Jurnal Pendidikan MIPA
e-ISSN: 2685-5488 | p-ISSN: 1411-2531
URL: https://jpmipa.fkip.unila.ac.id/index.php/jpmipa
Jounal's DOI: http://doi.org/10.23960/jpmipa

Vol. 27, No. 1, 2026
Article ID: 1281
Pages: 562-586

Bridging the Gap in Mathematics Education: The Efficacy of Deep Learning and STEM-Based PBL on Critical Thinking

Farikha Aulia Rahma, Megita Dwi Pamungkas*, & Fadhilah Rahmawati
Department of Mathematics Education, Universitas Tidar, Indonesia

*Corresponding email: megitadwip@untidar.ac.id

Received: 06 February 2026 *Accepted: 16 March 2026* *Published: 07 April 2026*

Abstract: Critical thinking skills are important for students to face challenges in the 21st century, but their development in mathematics learning still has limitations because the approach tends to use conventional methods whose center is the teacher. This study aims to analyze the effectiveness of deep learning methods in improving students' critical thinking skills in mathematics. The deep learning method, implemented with the principles of meaningful, mindful, and joyful learning, used a STEM-based Problem-Based Learning model. This method focused on analyzing real-world problems, working in groups, group discussions, and applying trigonometry concepts to real-world problems through project-based activities. The study used a quasi-experimental pretest–posttest control group design involving 71 students in class X at SMA Negeri 2 Magelang for the 2025/2026 school year. The experimental group received instruction using the deep learning method through the Problem-Based Learning model, based on STEM and trigonometry material, while the control group used conventional instruction. The instrument used in this study is a validated critical-thinking test with five factors: interpretation, analysis, evaluation, inference, and explanation. The results of the paired-sample t-test indicated a significant improvement in both groups ($p < 0.001$). The experimental group achieved a mean gain score of 38.43, while the control group recorded a mean gain of 30.97 points from the pretest to posttest. The independent-samples t-test also indicated a significant difference in posttest scores between the two groups ($p = 0.024$). These results indicate that the deep learning method is more effective than traditional learning in improving critical thinking in mathematics learning.

Keywords: critical thinking, deep learning approach, mathematics education.

Article's DOI: https://doi.org/10.23960/jpmipa.v27i1.pp562-586

INTRODUCTION

Critical thinking is a cognitive process used to achieve deep understanding, analysis, and evaluation of the meaning of an idea and the relationships between concepts, in order to produce accurate and rational conclusions (Ennis, 1962). This ability plays a very important role in cognitive and intellectual development and is recognized as the primary foundation for equipping students to face the various challenges of the 21st century. In the Regulation of the Minister of Education and Culture of the Republic of Indonesia Number 5 of 2022 concerning Competency Standards for Primary and Secondary Education Graduates, it is emphasized that every primary and secondary education graduate is required to be able to guide the ability to think and act creatively, productively, critically, independently, collaboratively, and communicati vely. According to the regulations, critical thinking skill is not a substitute for theorists, indicating that curriculum requirements must be implemented in classroom instruction. This skill is very important because it will be able to enable students to conduct a thorough problem analysis, integrate new information with pre-existing knowledge, identify possible solutions, face challenges with confidence, assess various points of view, and convey understanding effectively (Ennis, 1996; Facione, 1990; Kolstø et al., 2024; Zhu, 2023).

Mathematics plays an important role in education, as it contributes to the development of critical thinking skills in children. Through mathematics learning, students can improve their skills, especially in problem-solving, develop independence in thinking, and hone analytical skills to conclude, assess arguments, and make decisions more systematically (Putri et al., 2025; Youssef, 2024). As a field with an abstract nature, mathematics can help students make more logical assessments grounded in real-world knowledge (Zhu, 2023). However, students who often have difficulty developing critical thinking are characterized by low motivation to learn, namely a lack of confidence in expressing opinions, limitations in critically evaluating knowledge, and a lack of meaningful class discussions (Khurma & El Zein, 2024; Kolstø et al., 2024). Such patterns indicate a persistent misalignment between curricular expectations and classroom practices.

Challenges in developing critical thinking skills can arise when the educational environment has not provided adequate opportunities for students to practice them. This condition is further exacerbated by the design of classwork and discussion activities that are not optimal for developing students' critical thinking skills (Dessie et al., 2023; Hattori et al., 2025; Monteleone et al., 2023). Sachdeva & Eggen (2021) emphasized that the limitations of learning methods also contribute to low success in developing students' critical thinking skills. Ideally, mathematics education is designed to create a learning environment that allows students: 1) to be actively involved in building knowledge through investigation and discovery activities; 2) achieve meaningful learning by linking abstract concepts with real-life contexts; 3) develop metacognitive skills through reflection and self-assessment; and 4) establishing cooperation in solving complex problems (Lestari et al., 2023). Murphy et al. (2021), Yoshida et al. (2023), and Novianty et al. (2023) provide evidence that the central learning method among students who focus on deep learning has proven more effective than conventional methods in improving critical thinking skills.

Unfortunately, mathematics learning in schools still relies a lot on conventional approaches whose center is the teacher so that in the development of critical thinking skills becomes less optimal so that this model is more indicative of delivering material with teachers who have an active role in explaining while students are less involved in finding factors through interaction (Alqahtani & Alhamami, 2024; Golightly & Sprenger, 2024; Ulfatun et al., 2023). Research by Woods and Copur-gencturk (2024) indicates that with a teacher-centered approach to reduce opportunities for pedagogic knowledge development when compared to an approach that is centered on students, this condition is also seen at SMA Negeri 2 Magelang, where learning is still dominated by teachers, and student involvement in the critical thinking process still has limitations.

To overcome these difficulties, the Deep Learning approach is implemented as a holistic solution. In this research, deep learning does not refer to artificial intelligence or neural network systems. Instead, it refers to the pedagogical concept of the deep approach to learning introduced by Marton and Säljö (1976a, 1976b). This approach emphasizes the importance of students prioritizing a deep understanding of the learning material, rather than merely memorizing texts or superficially remembering facts (Marton & Säljö, 1976a, 1976b). In its own application, this methodology is prepared based on three main components, namely: 1) mindful learning which encourages adaptive thinking skills and active involvement of students in the learning process; 2) meaningful learning which focuses on the relationship between new concepts and knowledge that has been previously possessed;

and 3) joyful learning which gives an overview of the learning experience as a very pleasant process and provides comfort for students (Ausubel, 1963; Langer, 2000; Montessori, 1912).

Using deep learning aims to increase students' active involvement through several key factors, including tailoring instruction to students' needs, connecting material to real-life contexts, and developing global competencies such as character, citizenship, collaboration, communication, creativity, and critical thinking. In addition, this method can support inquiry-based learning to build skills, knowledge, and confidence related to new knowledge from previous experiences, and to create a safe and immersive learning environment (Fullan et al., 2018; Levin, 2024; Zhao et al., 2023). Several studies indicate the effectiveness of deep learning methods across various areas of education. The flipped classroom model has been shown to increase students' cognitive engagement (Hava, 2021) and is used to improve learning outcomes, especially in vocational education (Jing et al., 2024). In addition, establishing Curriculum Design Coherence (CDC) plays an important role in improving students' epistemic understanding (McPhail, 2020).

Despite the abundance of research and literature on PBL, STEM integration, and deep learning pedagogy, several research gaps remain. First, many studies examine these instructional approaches separately, without integrating a deep pedagogical orientation. Second, empirical research on secondary-level mathematics education remains relatively limited. Third, previous studies often measure general academic achievement rather than specific, measurable dimensions of critical thinking. Addressing these gaps, the present study proposes an integrated instructional model that combines the deep approach to learning with STEM-based PBL in mathematics classrooms. This integration is designed to ensure that problem-solving activities are not merely procedural but grounded in meaningful conceptual understanding and reflective reasoning. By embedding deep learning principles at each stage of the PBL process, the model aims to create a structured environment that systematically fosters measurable critical-thinking skills.

This study contributes to the literature in three significant ways. First, conceptually, it integrates the deep pedagogical approach within a STEM-based PBL framework in mathematics education. Second, theoretical, it positions deep learning as a mediating construct for the development of specific critical thinking dimensions. Third, methodologically, it provides empirical evidence through a controlled comparative design that measures indicators of critical thinking rather than relying solely on general academic performance.

Research Questions

Based on the theoretical and identified research gaps, this study addresses the following research questions.

1. Does the integration of the deep learning approach within a STEM-based PBL model significantly improve students' critical thinking skills in mathematics?
2. Is there a significant difference in critical thinking performance between students taught using the deep learning method and those receiving conventional instruction?

