations. Verbal sentences were generally used only to name or label concepts, such as domain, codomain, and range, without further elaboration of the rationale or conceptual relationships. This pattern indicates that verbal representations do not yet function as a means of developing mathematical arguments, but rather merely complement the terminologically dominant visual and symbolic processes. These findings indicate a general tendency for subjects to focus more on visual illustrations to demonstrate concepts, while procedures tend to use symbolic representations without verbal explanations of each step.

The third finding was that subjects with both visualizer cognitive styles tended to explain the solution process based on the visual representations they created on worksheets. This is evident in the tendency for subjects to more frequently point to parts of the drawings when explaining solution steps, rather than referring directly to written notation or symbolic models. Gestures when explaining something serve as a reflection of what someone is thinking (Weliweriya et al., 2018). The verbal explanations provided complemented the previously constructed visualizations, thus maintaining the primary focus on the images as a means of thinking. In contrast, subjects with a verbalizing cognitive style, both male and female, showed a different pattern, with explanations more focused on symbolic descriptions and written procedural steps. Verbalizing subjects tended to refer to notation or sequences of steps when explaining solution strategies, while the use of images was merely supportive. This difference suggests that cognitive style preferences influence how students construct verbal representations when reflecting on their work. The following interview excerpts demonstrate this focus on images.

R: How do you determine the domain of (f o g) (x) for x = {1,2,3,4}?

S: I'll draw the function f (x) first (pointing to the arrow diagram) because I don't know the function formula.

R: Why do you need to draw it again? Isn't the diagram already in the problem?

S: To make it easier for me to find the function formula.

R: How do you find the function formula?

S: By looking at the pattern in the diagram (pointing to the arrow diagram), 1 is mapped to 1, 2 is mapped to 3, 3 is mapped to 5, and 4 is mapped to 7.

R: So, how do you find the pattern?

S: By guessing.

Female subjects, with both visualizer and verbalizer cognitive styles, tended to provide more coherent and elaborate explanations when describing their solution processes. Female students are more able to elaborate in conveying arguments (Nugroho & Suseno, 2025). The explanations typically followed a systematic sequence of steps, accompanied by additional explanations of the rationale for the chosen strategy or the relationships among the representations used. In contrast, male subjects tended to provide shorter and more fragmented explanations, focusing on specific parts deemed important without detailing the entire process. This pattern indicates a difference in verbal communication styles in reflecting mathematical thinking, where female subjects place more emphasis on the completeness of the narrative, while male subjects are more direct in addressing the core procedures considered relevant. The following is an excerpt from an interview with a female subject.

R: How do you determine the composition formula for the function f (g(x))?

S: I'll first write down the formulas for the functions f and g (while pointing to the formulas for f (c) and g (x).

R: What happens next?

S: Substitute the formula for the function g into the formula for the function f, since f (g(x)) is required. This gives 2 (x^2 - 4 - 1, which equals $2x^2$ - 8 - 1. The result is $2x^{2-9}$.

R: How do you find the composition formula for the function 9 (f(x))?

S: It's the same as f (g(x)) , except I reverse it. For f (9(x)), I substitute 9(x) into f (x), and for 9 (g(x)), I substitute f (x) into 9 (x)

R: What happens next?

S: Since f (x) = 2x - 1 and 9 (x) = x^2,- 4 substituting 2x - 1 for in 9 (x) gives $(2x - 1)^2$ - 4. This is squared to 4 x^{2-} 4x + 1 - 4, so the final result is $4x^2$ - 4x - 3.

The following is evidence from an interview excerpt with a male subject.

R: Could you please explain how you obtained the formula for the composition of functions f (9(x))?

S: Here, we have the function f (x) = 2 - 1 and 9 (x) = 2 x^2 9, so f (9(x)) = $2x^2$ - 9.

R: Where did you get the formula for the function f (x)?

S: 1 multiplied by 2 *minus* 1 *equal* 1. |

R: What does that mean?

R: Because 2 *map to* 1, 2 *maps to* 3*, and maps to* 5*, this (the first set member) is multiplied by* 2 *and then subtracted by* 1.