METHOD

Participants

The population in this study included all students in class X of SMA Negeri 2 Magelang in the 2025/2026 school year. Administratively, the students were divided into eight classes. However, because it was not possible to randomly assign individual students to new groups within the school system, the sampling process used intact classes. To reduce potential differences due to teaching style, the sample was limited to classes taught by the same mathematics teacher. In this case, there were four grade X classes taught by the same teacher. These four classes were then considered as the sampling frame for the study.

Each of the four classes was first assigned a numerical code. A random number generator was then used to select two classes from the list. After the two classes were selected, an equivalence test was conducted using the students' pretest scores. The pretest data were analyzed through a homogeneity test to ensure that the two groups had relatively similar initial characteristics before the treatment was implemented.

Following this procedure, one of the selected classes was randomly assigned as the experimental group (n = 35), while the other served as the control group (n = 36). The participants were aged 15-16 years. The experimental group (Class X-7) comprised 16 male and 19 female students, while the control group (Class X-8) comprised 16 male and 20 female students. Students' prior academic achievement was also examined using their previous daily mathematics test scores. These scores were used as supporting baseline data to provide an overview of the students' initial academic level before the intervention.

Research Design and Procedures

This study employed a quasi-experimental quantitative research design. The research design used was a pretest–posttest control group design, as shown in Table 1.

Table 1. Research design

Class	Pretest	Treatment	Posttest
E	O_1	X	O_2
C	O_1	-	O_2

Source: Creswell & Creswell (2018)

Explanation:
E : Experimental group
C : Control group
O^1: Administration of pretest
X : Implementation of a deep learning approach in instruction
O^2 : Administration of the posttest

This study was conducted over a three-week intervention period consisting of six instructional sessions (90 minutes each). Treatment was given to the experimental group in the form of Deep Learning applications focused on trigonometry, accompanied by the use of its principles for problem-solving in real-life situations. At the initial stage, students were presented with contextual problems such as calculating angles of elevation and depression in practical situations, estimating the distance across a river, or determining the height of the school flagpole using angle measurements. These problems were intended to stimulate cognitive skills and encourage students to connect abstract trigonometric ratios with real-world phenomena.

During the conceptual exploration phase, students identified variables, constructed mathematical models using trigonometric ratios, and justified the selection of specific trigonometric principles. In the collaborative inquiry stage, students worked in small groups to design solution strategies, compare alternative approaches, and evaluate the logical consistency of their reasoning. The teacher facilitated metacognitive questioning to deepen understanding, encouraging students to reflect on assumptions, measurement accuracy, and the appropriateness of the mathematical model used. The fidelity of implementation was monitored using structured observation checklists completed during each session to ensure consistent application of deep learning principles throughout the intervention period. The control group, conversely, was instructed using conventional pedagogical techniques. The procedural steps for the research implementation are presented in the following flowchart.

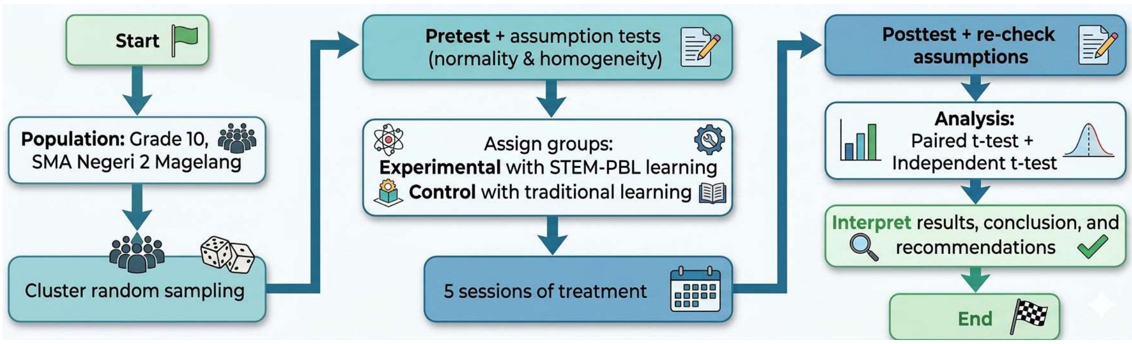


Figure 1. Research procedure flowchart illustrating the quasi-experimental design

Instrument

The instrument consisted of six essay questions contextualized in real-life trigonometric situations, such as determining heights or distances using angle measurements. This assessment was designed to measure the elements of critical thinking as formulated in Facione's (1990) framework. The indicators of critical thinking skills used in this study are presented in Table 2.

Based on these indicators, students were required not only to perform calculations but also to articulate assumptions, justify chosen trigonometric ratios, and evaluate alternative solution methods. The development process followed systematic stages, including specification of indicators based on theoretical constructs, item drafting, expert validation, revision, and pilot testing.

Table 2. Critical thinking ability indicators according to Facione (1990)

Aspect	Indicator
Interpretation	The ability to identify the meaning of the information provided
Analysis	The ability to recognize arguments or relationships between statements, questions, and concepts
Evaluation	The ability to assess the credibility of the arguments and claims presented
Inferences	The ability to draw logical conclusions based on the availability of information
Explanation	The ability to review or adjust initial conclusions based on understanding the information obtained

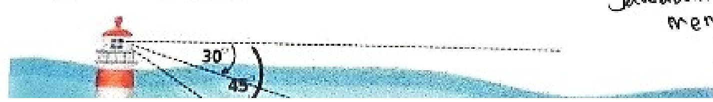
Prior to its use with the research sample, the test instrument was evaluated by three experts, including two mathematics education lecturers and a mathematics teacher, using Aiken's V coefficient to assess content validity. The Aiken's V values for individual items ranged from 0.89 to 1.33, indicating high content validity. The overall Aiken's V coefficient for the instrument was 0.83, which falls within the "high validity" category. The test results indicated that the instrument was appropriate for use with certain modifications. These revisions primarily involved clarifying wording, improving the precision of mathematical terminology, and refining contextual problem statements to enhance clarity and alignment with the intended critical thinking indicators. All suggested modifications were incorporated prior to the pilot testing stage to ensure the instrument's conceptual accuracy and comprehensibility. One validator offered suggestions to improve the clarity of the wording of the question items, as shown in Table 3.

Table 3. Recommendations for enhancing the inquiry from the validator

Original document

e. Untuk mendapatkan hasil pengukuran yang akurat, anak tersebut bergerak mendekati pohon sehingga jaraknya menjadi 6 m dan setelah diukur lagi sudut pandangnya menjadi 45°. Apakah hasil tinggi pohon sama atau berbeda? Jika berbeda, apakah hal tersebut menunjukkan sebuah masalah atau memberikan wawasan baru? → dapat diperjelas agar jawaban siswa mengarah pada jawaban yang diharapkan.

Perhatikan gambar di bawah ini!



Convert

e. Untuk mendapatkan hasil pengukuran yang akurat, anak tersebut bergerak mendekati pohon sehingga jaraknya menjadi 6 m dan setelah diukur lagi sudut pandangnya menjadi 45°.*Apakah hasil tinggi pohon sama atau berbeda? Jika berbeda, apakah hal tersebut menunjukkan sebuah masalah atau memberikan wawasan baru?*

- *dapat diperjelas agar jawaban siswa mengarah pada jawaban yang diharapkan*

Translate

e. To obtain accurate measurement results, the child moved closer to the tree, reducing the distance to 6 m. After measuring again, the viewing angle became 45 degrees. Is the height of the tree the same or different? If it is different, does that indicate a problem or provide new insights?

- It can be clarified so that the students' answers lead to the expected answer.

The question in the exam has been revised in compliance with the proposed enhancements as follows:

Table 4. Enhancement of inquiries based on validator recommendations

Original document

e. Jika Rendi bergerak mendekati pohon sehingga jarak antara Rendi dan pohon menjadi 6 m dan dia mengukur sudut elevasi menjadi 45°, apakah hasil perhitungan tinggi pohon akan sama dengan hasil sebelumnya? Jika berbeda, apa yang dapat disimpulkan dari perbedaan tersebut?

Convert

e. Jika Rendi bergerak mendekati pohon sehingga jarak antara Rendi dan pohon menjadi 6 m dan dia mengukur sudut elevasi menjadi 45°, *apakah hasil perhitungan tinggi pohon akan sama dengan hasil sebelumnya? Jika berbeda, apa yang dapat disimpulkan dari perbedaan tersebut?*

Following the revision process informed by expert suggestions, the instrument was piloted with 35 students outside the research sample to assess

Translate
e. If Rendi moves closer to the tree so that the distance between Rendi and the tree becomes 6 m and he measures the angle of elevation to be 45°, will the calculated height of the tree be the same as the previous result? If they differ, what can be concluded from those differences?

its reliability. This pilot test was conducted in accordance with the coefficient criteria proposed by Arikunto (2018). A test instrument is regarded as having a high level of reliability if $r_{xy} > 0.60$. The reliability of the instrument was analyzed using Cronbach's Alpha to determine the internal consistency of the items presented in Table 5.

Table 5. Reliability coefficients of the instrument

Test type	Number of items	Cronbach's Alpha	Interpretation
Pretest	3	0.9268	Very high
Posttest	3	0.9238	Very high

As shown in Table 5, the Cronbach's Alpha coefficients for both the pretest and posttest exceed 0.90, indicating very high internal consistency. An analysis of item difficulty and discrimination power was also conducted to assess the feasibility of each item. The results of this analysis are presented in Table 6.

Based on Table 6, the item analysis results show distinct characteristics for each question in both the pretest and posttest instruments. For the

Table 6. Item analysis results of the pilot test

Item	Difficulty Index	Discrimination Index	Decision
Pretest 1	0.64	0.50	Retained
Pretest 2	0.77	0.23	Eliminated
Pretest 3	0.75	0.41	Retained
Posttest 1	0.64	0.47	Retained
Posttest 2	0.68	0.32	Eliminated
Posttest 3	0.64	0.64	Retained

pretest, item 1 obtained a difficulty index of 0.64, which falls into the moderate category, and a discrimination index of 0.50, which is classified as good. Similarly, item 3 showed moderate difficulty, with a difficulty index of 0.75 and a discrimination index of 0.41, indicating good discriminative power. These results suggest that both items function well in measuring students' abilities and can distinguish between students with higher and lower performance levels. Therefore pretest items 1 and 3 were retained in the final instrument.

In contrast, pretest item 2 had a difficulty index of 0.77, indicating that the item was relatively easy for students. Its discrimination index was 0.23, which only falls into the fair category. This relatively low discrimination value suggests that the item was less effective at differentiating students by ability level. For this reason, pretest item 2 was not included in the final version of the instrument.

A similar pattern can be observed in the posttest items. Posttest item 1 showed a moderate difficulty level (0.64) and good discrimination power (0.47). Posttest item 3 also showed a moderate difficulty level (0.64) but with a higher discrimination index of 0.64. Both items, therefore, met the required criteria and were retained. On the other hand, posttest item 2 had a moderate difficulty index (0.68) but a relatively lower discrimination index of 0.32. Although the value still falls within an acceptable range, it was not considered as strong as the other items and was therefore excluded from the final instrument.

Based on these considerations, the final instrument used in both the pretest and posttest consisted of two essay questions, namely items 1 and 3. These items were selected because they demonstrated appropriate levels of difficulty and satisfactory discrimination power in the pilot test. The reliability of the selected items was then examined, and the results are presented in Table 7.

After refining the items based on difficulty and discrimination analysis, the reliability coefficients increased further, demonstrating excellent reliability. These findings indicate that the instrument consistently measures students' critical thinking skills in trigonometry.

Table 7. Reliability coefficients of the final selected instrument

Test type	Number of items	Cronbach's Alpha	Interpretation
Final selected items (pretest)	2	0.9603	Excellent
Final selected items (posttest)	2	0.9529	Excellent

Data Analysis

After the pretest results are obtained, the posttest is conducted in the final stage of the research. The results are then used as the primary basis for testing research hypotheses. Prior to further analysis, the data from the mathematical critical thinking exam results must undergo tests of normality and homogeneity to confirm that the requirements for analysis are met. Once the criteria for both tests are met, the next step is to conduct a paired-samples t-test to draw conclusions from the study. The theories suggested in this study are as follows.

$H_0: \mu_1 \leq \mu_2$

(The implementation of the deep learning approach in mathematics education does not yield a notable enhancement in critical thinking abilities among 10th-grade students at SMA Negeri 2 Magelang.)

$H_1: \mu_1 > \mu_2$

(The implementation of the deep learning approach in mathematics education has markedly enhanced critical thinking abilities among 10th-grade students at SMA Negeri 2 Magelang.)

The testing conditions indicate that the null hypothesis is rejected if the *sig.*(2-tailed) < *a,* where the significance level a = 5% and the degrees of freedom (*df*) = *n* - 1, with *n* being the number of data pairs.

In addition to examining the overall critical thinking scores, this study also analyzed students' performance on each indicator. Differences between the experimental and control groups were tested using the Mann-Whitney U test, and pretest scores were included to account for students' initial levels of critical thinking.

Beyond the statistical analysis, students' written responses were also examined to better understand how they expressed their reasoning. A qualitative content analysis was conducted using a deductive coding approach. Each response was reviewed to identify how the indicators appeared in students' answers and to observe the depth and clarity of their reasoning.

To maintain consistency in the analysis, all responses were read several times and then coded using criteria derived from the operational definitions of each indicator. During the coding process, responses exhibiting similar reasoning patterns were grouped to identify common tendencies across students' answers. This procedure helped ensure that the analysis reflected patterns found across the dataset rather than relying on a few selected responses.

RESULT AND DISCUSSION

Implementation Fidelity of the Deep Learning Model

Structured observation checklists were used during the learning process in the experimental class to monitor the implementation of the deep learning model throughout the intervention. Observations were conducted in each session to document how the model's main components appeared during classroom activities. The checklist focused on several key aspects of deep learning, including meaningful learning, mindful learning, and joyful learning, which represent the central features of the instructional approach applied in this study.

Through this structured observation, the researcher was able to record more systematically how the planned learning activities were carried out by the teacher and how students responded to them during the lesson. Each indicator on the checklist was designed to capture observable classroom practices aligned with the principles of deep learning. These included connecting mathematical concepts with real-life situations, guiding students to examine problems more carefully, encouraging reflection on their reasoning, and supporting active participation in group discussions and collaborative tasks. A summary of these observation results is presented in Table 8.

Table 8. Implementation fidelity of the deep learning approach based on the observation checklis

Deep Learning Component	Observation Indicators	M1	M2	M3	M4	Total	Percentage (%)
Meaningful learning	Learning materials are connected to students' prior knowledge or experience	✓	✓	✓	✓	4	100
	The examples or problems used are contextual and related to students' daily life	✓	✓	✓	✓	4	100
	Learning activities facilitate students to independently discover concepts or meaning from the material		✓	✓	✓	3	75
	Knowledge acquired by students is applied through assignments, case studies, or learning projects	✓		✓	✓	3	75
	The learning material helps students understand the relationship between the concepts learned and real-life situations in their environment	✓	✓	✓	✓	4	100
Mindful learning	Learning objectives are clearly communicated so that students understand the direction of the learning activities	✓	✓	✓	✓	4	100
	Guiding questions are provided to encourage students to think critically	✓	✓	✓	✓	4	100
	Students are actively involved in discussions or activities that require thinking processes	✓	✓	✓	✓	4	100
	Reflection activities are conducted to help students understand the learning process and outcomes	✓	✓	✓	✓	4	100

	Students demonstrate awareness of their thinking process by explaining their reasoning or steps in solving problems		✓		✓	2	50
Joyful learning	The classroom atmosphere during the learning process is positive and enjoyable	✓	✓	✓	✓	4	100
	The teaching methods or learning media used are varied and attract students' attention	✓		✓	✓	3	75
	Students show enthusiasm and engagement during learning activities	✓		✓	✓	3	75
	Learning activities involve interactive elements such as educational games, simulations, or ice breakers	✓	✓	✓	✓	4	100
	Positive interactions among students are observed during learning activities, creating a comfortable learning environment	✓		✓	✓	3	75
					Total:	53	83.33

Explanation:
M1: Meeting 1
M2: Meeting 2
M3: Meeting 3
M4: Meeting 4

Based on Table 8, observations from four meetings indicate that the learning process was implemented well overall, with an overall achievement level of 83.33% across all observed indicators. Most indicators of meaningful, mindful, and joyful learning appeared consistently throughout the lessons. The material was often linked to students' prior knowledge and explained using examples from daily life. The learning objectives were clearly stated at the beginning of each meeting, and guiding questions were used to encourage students to think more critically. Students were also actively involved in discussions and other learning activities, and reflection sessions were held at the end of lessons to help them review what they had learned.

Interactive activities such as educational games, simulations, and ice-breakers were used to create a more engaging learning atmosphere. These activities helped maintain students' participation during the lessons. However, the indicator related to students' ability to explain the reasoning when solving problems showed the lowest result, appearing in only two meetings. This suggests that although students were able to solve problems, they were not yet used to explaining their thinking process.