There are differences in the conversations of female and male subjects, both visualizers and verbalizers. The tendency toward longer explanations is not influenced by cognitive style; rather, gender differences play a more significant role. Male subjects need to be guided with questions to encourage them to provide explanations. However, when asked, female subjects provide quite lengthy explanations. For ease of reading, all findings are presented in Table 5.

Based on the research data, a process flowchart can be created for each research subject. This diagram maps the steps the subject takes from start to finish in solving a mathematical problem, specifically the function composition, as seen in Figures 9 through 12.

Table 5. Profile of mathematics representation based on cognitive style and gender

Representations		Visualizer		Verbalizer	
		Male	Female	Male	Female
Visual	Questions with drawing instructions	- Presenting known information in the form of images	- Does not represent known information in pictorial form	- Presenting known information in the form of images	- Does not represent known information in pictorial form
		- Representing in the form of images in solving problems	- Representing in the form of images in solving problems	- Representing in the form of images in solving problems	- Representing in the form of images in solving problems
	Questions without drawing instructions	- Start by making a picture before moving on to symbolic representation	- Start by making a picture before moving on to symbolic representation	- Do not start by making a picture before moving on to symbolic representation	- Do not start by making a picture before moving on to symbolic representation
Symbolic		- Using pictorial representation to facilitate the move towards symbolic representation	- Using pictorial representation to facilitate the move towards symbolic representation	- Not using pictorial representation to facilitate the direction of symbolic representation	- Not using pictorial representation to facilitate the direction of symbolic representation
		- Using less systematic symbols	- Using more systematic symbols	- Using less systematic symbols	- Using more systematic symbols
		- Experiencing errors in algebraic operations	- Experiencing errors in algebraic operations	- No errors in algebraic operations	- No errors in algebraic operations
Verbal		- The use of verbal representation is very limited	- The use of verbal representation is very limited	- The use of verbal representation is very limited	- The use of verbal representation is very limited
		- In explaining, the focus tends to be on the picture	- In explaining, the focus tends to be on the picture	- In explaining, it tends to focus on symbols	- In explaining, it tends to focus on symbols

Figure 9 (on the left) shows the subject's cognitive process for solving a problem involving drawing instructions. The student first represents known information as an arrow diagram, then develops a three-set structure to map function composition. The next stage shows the transition from visual to symbolic representation through notational labeling. This diagram illustrates the progression from relation visualization to symbolic formulation, showing the interaction and coordination between visual and symbolic representations in building an understanding of function composition. Meanwhile, the figure on the right shows the problem-solving process without drawing instructions. The subject first interprets the function information provided in the form of an arrow diagram and a formula, then understands the question about function composition. Although not asked to draw, the student still uses the visual representation as a bridge to the symbolic representation. Although the symbolic steps are less systematic, the visual representation continues to serve as a facilitator before the student moves on to symbolic manipulation, the final form of the solution.

Figure 10, the flowchart on the left, illustrates the Female Visualizer subject's thought process and representation of a problem with drawing instructions. The process begins with the problem interpretation stage. The subject does not transform the information in the problem into a visual representation. The initial visual representation is used directly in the problem-solving process. In the final stage, an integration

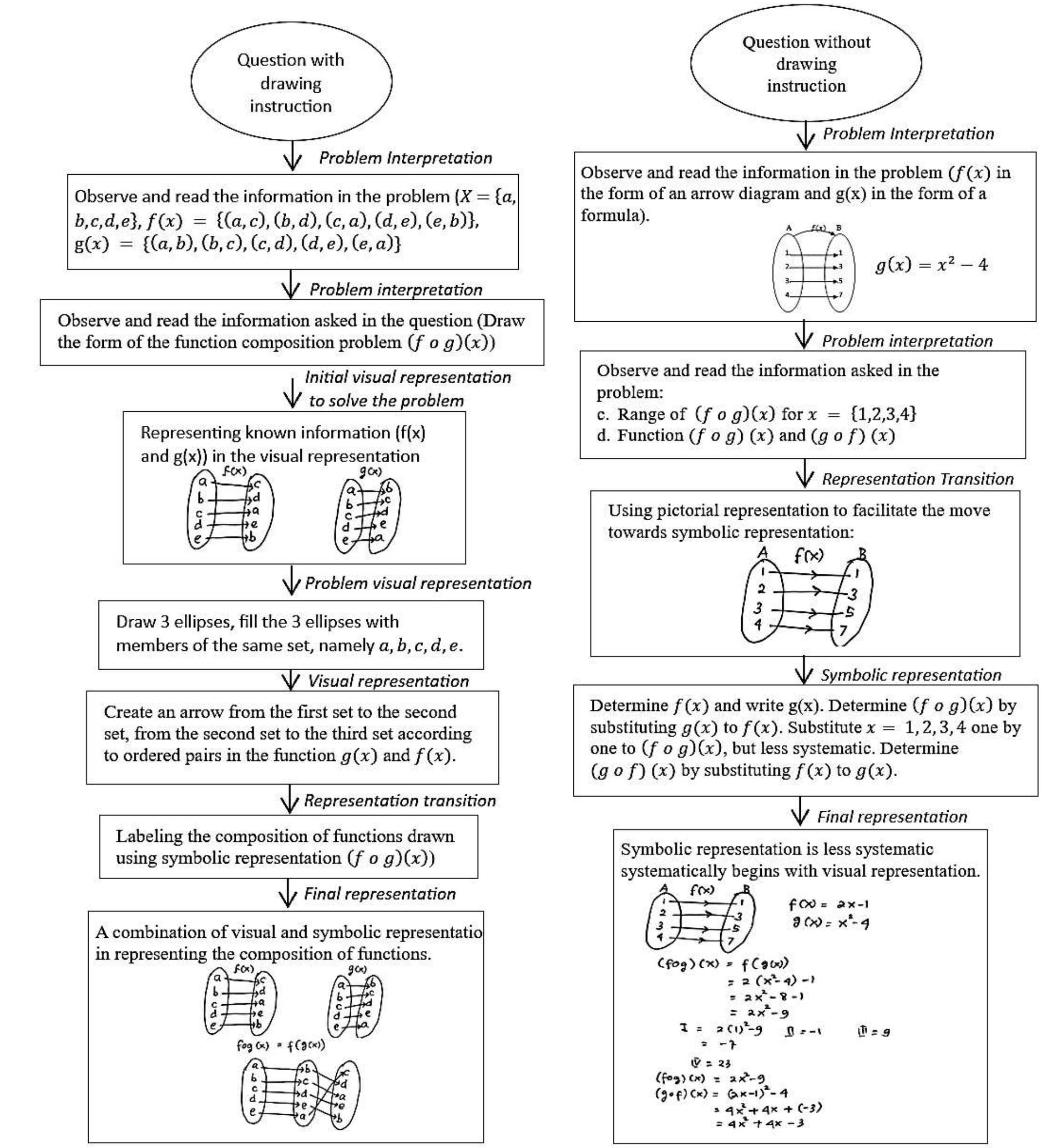

Figure 9. Flowchart of the thought process and representation of the subject, male visualizer

of visual and symbolic representations is evident in the presentation of the functional composition, indicating the dominance of the visual approach in the subject's thought process. Meanwhile, the image on the right shows the thought process flow and representation of the Female Visualizer subject on a problem without drawing instructions. The process begins with the interpretation of information. Although not asked to draw, subject still uses the visual representation as a bridge before moving on to symbolic manipulation. The final representation shows symbolic dominance supported by the initial visual structure, confirming that visualization still functions as a foundation in the reasoning process before bei systematically formalized symbolically.