Another point worth noting is the decline in several indicators during the second meeting. Students appeared less enthusiastic, and interaction was also lower compared to other meetings. This may be related to the timing of the lesson, which took place in the afternoon after several previous classes. Many students seemed tired, which affected their focus and participation. Even so, the learning process improved in the following meetings. Overall, the results indicate that the deep learning approach was implemented well, although students still need more opportunities to practice explaining their reasoning when solving problems.

Differences Between the Experimental and Control Groups

The research findings demonstrate that students' critical thinking skills in trigonometry, when taught using a deep learning approach, are notably superior to those taught via conventional approaches. Evidence for this was obtained from students' pretest and posttest results. Prior to conducting parametric analysis, the students' score data were assessed for normality and homogeneity of variance. This study uses the Shapiro–Wilk test to assess the normality of the data distribution at the 5% significance level. The results indicated that the pretest of the experimental group had a significance value of 0.210 $(p > 0.05)$ while the control group had a significance value of 0.369 . In the posttest data, the experimental group obtained a significance value of 0.055 and the control group 0.076 .

The next stage is carried out to be able to test homogeneity in both classes. Based on the analysis of the pretest scores, the significance value was 0.125 , indicating that the variance between the two groups was homogeneous. Similarly, the results of the posttest score analysis with a significance value of 0.553 , indicate that the assumption of homogeneity of variance was met and that parametric statistical analysis could be appropriately conducted. The results of the test indicate that students' critical thinking ability has the same variance, or is homogeneous.

The results of the Shapiro-Wilk test indicated that all pretest and posttest data were normally distributed (). Levene's test further confirmed the homogeneity of variances (). Therefore, the assumptions for parametric analysis were satisfied. Once the requirements of normality and homogeneity are met, parametric statistical analysis can be resumed.

Statistical analysis was conducted using an independent-samples t-test to assess the significance of differences in the effectiveness of deep learning approaches versus conventional learning. This test specifically focused on the differences in posttest results between the experimental and control groups. The test results are then summarized and presented in Table 9.

Table 9. Independent samples t-test result

Comparison	t	df	p	Cohen's d
Experimental vs Control	2.305	69	0.024	0.55

The independent-samples t-test showed a statistically significant difference in posttest scores between the experimental and control groups (t = 2.305, p =0.024). The effect size, calculated as Cohen's d, was 0.55, which falls in the moderate category. This result indicates that the learning approach applied in this study had a measurable impact on students' critical thinking skills. Since the null hypothesis was rejected at the 0.05 significance level ($p < 0.05$), the improvement observed in the experimental group can reasonably be associated with the instructional treatment that was implemented. This improvement reflects a meaningful development in students' ability to analyze and justify mathematical reasoning within trigonometric contexts.

Furthermore, both groups were statistically equivalent at the pretest stage, and the assumptions of normality and homogeneity were met. The posttest difference provides stronger internal validity for attributing the improvement to the treatment. Therefore, the deep learning approach can be considered an effective instructional strategy for fostering critical thinking skills in secondary mathematics education.

Improvement Within Each Group

To provide a clearer illustrate the pattern of students' learning progress, scatter plots were constructed to visualize the distribution of individual pretest and posttest scores in both the experimental and control classes in the following Figure 2.

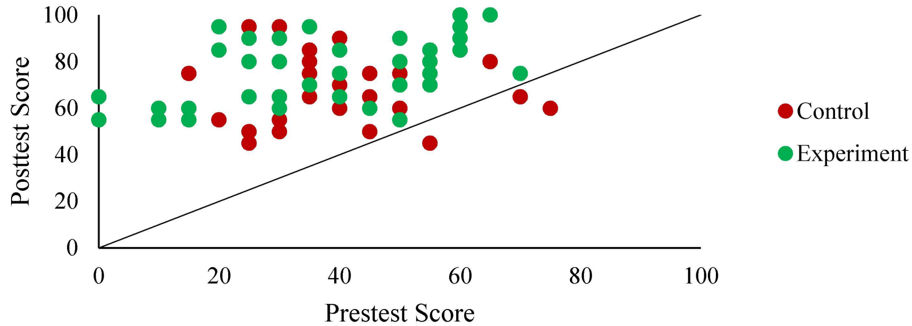


Figure 2. Scatter plot comparison of pretest and posttest scores for the experimental and control groups

To provide a clearer picture of students' learning progress, a scatter plot was used to compare the pretest and posttest scores of both groups, as shown in Figure 2. The diagonal line represents equal values between pretest and posttest scores. Points above this line indicate that students scored higher on the posttest than on the pretest. In the figure, the red points represent students in the control group, while the green points represent students in the experimental group.

As shown in Figure 2, most data points from both groups lie above the diagonal line, indicating that many students improved after the learning process. However, the green points representing the experimental group tend to be positioned higher above the diagonal line than the red points from the control group. This suggests that the increase in scores was generally greater among students in the experimental class. This visual pattern is consistent with the statistical analysis, which indicates that the deep learning approach implemented in the experimental group had a stronger effect on the development of students' critical thinking skills than the conventional learning applied in the control group.

To complement the scatter plot, a box plot was also used to compare the distributions of pretest and posttest scores for both groups, as shown in Figure 3.

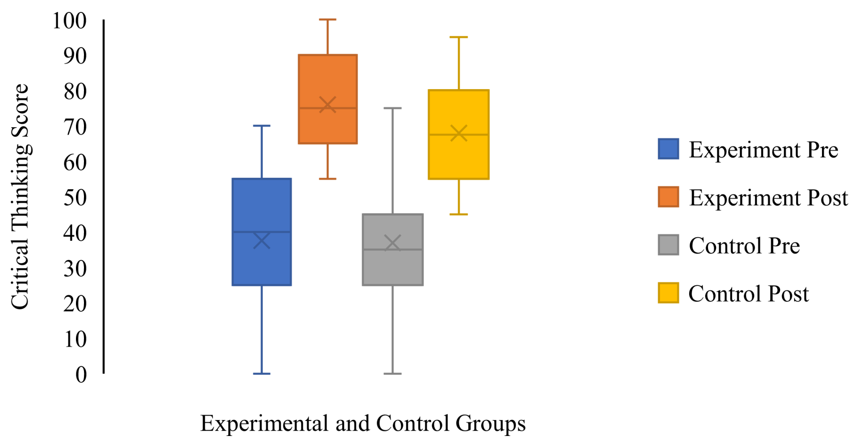


Figure 3. Boxplot comparison of pretest and posttest critical thinking scores between the experimental and control groups

Figure 3 illustrates how the scores are distributed by showing the median, the interquartile range, and the overall spread of the data. From the box plot, it is evident that the experimental group's median score increased noticeably from the pretest to the posttest. The control group also shows an improvement, but the shift in the median and the spread of scores is not as large as in the experimental group. This difference suggests that the students in the experimental class tended to experience a greater improvement after the learning intervention.

To further observe how students' scores changed individually, a slope graph was used to connect each student's pretest and posttest scores, as presented in Figure 4.

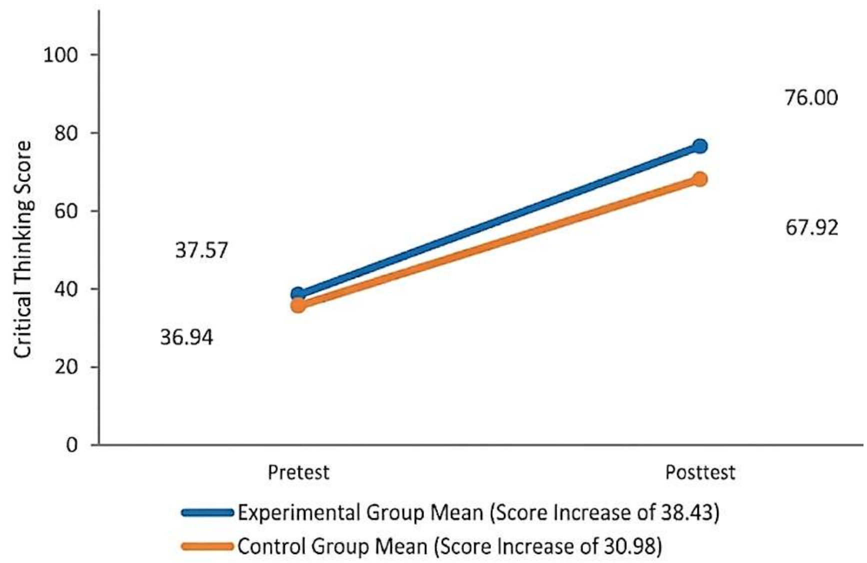


Figure 4. Individual score changes from pretest to posttest in the experimental and control groups

Figure 4 presents a slope graph showing the change in students' critical thinking scores from pretest to posttest for both the experimental and control groups. In the figure, the thin blue line represents individual students in the experimental group, while the thin yellow lines represent those in the control group. Most of the lines in both groups slope upward, indicating that students' critical thinking scores generally improved after the learning activities. Even so, the increase appears stronger in the experimental group. The mean score in this group rose from 37.57 in the pretest to roughly 76 in the posttest. In comparison, the control group's mean score increased from 36.94 to 67.92. Overall, this pattern suggests that students in the experimental group tended to show greater improvement in their critical thinking skills than those in the control group.