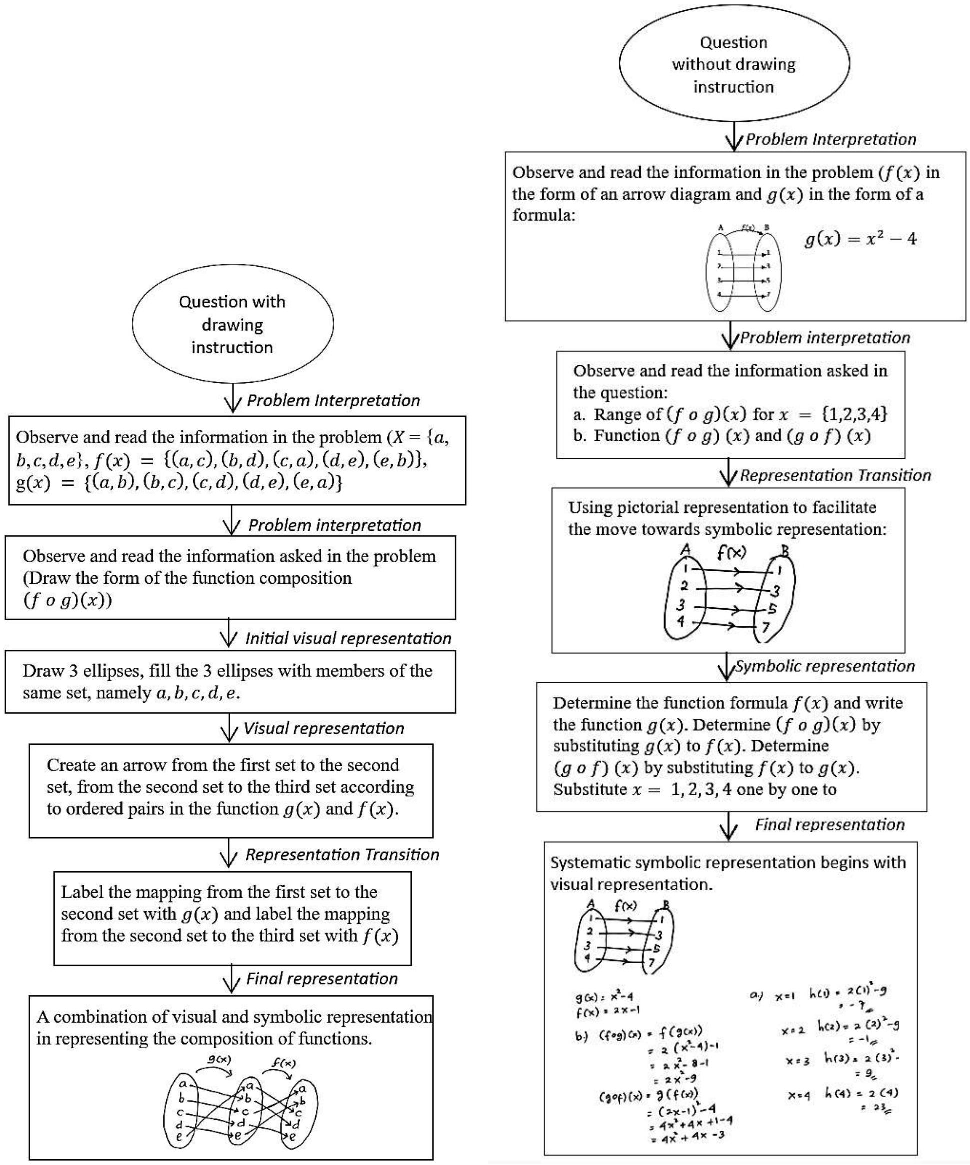

Figure 10. Flowchart of the thought process and representation of the subject female visualizer

Figure 11, on the left, shows the thought process and representation of a Male Verbalizer subject in a problem with drawing instructions. The process begins with interpreting the information in the problem. Next, they represent known information using visual representations, then use them to solve the problem. Symbolic labeling of the images completes the problem, and in the final stage, visual and symbolic integration is evident. The image on the right depicts the thought process and representation of a Male Verbalizer subject on a problem without drawing instructions. The process begins with interpreting the information in the problem. The problem is solved directly using symbolic representation, although less systematically.