Furthermore, a paired-samples t-test was conducted to assess performance improvement within each group. This test aims to assess the development of students' abilities in each group from the pretest stage to the posttest. The test results are then summarized and presented in Table 10.

Table 10. Paired samples t-test results for pretest and posttest scores

Group	t	df	p	Cohen's d
Experimental	−13.10	34	< 0.001	2.21
Control	−8.67	35	< 0.001	1.44

Table 10 shows that both groups experienced a significant increase in their scores from pretest to posttest. In the experimental group, the improvement was substantial ($t = 13.10$, $p < 0.001$), with a Cohen's d of 2.21, indicating a very large effect size. The control group also showed a significant improvement ($t = 0.87$, $p < 0{,}001$). The calculated Cohen's d was 1.44, which also falls into the very large effect size category. Although both groups improved over time, the magnitude of the improvement was noticeably greater in the experimental group. This result suggests that the deep learning intervention contributed more strongly to the development of students' critical thinking skills.

Analysis of Improvement in Each Critical Thinking Indicator

To examine the improvement in students' critical thinking in more detail, each indicator was analyzed separately. The improvement in each indicator was measured using gain scores, calculated as the difference between posttest and pretest scores. Before conducting further analysis, the data distribution was first checked. The results showed that several indicators did not meet the normality assumption. For this reason, a non-parametric test was used. The Mann-Whitney U test was then conducted to compare the gain scores of the experimental and control groups for each indicator. Since the analysis involved multiple tests, a Bonferroni correction was applied to minimize the possibility of Type I error. After the correction, the level of significance was set at $a = 0.01$. The results of the Mann-Whitney U test for each indicator are presented in Table 11.

The findings indicate significant differences between the experimental and control groups in the interpretation ($p = 0.002$) and inference ($p = 0.003$) indicators, as both p-values are lower than the adjusted significance level. This result suggests that the learning intervention helped students improve their ability to interpret information and draw conclusions from the data or problem situations provided. These abilities are important in understanding contextual problems and identifying relevant information before deciding on a possible solution.

Table 11. Summary of Mann-Whitney U Test results for the improvement of each critical thinking indicator

Indicator	U	Z	p-value	Result
Interpretation	396.00	-3.053	0.002	Significant
Analysis	577.50	-0.633	0.527	Not significant
Evaluation	624.50	-0.072	0.942	Not significant
Inference	372.50	-2.988	0.003	Significant
Explanation	554.00	-0.920	0.358	Not significant

On the other hand, no significant differences were found in the analysis ($p = 0.527$), evaluation ($p = 0.942$), and explanation ($p = 0.358$) indicators. Although students in both groups showed some improvement in these aspects, the differences between the groups were not statistically significant. This means that the intervention primarily supported the development of interpretation and inference skills, while improvements in analysis, evaluation, and explanation were similar across both groups.

Deep Learning in Developing Critical Thinking

Math learning not only focuses on the accuracy of the final answer but also gives direction for the development of a mindful, meaningful, and joyful learning environment. The research conducted in both classes serves as a guide to applying a problem-solving learning paradigm grounded in Science, Technology, Engineering, and Mathematics (STEM) to strategically improve students' critical thinking skills. Therefore, the experimental group received treatment through the application of the STEM-based Problem-Based Learning (PBL) model to create a more contextual, meaningful, and joyful learning experience. Meanwhile, the control group continued to use conventional learning methods.

The initial stages of learning in both classes indicate that students are asked to identify trigonometric problems in daily life and develop ways to overcome them. This stage allows students to connect the basic trigonometry knowledge they already have to the problem section, improving their understanding of the material. However, students in the experimental group reported higher enthusiasm because they were able to follow a new STEM-based Project-Based Learning model. The learning activities can be defined as the determination of the method, Deep Learning, entirely through the PBL model, whose basis is STEM.

To ensure that the claims of meaningful, mindful, and joyful learning were empirically grounded, these aspects were systematically measured during the intervention. A structured observation sheet was used to capture students' learning experiences. Meaningful learning was evident in students' ability to connect trigonometric concepts to real-life situations and to explain their reasoning clearly. Mindful learning was reflected in their metacognitive behaviors, such as questioning assumptions, evaluating alternative solutions, and reflecting on their strategies. Joyful learning was observed through active participation, collaborative interaction, and positive enthusiasm during problem-solving activities. The results of this observation are summarized in Table 12.

Table 12. Analysis of the implementation of meaningful, mindful, and joyful learning in instruction

Aspect	Learning Activities
Meaningful Learning	Students determine the heights of the flagpole and basketball hoop using a rudimentary homemade clinometer.
Mindful Learning	Assignment of roles within the group (observer, distance measurer, and data recorder). Group discussion to ascertain the object's height.
Joyful Learning	Practical activities include constructing a clinometer, taking measurements outdoors, collaborating in small groups, and presenting results.

The application of deep learning in project-based learning to measuring the height of an object using a simple clinometer indicates a high level of student engagement. In the context of meaningful learning itself, students can apply trigonometry to determine the heights of the flagpole and basketball hoop. Through this activity, the relationship between mathematical concepts and real-world problems in the learning environment can be built, improving students' conceptual understanding and making learning materials more relevant to their daily lives. This practical activity provides a hands-on learning experience that supports meaningful learning, making the resulting understanding more profound than that from memorization-focused activities (Kostiainen & Pöysä-Tarhonen, 2025).

With this practical activity, students can connect new knowledge to real experiences and aspects of life, making learning more meaningful and relevant. Activities that solve real problems can help students understand the learning process and become more aware of their strengths and weaknesses (Rincón et al., 2024). In addition, connecting initial knowledge to real problems has also been associated with increased understanding and improved memory for the material (Tulak et al., 2024).

The division of tasks within groups encourages students who are actively involved to engage in critical thinking by taking responsibility for their roles and participating in problem-solving discussions. Group work and role clarity indicate that student-centered learning can support mindful learning. It is in this environment that students can be encouraged to develop self-awareness, manage emotions well, and have social skills (Li, 2025). Mindful learning involves actively managing cognitive, emotional, and social processes rather than passively accepting knowledge (Wang & Zhang, 2019).

Understanding the role of the group can help students in accepting new experiences, managing emotions well, and being able to be effectively involved in learning activities, so that with the active involvement of students in the group, they can support awareness to contribute and carry out their responsibilities to achieve goals simultaneously (Feriyanto & Anjariyah, 2024). In addition, having role awareness makes it easier for students to understand and accept each member's contributions without causing pressure, thereby increasing learning effectiveness (Zuo & Wang, 2023).

Meaningful, mindful learning elements have given rise to this activity, which aligns with Piaget's constructivist theory. This theory highlights that meaningful learning occurs when students actively build their own understanding through direct experience rather than simply receiving information from the teacher (Piaget, 1959). In this project, students first explored trigonometric concepts and then applied them to estimate the height of real objects using a simple clinometer. This activity required them to rethink and reorganize their prior knowledge of trigonometric ratios and angle relationships within a real-life context. As students compared their initial assumptions with actual measurement results, they sometimes encountered discrepancies that prompted reflection and revision of their reasoning. This process reflects Piaget's concepts of assimilation and accommodation, where new experiences are integrated into existing cognitive structures. In this way, the observed improvement in critical thinking skills can be explained by the active, reflective knowledge construction process fostered through contextual application, which helped students develop a deeper, more durable conceptual understanding.

Çibukçiu (2025) revealed that constructivism can encourage students to use their own strategies to solve mathematical problems, thereby increasing learning motivation and independence and fostering critical thinking and creativity. This theory can also be implemented through an active learning model, enabling students to build their own knowledge through interaction with the existing learning environment (Mariam et al., 2026). Therefore, implemented learning can increase students' overall cognitive, emotional, and behavioral involvement.