Figure 12, on the left, shows the thought process and representation of a Female Verbalizer subject in a problem with drawing instructions. The process begins with interpreting the information in the problem. The information is not transformed into a visual representation. The were

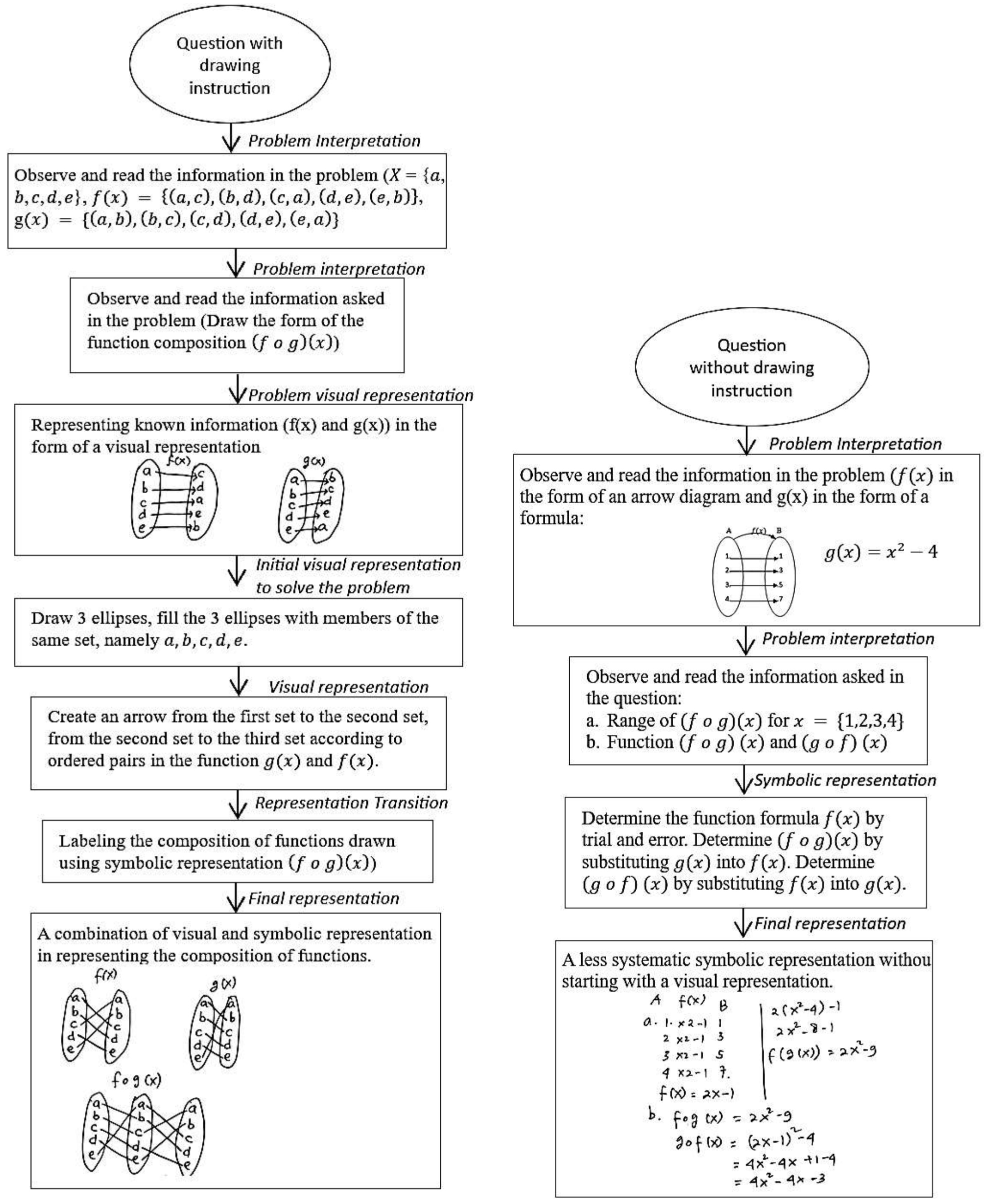

Figure 11. Flowchart of the thought process and representation of the subject, male verbalizer

visual representation is used directly in problem-solving. Symbolic representations complement the visual representations created. The image on the right depicts the thought process and representation of a Female Verbalizer subject on a problem without drawing instructions. The process begins with interpreting the information in the problem. The subject immediately uses symbolic representations, culminating in a final, systematically structured solution.

Another interesting finding was the discovery of a common error experienced by all subjects in this study. This error lies in the concept of determining the codomain in a function composition. The instrument used in this study, specifically in determining function composition, used a single set, X = $\{a,b,c,d\}$. Analysis of the worksheet results, followed by interviews, showed that all subjects, with both visualist and verbalist cognitive styles, and across both

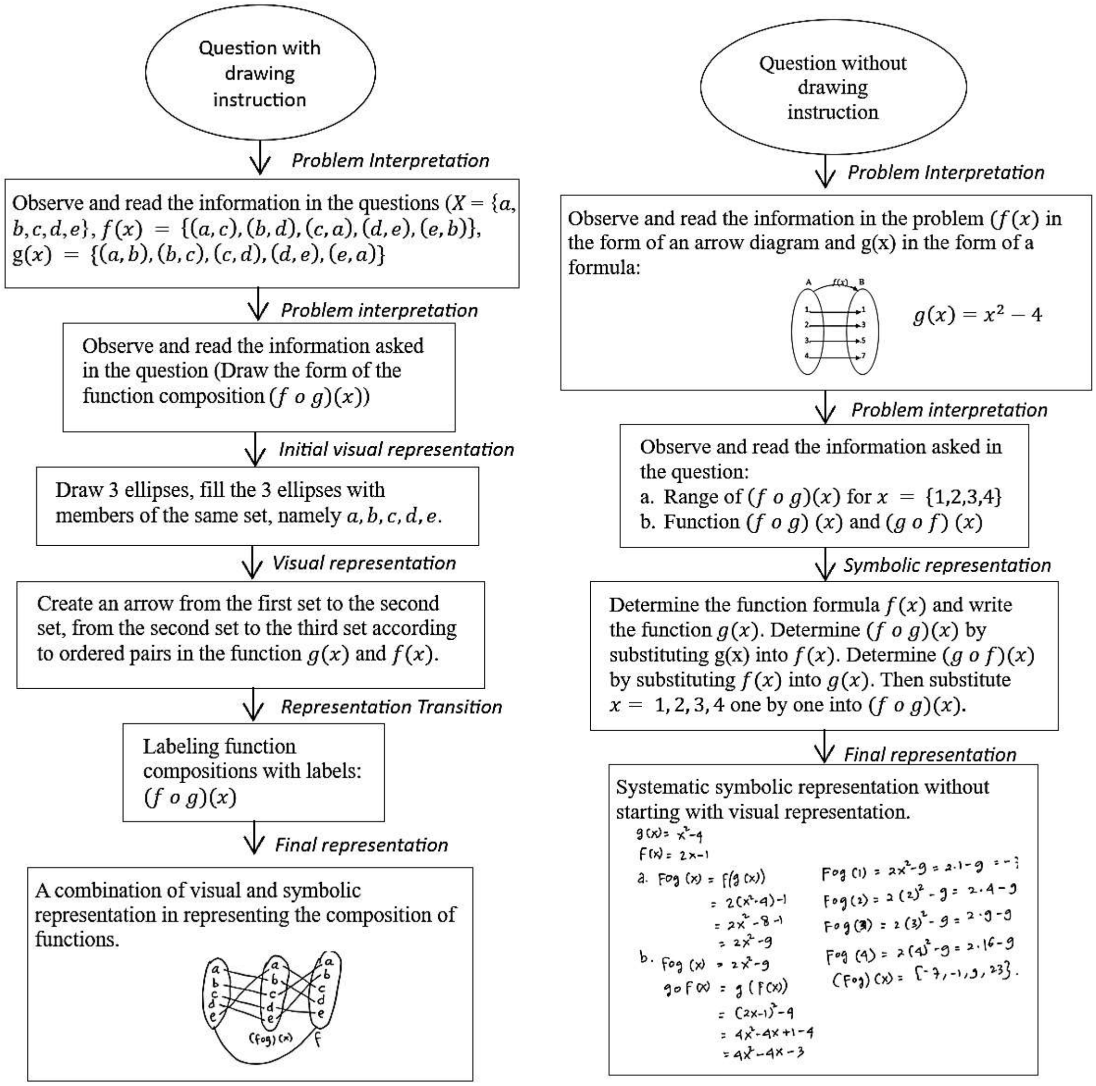

Figure 12. Flowchart of the thought process and representation of the subject female verbalizer

genders, experienced a conceptual error in determining the codomain. This error appeared consistently across all subjects. The errors were not visible on the worksheet, as they wrote the same set members for the domain, codomain, and range: $\{a, b, c, d, e\}$. This did not explain the differences between the domain, codomain, and range of the first, second, or third sets of a function composition they drew. To ensure their understanding of the domain, codomain, and range in function composition, researchers conducted in-depth interviews and obtained data indicating that all subjects stated that the codomain is the second set in a function composition. Interestingly, the pattern did not show significant differences based on cognitive style or gender, indicating that the source of the difficulty does not lie solely in individual characteristics. This pattern indicates the presence of cross-category conceptual barriers, likely related to conceptual understanding that remains procedurally oriented rather than conceptually oriented in connecting various mathematical representations. This suggests that conceptual understanding is related to its representation (Kowiyah et al., 2019).

The following is evidence of the subjects' work (seen in Figure 9) in determining the domain, codomain, and range. All subjects wrote the same thing, without any verbal explanation on the worksheet.

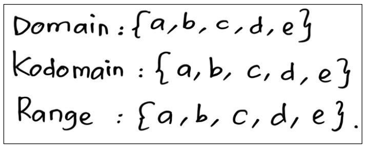

Figure 13. The subjects' work in determining the domain, codomain, and range

To confirm whether misconceptions existed, the researchers conducted interviews and found misconceptions regarding the concept of codomain. They stated that the codomain of a function composition is the second set. The following is an excerpt from the interview with the subjects.

R: Let's state the domain of f(g(x)).
S: The domain is set A.
R: Then what is the codomain?
S: The codomain is set B.