Conversely, Vygotsky (1978) indicates that learning occurs through social interaction and dialogue in a group, as seen in the practice of implementation, mindful learning, where role sharing and active discussion can support the simultaneous exchange of ideas, communication, and understanding. Through this interaction, students learn within the Zone of Proximal Development (ZPD), a condition in which they can achieve a higher level of understanding than with the help of peers or teachers.

In addition to meaningful and mindful learning, joyful learning is also clearly evident through practical activities, such as creating a simple clinometer, where learning outside the classroom involves collaborating in groups and presenting results. This activity takes place in a relaxed and fun learning atmosphere. These creative activities can increase curiosity. This can be seen in the students' enthusiasm for problem-solving. Students can also convey that learning mathematics becomes more fun and interesting, so that it does not make you bored, because there is a new learning experience.

Experience-based activities can create a fun learning atmosphere that helps students feel comfortable and motivated to be actively involved (Feriyanto & Anjariyah, 2024). This supportive environment can help students understand the material, deepen their understanding, and encourage their courage to speak up. This experience-based learning invites students directly into fun activities, making learning more real and lasting longer (Jeet & Pant, 2023). In addition, implementation activities can provide opportunities for students to create imaginative works and think independently, ultimately increasing their confidence and satisfaction with learning (Cronqvist, 2021).

Elements of joyful learning. This learning aligns with humanistic thinking, in which Maslow argues that learning is meaningful when students experience emotional changes and a willingness to learn through deep experiences, rather than focusing solely on cognitive abilities (Maslow, 1970). This exploratory activity fosters a sense of self-satisfaction in students, increasing their motivation to learn and fostering curiosity, responsibility, and cooperation, which, in turn, support their development. This is in line with the views of Carl Rogers, who emphasized the importance of a warm, supportive learning atmosphere (Rogers, 1961). In practice, where teachers have an important role as teachers and facilitators by creating a conducive learning environment, can appreciate students' contributions, and encourage them to manage their own learning process, so that a humanistic approach in education can emphasize overall student development, activation of self-potential, and positive interaction between teacher and student (Li, 2025).

Consequently, all three dimensions of deep learning have been comprehensively executed, demonstrating the applicability of constructivist and humanistic theories. This stems from an educational approach that positions students as active agents in constructing information through experiential learning, social interaction, and critical thinking. To further explore differences in students' response quality, a deductive content analysis was conducted using predefined critical-thinking aspects. The coding framework used in this analysis is presented in Table 13.

Table 13. Coding scheme for students' critical thinking responses

Aspect	Coding Criteria
Interpretation	Correctly recognizes given data and context
Analysis	Connects trigonometric concepts logically
Evaluation	Provides justified reasoning
Inference	Produces logical and consistent conclusions
Explanation	Gives structured and systematic reasoning

To provide a more concrete illustration of the differences in students' critical thinking development between the two groups, examples of students' work from the experimental and control groups are presented in Table 14.

Table 14. Comparison of student performance outcomes in the inference aspect

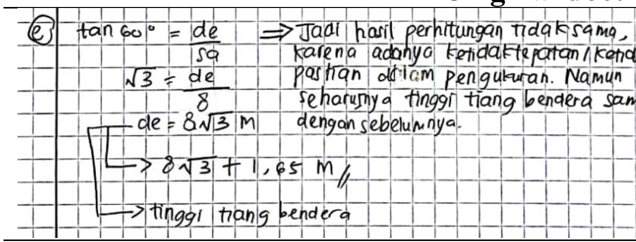
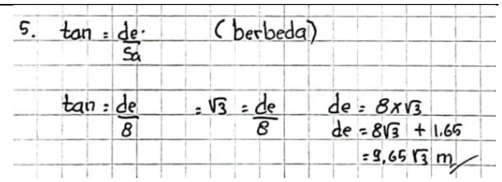
Original document	
	
Convert	
$\tan 60° = \frac{de}{sa}$ $\sqrt{3} \quad = \frac{de}{8}$ $de = 8\sqrt{3}\, m$ ➢ $8\sqrt{3} + 1.65\, m$ ➢ *Tinggi tiang bendera* *Jadi, hasil perhitungan tidak sama, karena adanya ketidaktepatan/ketidakpastian dalam pengukuran. Namun, seharusnya tinggi tiang bendera sama dengan sebelumnya.*	$tan = \frac{de}{sa}$ (berbeda) $tan = \frac{de}{8} = \sqrt{3} = \frac{de}{8}$ $de = 8 \times \sqrt{3}$ $de = 8\sqrt{3} + 1.65$ $= 9.65\sqrt{3}\, m$
Translate	
$\tan 60° = \frac{opposite}{adjacent}$ $\sqrt{3} \quad = \frac{opposite}{8}$ $opposite = 8\sqrt{3}\, m$ ➢ $8\sqrt{3} + 1.65\, m$ ➢ Height of the flagpole So, the calculated results differ due to inaccuracies or uncertainties in the measurements. However, the height of the flagpole should be the same as before.	$tan = \frac{opposite}{adjacent}$ (different) $tan = \frac{opposite}{8} = \sqrt{3} = \frac{opposite}{8}$ $opposite = 8 \times \sqrt{3}$ $opposite = 8\sqrt{3} + 1.65$ $= 9.65\sqrt{3}\, m$

Table 14 indicates a difference in the ability to think critically, particularly in inference. The analysis of point e in the experimental group and point 5 in the control group indicated that students in the experimental group could perform precise calculations using the available numerical data. This ability indicates that they can combine their knowledge into a coherent array of arguments and relate facts to trigonometric theory in context, but, conversely, the answers in the control group still indicate weaknesses in drawing conclusions, inconsistencies in calculations, and a lack of logical reasoning. These results are consistent with the findings of Rehman et al. (2023), who found that active learning can help students analyze data, draw conclusions, express ideas, and assess claims in real-world problems.

On the other hand, the students' answers in the control group are more likely to exhibit mechanical and procedural properties, indicating that their understanding remains shallow, as they often immediately use formulas without going through the necessary analysis stages in solving the problem. This pattern indicates that the habit of imitating learning examples without encouragement to analyze, evaluate, and develop problem-solving strategies independently results in students finding it difficult to recognize important information by connecting concepts to context and to explain the reasons for answers to questions that require critical thinking. These results are in line with research by Arisoy & Aybek (2021), which indicates that using active learning methods is more effective at improving critical thinking than conventional approaches.

To obtain a more systematic picture of how students demonstrated the critical thinking indicators, a qualitative analysis was also conducted. Students' responses were coded using a rubric based on the predetermined critical thinking indicators. Each response was then classified into one of three levels of achievement, such as not evident, partially evident, or comprehensively evident. After the coding process, the frequencies and percentages of students in each category were calculated for each critical thinking indicator in both the experimental and control groups. The summary of this distribution is presented in Table 15.

Table 15. Distribution of students' critical thinking aspects

Aspect	Level	Experimental group n (%)	Control group n (%)
Interpretation	Not evident	2 (6%)	5 (14%)
	Partially evident	11 (31%)	11 (31%)
	Comprehensively evident	22 (63%)	20 (56%)
Analysis	Not evident	1 (3%)	5 (14%)
	Partially evident	13 (37%)	19 (53%)
	Comprehensively evident	21 (60%)	12 (33%)
Evaluation	Not evident	3 (9%)	6 (17%)
	Partially evident	4 (11%)	9 (25%)
	Comprehensively evident	28 (80%)	21 (58%)
Inference	Not evident	3 (9%)	6 (17%)
	Partially evident	14 (40%)	20 (56%)
	Comprehensively evident	18 (51%)	10 (28%)
Explanation	Not evident	3 (9%)	11 (31%)
	Partially evident	6 (17%)	5 (14%)
	Comprehensively evident	26 (74%)	20 (56%)

Based on Table 15, the results show that students in the experimental group more frequently demonstrated critical thinking at the comprehensively evident level across most aspects than those in the control group. For example, in the inference aspect, more than half of the students (51%) in the experimental group were able to draw appropriate conclusions from the information provided in the problem. Their responses generally showed a clear reasoning process, in which the students connected the given data to relevant trigonometric concepts before concluding. In contrast, only a smaller proportion of students in the control group (28%) reached this level, while a larger number of their responses were categorized as partially evident. These patterns indicate that the learning approach used in the experimental group provided students with greater opportunities to demonstrate higher levels of interpretation, analysis, evaluation, inference, and explanation when solving trigonometric problems.