R: Why do you think the codomain is set B?
S: Because set B is related to the second function, g(x).
R: Then what is the range?
S: The range is set to C.

The finding that all subjects made errors in determining the codomain suggests that the barriers that emerged were not solely related to cognitive style preferences or gender, but were rooted in an incomplete conceptual understanding of function composition. This indicates that representational preferences do not automatically guarantee conceptual accuracy. In other words, when a basic conceptual understanding of a concept has not yet formed, various forms of representation tend to reproduce the same misconceptions. This finding suggests that differences in representation influenced by cognitive style may be less significant when students' conceptual foundations are not yet strong. Representations can facilitate understanding but cannot replace the construction of in-depth concepts. Therefore, the results of this study emphasize the importance of strengthening conceptual understanding as a foundation before optimizing representation variations based on students' cognitive characteristics.

Another finding related to the dominance of subjects in the negligible category (84% of participants) is significant and has important theoretical implications. Although the previous section explained that this contributed to the high number of these categories, this phenomenon can also be interpreted as indicating that most students do not exhibit extreme cognitive preferences. In other words, the majority of students in this study tended to have relatively balanced cognitive styles between visual and verbal. This finding raises questions about the relevance of the visualizer-verbalizer dichotomy in mathematics education, especially in populations with relatively homogeneous learning experiences. If most students fall somewhere in the middle of the spectrum, then approaches that overemphasize binary categorization of cognitive styles may not reflect students' cognitive realities. This aligns with the view that cognitive styles are better understood as a continuum rather than as distinct categories (Coffield et al., 2004). In the context of mathematics learning, this finding suggests that the flexibility of representation may outweigh any single preference, making instruction that integrates visual, symbolic, and verbal representations in a balanced manner potentially more relevant than approaches rigidly tailored to cognitive style categories.