All five critical thinking indicators, such as interpretation, analysis, evaluation, inference, and explanation, were examined through the deductive content analysis. The results showed that differences between the experimental and control groups were observable across all indicators. However, the most substantial contrasts consistently appeared in the dimensions of inference. In the experimental group, students more frequently demonstrated integrated reasoning, coherent argumentation, and logical conclusion drawing. At the same time, responses in the control group tended to reflect procedural and fragmented reasoning. While aspects of interpretation, analysis, evaluation, and explanation were present in both groups, the qualitative differences were less pronounced. Therefore, the discussion emphasizes the one aspect that exhibited the most distinctive transformation patterns, while acknowledging improvements across all measured dimensions.

The results of this study indicate that, by using Deep Learning, not only can learning outcomes be improved statistically, but the quality of students' thinking processes can also be improved. Learning that has meaning can help students build a complete understanding by relating new knowledge to existing cognitive structures. In comparison, mindful learning can encourage students to be active, reflective, and critical at every stage of learning, while joyful learning fosters a learning atmosphere that supports exploration, enabling students to dare to try several solutions. Research by Xu et al. (2023) indicates that active learning, including problem-solving, is effective in improving critical thinking skills because students are directly involved in solving problems through group work, discussions, questions and answers, and reflection, thereby training the ability to interpret, analyze, synthesize, and evaluate.

The benefits of applying a deep learning approach in this study are also evident in the active student involvement in the learning process. Through practical activities such as making a simple clinometer and measuring vertical objects in the classroom, students can engage in understanding, analyzing, and evaluating the application of trigonometric formulas in real-world contexts. Utami & Putri (2023) emphasized that, under the Independent Curriculum, active learning encourages students to go beyond mere memorization and emphasizes critical understanding, analysis, and assessment of knowledge. In addition, Utami & Pramudiani (2024) focus on how incorporating empirical context in education can support improvements in students' ability to understand and use mathematical concepts critically.

This study provides important empirical data on the effectiveness of the deep learning approach in improving students' critical thinking skills in mathematics. These findings have significant practical implications for mathematics educators in schools, particularly regarding the implementation of the Independent Curriculum, which focuses on holistic learning and 21st-century skill development. Therefore, the deep learning approach warrants consideration as a strategic alternative for developing critical thinking skills, replacing the conventional learning model predominantly used in mathematics education.

Research Limitations

This research has several methodological limitations that need to be addressed. The research focus that is so narrow on one particular field, namely trigonometry, is a clue to the results that have not been widely applied to other mathematical disciplines. In addition, the limited duration of the intervention with only five meetings per class will limit the ability of researchers to be able to assess the long-term impact of the application of deep learning approaches on improving students' critical thinking skills. Although a deductive content analysis was conducted, the qualitative examination relied on the researcher's interpretation and did not involve inter-rated reliability testing. Future studies are recommended to incorporate multiple coders to strengthen analytical reliability.

CONCLUSION

The results of the study provided insight into how applying a deep learning approach to mathematics learning can significantly improve critical thinking skills among grade 10 students at SMA Negeri 2 Magelang. This increase in ability is evident in the analysis of the difference between pretest and posttest scores using a paired-samples t-test. Although both groups showed an increase in scores, the experimental group experienced a much greater improvement than the control group. The results of the independent samples t-test confirmed a significant difference in posttest scores between the two groups, providing evidence of the effectiveness of the deep learning approach in developing critical thinking skills. The application of the deep learning approach through the STEM-based Problem-Based Learning model was shown to significantly improve students' critical thinking skills. The learning activities designed within this approach were aligned with the principles of mindful, meaningful, and joyful learning, which may have contributed to the observed improvement in students' reasoning quality. Therefore, students not only gain a theoretical understanding of trigonometric ratios but also internalize and apply these principles in practical problem-solving, thereby significantly improving their critical thinking skills.

Mathematics educators are strongly encouraged to apply learning methods that focus on knowledge production in accordance with deep learning principles. The creation of relevant and contextual learning materials is essential to ensure that students have meaningful learning experiences. Future research should expand the study's scope, use a more representative sample size, and extend the treatment period. This endeavor is essential to enhance the validity of the generalization of the findings and to offer a more thorough comprehension of the long-term effects of this approach.

DECLARATION OF GENERATIVE AI USAGE IN THE WRITING PROCESS

During the writing of this manuscript, the authors employed ChatGPT, DeepSeek, Humata, and QuillBot to facilitate brainstorming and assist with language editing, including grammar and style refinement. Any content produced or suggested by an AI tool has been thoroughly reviewed, verified, and edited by the authors. The authors take complete responsibility for the originality, accuracy, and integrity of the final published manuscript.

REFERENCES

Alqahtani, R. M., & Alhamami, M. (2024). English as a foreign language teaching approaches in Saudi K–12 education: Teacher-centered or student-centered. *Journal of Education and Learning*, *18*(3), 817–824. https://doi.org/10.11591/edulearn.v18i3.21303

Arikunto, S. (2018). *Dasar-dasar Evaluasi Pendidikan*. PT Bumi Aksara.

Arisoy, B., & Aybek, B. (2021). The effects of subject-based critical thinking education in mathematics on students' critical thinking skills and virtues*. *Eurasian Journal of Educational Research*, *2021*(92), 99–120. https://doi.org/10.14689/ejer.2021.92.6

Ausubel, D. P. (1963). *The psychology of meaningful verbal learning*. Grune & Stratton, Inc.

Çibukçiu, B. (2025). The impact of constructivist methods on students' mathematical problem-solving. *Discover Education*, *4*(1). https://doi.org/10.1007/s44217-025-00475-w

Creswell, J. W., & Creswell, J. D. (2018). Research design: qualitative, quantitative, and mixed methods approaches, fifth edition. In *Introducing English Language*. SAGE Publications, Inc. https://doi.org/10.4324/9781315707181-60

Cronqvist, M. (2021). Joy in Learning. *Educare*, *3*, 54–77. https://doi.org/10.24834/educare.2021.3.3

Dessie, E., Gebeyehu, D., & Eshetu, F. (2023). Enhancing critical thinking, metacognition, and conceptual understanding in introductory physics: The impact of direct and experiential instructional models. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(7). https://doi.org/10.29333/ejmste/13273

Ennis, R. H. (1962). A Concept of Critical Thinking. *Harvard Educational Review*, *32*(1), 81–111. https://criticalthinking.net/wp-content/uploads/2024/04/Ennis_HER_v32n1_pp81-111.pdf

Ennis, R. H. (1996). Critical thinking dispositions: their nature and assessability. *Informal Logic*, *18*(2), 165–182. https://doi.org/10.22329/il.v18i2.2378

Facione, P. A. (1990). *Critical Thinking: A Statement of Expert Consensus for Puporses of Educational Assessment and Instruction*. https://files.eric.ed.gov/fulltext/ED315423.pdf

Feriyanto, F., & Anjariyah, D. (2024). Deep learning approach through meaningful, mindful, and joyful learning: a library research. *Electronic Journal of Education, Social Economics and Technology*, *5*(2), 208–212. https://doi.org/10.33122/ejeset.v5i2.321

Fullan, M., Quinn, J., & McEachen, J. (2018). *Deep learning: engage the world change the world*. SAGE Publications, Ltd.

Golightly, A., & Sprenger, S. (2024). A balancing act: South African geography teachers' implementation of teacher-centered and learner-centered instructional strategies in their classrooms. *Journal of Geography*, *123*(1), 3–13. https://doi.org/10.1080/00221341.2024.2307051

Hattori, Y., Inoue, Y., Matsubara, K., Hakamata, R., & Hisadomi, Y. (2025). Enhancing critical thinking in mathematics education: A rubric for students' social values. *International Electronic Journal of Mathematics Education*, *20*(3), em0830. https://doi.org/10.29333/iejme/16186

Hava, K. (2021). The effects of the flipped classroom on deep learning strategies and engagement at the undergraduate level. *Participatory Educational Research*, *8*(1), 379–394. https://doi.org/10.17275/per.21.22.8.1

Jeet, G., & Pant, S. (2023). Creating Joyful Experiences to enhance meaningful learning and integrate 21st-century skills. *International Journal of Current Science Research and Review*, *06*(02), 900–903. https://doi.org/10.47191/ijcsrr/v6-i2-05

Jing, C., Kaur, C., & Singh, S. (2024). Enhancing academic achievement in computer science students/ : An experimental study on the efficacy of deep learning teaching methods in higher vocational education. *International Journal of Education and Humanities*, *15*(2). https://doi.org/10.54097/17sxfz44

Khurma, O. A., & El Zein, F. (2024). Inquiry skills teaching and its relationship with UAE secondary school students' critical thinking: Systematic review of science teachers' perspectives. *Eurasia Journal of*

Mathematics, Science and Technology Education, *20*(2). https://doi.org/10.29333/ejmste/14155

Kolstø, S. D., Paulsen, V. H. P., & Mestad, I. (2024). Critical thinking in the making: students' critical thinking practices in a multifaceted SSI project. *Cultural Studies of Science Education*, *19*(4), 499–530. https://doi.org/10.1007/s11422-024-10217-3

Kostiainen, E., & Pöysä-Tarhonen, J. (2025). Meaningful learning over the course of teacher education: Students' reflections. *Teaching and Teacher Education*, *168*(April), 105241. https://doi.org/10.1016/j.tate.2025.105241

Langer, E. J. (2000). Mindful learning. *Current Directions in Psychological Science*, *9*(6), 220–223. https://doi.org/10.1111/1467-8721.00099

Lestari, R., Prahmana, R. C. I., Chong, M. S. F., & Shahrill, M. (2023). Developing Realistic Mathematics Education-based worksheets for improving students' critical thinking skills. *Infinity Journal*, *12*(1), 69–84. https://doi.org/10.22460/infinity.v12i1.p69-84

Levin, O. (2024). Simulation as a pedagogical model for deep learning in teacher education. *Teaching and Teacher Education*, *143*(November 2023), 104571. https://doi.org/10.1016/j.tate.2024.104571

Li, X. (2025). The curriculum planning and implementation for mindfulness education and diversified humanism based on big data. *Scientific Reports*, *15*(1), 1–15. https://doi.org/10.1038/s41598-025-95491-z

Mariam, M., Surjono, H. D., Wijanarka, B. S., & Maulana, I. (2026). Constructivist-based flipped classroom in e-learning: Enhancing student engagement and learning outcomes. *Multidisciplinary Science Journal*, *8*(2). https://doi.org/10.31893/multiscience.2026028

Marton, F., & Säljö, R. (1976a). On qualitative differences in learning II Outcome as a function og the learner's conception of the task. *British Journal of Educational Psychology*, *46*(2), 115–127. https://doi.org/10.1111/j.2044-8279.1976.tb02304.x

Marton, F., & Säljö, R. (1976b). On qualitative differences in learning: I—Outcome and process. *British Journal of Educational Psychology*, *46*(1), 4–11. https://doi.org/10.1111/j.2044-8279.1976.tb02980.x

Maslow, A. H. (1970). *Motivation and Personality*. Harper & Row, Publishers, Inc.

McPhail, G. (2020). The search for deep learning: a curriculum coherence model. *Journal of Curriculum Studies*, *53*(4), 1–15. https://doi.org/10.1080/00220272.2020.1748231

Monteleone, C., Miller, J., & Warren, E. (2023). Conceptualising critical mathematical thinking in young students. *Mathematics Education Research Journal*, *35*(2), 339–359. https://doi.org/10.1007/s13394-023-00445-1

Montessori, M. (1912). *The Montessori Method*. Frederick A. Stokes Company.

Murphy, L., Eduljee, N. B., & Croteau, K. (2021). Teacher-centered versus student-centered teaching: Preferences and differences across academic majors. *Journal of Effective Teaching in Higher Education*, *4*(1), 18–39. https://doi.org/10.36021/jethe.v4i1.156

Novianty, A., Chasanah, A. N., & Pamungkas, M. D. (2023). Pengaruh pembelajaran matematika model problem based learning dan discovery learning terhadap kemampuan berpikir kritis siswa kelas

VIII. *Jurnal Lebesgue/ : Jurnal Ilmiah Pendidikan Matematika, Matematika Dan Statistika*, *4*(1), 596–602. https://doi.org/10.46306/lb.v4i1.303

Piaget, J. (1959). *The Language and Thought of the Child*. Routledge & Kegan Paul Ltd.

Putri, A., Nusantara, T., Purwanto, & As'ari, A. R. (2025). The contribution of critical thinking skills in rich mathematical problem completion: Insights from pre-service mathematics teachers. *Eurasia Journal of Mathematics, Science and Technology Education*, *21*(2). https://doi.org/10.29333/ejmste/15931

Regulation of the Minister of Education and Culture Number 5 of 2022 Concerning Graduate Competency Standards, Ministry of Education and Culture of the Republic of Indonesia (2022).

Rehman, N., Zhang, W., Mahmood, A., Fareed, M. Z., & Batool, S. (2023). Fostering twenty-first century skills among primary school students through math project-based learning. *Humanities and Social Sciences Communications*, *10*(1). https://doi.org/10.1057/s41599-023-01914-5

Rincón, Y. R., Munárriz, A., & Ruiz, A. M. (2024). A new approach to continuous assessment: Moving from a stressful sum of grades to meaningful learning through self-reflection. *International Journal of Management Education*, *22*(3), 1–17. https://doi.org/10.1016/j.ijme.2024.101072

Rogers, C. R. (1961). *On Becoming a Person*. Houghton Mifflin Company.

Sachdeva, S., & Eggen, P.-O. (2021). Learners' Critical Thinking About Learning Mathematics. *International Electronic Journal of Mathematics Education*, *16*(3), em0644. https://doi.org/10.29333/iejme/11003

Tulak, T., Tangkearung, S. S., Tulak, H., & Paseno, E. W. (2024). *Application of Meaningful Learning Model to Improve Student's Learning Outcomes*. Atlantis Press SARL. https://doi.org/10.2991/978-2-38476-108-1_66

Ulfatun, T., Suyatmini, Kusumaningtyas, A., & Setiyawan, Y. A. (2023). Teacher's understanding of teaching models and students' human literacy. *International Journal of Evaluation and Research in Education*, *12*(4), 1925–1933. https://doi.org/10.11591/ijere.v12i4.25618

Utami, A. M., & Pramudiani, P. (2024). Exploring students' critical thinking skills in Geometry through the context of betawi culture: damdas 3 batu game. *Mathematics Education Journal*, *18*(3), 469–484. https://doi.org/10.22342/jpm.v18i3.pp469-484

Utami, M. R. P., & Putri, R. I. I. (2023). Students' critical thinking skills in solving pisa-like questions in the context of the jakabaring palembang tourism. *Journal of Mathematics Education*, *17*(2), 135–148.

Vygotsky, L. S. (1978). *Mind in Society: The Development of Higher Mental Processes*. Harvard University Press.

Wang, S., & Zhang, D. (2019). Student-centred teaching, deep learning and self-reported ability improvement in higher education: Evidence from Mainland China. *Innovations in Education and Teaching International*, *56*(5), 581–593. https://doi.org/10.1080/14703297.2018.1490662

Woods, P. J., & Copur-gencturk, Y. (2024). Examining the role of student-centered versus teacher-centered pedagogical approaches to self-directed learning through teaching. *Teaching and Teacher Education*, *138*(2024), 1–16. https://

doi.org/10.1016/j.tate.2024.104688

Xu, E., Wang, W., & Wang, Q. (2023). The effectiveness of collaborative problem solving in promoting students' critical thinking: A meta-analysis based on empirical literature. *Humanities and Social Sciences Communications*, *10*(1), 1–11. https://doi.org/10.1057/s41599-023-01508-1

Yoshida, F., Conti, G. J., Yamauchi, T., & Kawanishi, M. (2023). Learner-Centeredness vs. Teacher-Centeredness: How Are They Different? *Journal of Education and Learning*, *12*(5), 1. https://doi.org/10.5539/jel.v12n5p1

Youssef, N. H. (2024). The level of fulfillment of mathematics teaching requirements among teachers in the intermediate stage to develop higher-order thinking skills. *Eurasia Journal of Mathematics, Science and Technology Education*, *20*(9). https://doi.org/10.29333/ejmste/15025

Zhao, Y., Gao, W. W., & Ku, S. S. (2023). Optimization of the game improvement and data analysis model for the early childhood education major via deep learning. *Scientific Reports*, *13*(1), 1–14. https://doi.org/10.1038/s41598-023-46060-9

Zhu, H. (2023). Transforming mathematics education: fostering critical thinkers. *Advances in Social Science, Education and Humanities Research Proceedings of the 2023 International Conference on Applied Psychology and Modern Education (ICAPME 2023)*, 108–114. https://doi.org/10.2991/978-2-38476-158-6_16

Zuo, H., & Wang, L. (2023). The influences of mindfulness on high-stakes mathematics test achievement of middle school students. *Frontiers in Psychology*, *14*. https://doi.org/10.3389/fpsyg.2023.1061027