CONCLUSION

This section will present conclusions about the profiles of visual, symbolic, and verbal representations used to solve mathematical problems. Visual representations by male and female visualist and verbalist students in solving mathematical problems are shown as pictures. Both male visualist and verbalist students use visual representations to represent information in the problem to facilitate understanding. In problems without picture instructions, both male and female visualist students continue to use visual representations before switching to symbolic representations. Although the intensity of visual representation use differs (visualizers more than verbalizers), there is no variation in the visual representations used by the subjects; the pictures used tend to be copied from their teachers, without further exploration.

Both male and female visualist students use visual representations to facilitate the transition to symbolic representations. Meanwhile, neither male nor female verbalist students use pictorial representations to transition to symbolic representations. Male visualist and verbalist students use symbolic representations less systematically than female visualist and verbalist students. In symbolic representation, visualist students make errors at the algebraic operation stage, resulting in symbol errors. Verbal students, on the other hand, do not make errors.

All subjects used verbal representations, but to a very small extent. Verbal representations were not used to explain the steps for completing the worksheets; they were only used to write the names of concepts. Visualists, both male and female, tended to point to pictures when explaining their work processes, whereas verbalists tended to point to symbols. Female visualists and verbalists were more coherent in explaining the step-by-step process of the work they had completed, while male visualists and verbalists were less coherent.

Another finding in this study was that conceptual misconceptions override differences in representation, that representational preferences become irrelevant if conceptual understanding is flawed, and that it is important to strengthen conceptual understanding as a foundation before optimizing representation variations based on students' cognitive characteristics and gender. In other words, when a basic understanding of a concept has not yet been conceptually established, various forms of representation tend to reproduce the same misunderstandings. Another finding in this study was the dominance of students categorized as Negligible, which implies that most students do not exhibit extreme cognitive preferences, and that representational flexibility may be more dominant than a single preference.

Based on the findings of this study, it is recommended that teachers facilitate the transition from visual to symbolic representations, as visual learners tend to rely on visual cues before moving on to symbolic ones. Teachers should emphasize the use of ordered, detailed symbols in learning because visual learners tend to be less systematic when solving symbolic problems. Teachers should provide more examples of diverse visual forms and provide opportunities for students to explore visual representations throughout the lesson. Verbal discussion activities should be reinforced to help students articulate solution strategies, particularly in writing step-by-step reasoning.

Based on the findings of this study, mathematics learning should prioritize strengthening conceptual understanding as the primary foundation for developing accurate representations. Given that conceptual errors occur universally across subjects, regardless of cognitive style or gender, teachers need to emphasize conceptual understanding before developing a variety of representations. Although students with a visualizing cognitive style tend to begin problem-solving with visual representations, learning should not simply encourage the transition from visual to symbolic; it should also facilitate flexible coordination among visual, symbolic, and verbal representations.

Furthermore, the dominance of the balanced (negligible) cognitive style category indicates that most students do not have an extreme preference for one particular type of representation. Therefore, learning strategies should not overemphasize rigid differentiation based on cognitive style categories, but rather provide a variety of representations and allow students to elaborate and develop their own visual representations, rather than simply copying the teacher's model. A deeper understanding of symbolic representations and verbal discussion activities should also be strengthened to help students articulate reasoning and coherent solution steps.

Recommendations for further research include exploring the relationship between conceptual understanding and representational fluency in greater depth. Research with a larger number of participants is needed to test the consistency of patterns across genders and cognitive styles. Future research could combine qualitative analysis with quantitative approaches.

■ DECLARATION OF GENERATIVE AI USAGE IN THE WRITING PROCESS

During the writing process, the author used Google Translate and ChatGPT to assist with language refinement and structural improvement. All AI-generated suggestions were critically reviewed, revised, and validated by the author. The author is solely responsible for the final content of the published article.

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