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The Exploration of APOS-based PBL Model to Improve the Students' Mathematics Problem-Solving Skills

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Abstract: Math majors must be excellent problem solvers because problem-solving is the foundation of teaching mathematics. Their problem-solving skills will help them create service initiatives as math teachers. This research investigated how well the APOS-based PBL model enhances students' skills in solving mathematical problems. This quasi-experimental design used a non-equivalent post-test-only control group, which is quantitative. Students at a university in Central Java, Indonesia, participated in the study. Tests of problem-solving skills were useful to collect the data. The results of the study showed that (1) students' problem-solving skills lead to learning completion (71); (2) students taught using a modified PBL model based on APOS theory have a different proportion of learning completeness than students taught using an average class using a direct learning model; (3) students' average problem-solving skills taught using the modified PBL model based on APOS theory are superior to those taught in the average class using a direct learning model; (4) increasing students' problem-solving skills taught using a modified PBL model based on APOS theory is preferable to improving students' skills taught using a direct learning model. This study offers insightful insights into the PBL model's implementation, incorporating APOS theory at every stage. These results suggest that to maximize students' problem-solving skills, consideration of the theory of thinking processes must be given when putting the learning model into practice

Keywords: APOS, mathematics, PBL, problem-solving skills.

▪ INTRODUCTION

Problem solving is one of the key components of mathematical thinking (Drijvers et al., 2019). Problem solving is crucial and should not be isolated from the study of mathematics since it is an essential component of the subject (NCTM, 1989; Rahayu et al., 2014). It is crucial for students to acquire the complex skill of problem solving (Pakarinen & Kikas, 2019). This viewpoint leads one to the conclusion that solving problems is a crucial component of learning mathematics and that it is crucial for college students to encounter difficulties. One aspect of thinking is problem solving, or the capacity to solve problems (Marzano et al., 1988). One approach to tracking a person's cognitive process is through the application of the APOS theory. Anwar et al. (2013) explain the APOS theory is a highly useful tool for comprehending how college students learn a variety of mathematics topics. Olesova & Borisova (2016) explain the APOS mental structure depicts a person's way of thinking when they solve mathematical problems. The APOS mental structure is applicable to realize and form the process of solving mathematical problems (Borji et al., 2018; Hidayatullah, 2019; Sutarto et al., 2018).

The mental structure construction of Action, Process, Object, and Schema is abbreviated as APOS. Dubinsky & Mcdonald (2001) explain the APOS theory describes students' mathematical knowledge as a propensity to construct or reconstruct actions, processes, and objects, organizing them into schemes to describe problems, and reflecting

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on their problems and solutions in a social context in response to mathematical situations. Arnon et al. (2014) explain that APOS theory is composed of the mental processes of interiorization, coordination, reversal, encapsulation, de-encapsulation, and thematization.

Since problem solving is the cornerstone of teaching mathematics, it is critical that math majors possess strong problem-solving skills (Avcu & Avcu, 2010). When they become math teachers, their capacity to comprehend and apply problem-solving techniques will be useful in creating service initiatives. Despite the importance of taking ownership of problem solving, the majority of aspiring teachers struggle with comprehension, accurately applying strategies, and handling non-routine problems (Güner & Erbay, 2021).

This gap creates problems when prospective teacher students carry out their duties in the field as teachers to teach about problem-solving. Prospective teacher students must equip themselves to teach problem-solving skills to their students, thereby minimizing this gap. One of the courses, Introduction to Probability, provides this material to prospective teacher students. This learning will enhance the problem-solving skills of prospective mathematics teachers. The objective of the probability theory course is for students to understand and master the material on sample space and events, the multiplication principle (basic rules for calculating sample points), to comprehend and apply permutations and combinations of an event, the probability of events and Bayes' theorem, random variables and probability distributions, expectation and variance.

The probability theory course serves as a prerequisite for the introduction to mathematical statistics. The topics surrounding probability theory are quite complex, particularly the material on probability. Misconceptions about probability can lead to errors in probabilistic reasoning (Hirsch & O'Donnell, 2001). The material on probability is filled with various types of problems in everyday life. Therefore, students are required to be able to solve problems related to opportunities. Exploring the problem-solving process in probability theory, particularly in relation to probability material, is crucial as it equips students to apply mathematical thinking patterns in various disciplines and daily life. Additionally, students will utilize their understanding of opportunities in their future careers as teachers in educational institutions.

Although having problem-solving skills is critical, many students still face difficulties. Despite consistent practice of problem-solving exercises, the experience of teaching the probability theory course reveals that students' problem-solving skills remain inadequate. The difficulties experienced by students include breaking down the words in a problem, understanding the intent and all the information from the problem, confusion in digesting information, imagining the context, creating a mathematical model from the problem, using accurate methods in calculations, making errors during calculations, and interpreting answers according to the context of the question (Hidayat & Sariningsih, 2018; Jourbert, 2009; Phonapichat, P., Wongwanich & Sujiva, 2014).

Problem-solving plays an important role in probability learning, so there is a need for an effort in the form of an appropriate learning model. The learning model is one factor that can influence the learning process (Simbolon & Koeswanti, 2020; Syah, 2010). Student-centered learning can improve their problem-solving skills (Wijayanti et al., 2017). Problem-based learning (PBL) is the applicable learning model. PBL is a learning approach that uses problems to guide the learning process (Kazemi & Ghoraishi, 2012).

The learning process commences with a scenario involving real-life problems that require resolution (Kim, 2021). Students solve problems using the necessary knowledge and information they have acquired. Based on the research findings, learning with the PBL model effectively enhances students' mathematical problem-solving skills. The research findings support this claim (Argaw et al., 2017; Hendriana et al., 2018; Kadir et al., 2016; Ulger, 2018).

Using the PBL model to teach mathematics enables lecturers to concentrate more on comprehending the mental mechanisms and structures outlined in the APOS theory, which students are likely to encounter in the classroom (Mudrikah, 2016). As a result, lecturers can make informed decisions when addressing the problems that arise in mathematics learning in the classroom.

This research uses the APOS-based PBL model, which aligns with Arends' (2008) proposed PBL stages: orientation to the problem, student organization, individual and group investigation, presentation, analysis, and evaluation of the problem-solving process. We modify the learning activities by integrating the APOS theory into every stage of PBL. This research differs from previous studies, such as the study by Eviliasani et al. (2022). Those researchers implemented the Modified APOS (M-APOS) model in learning for students, focusing on increasing students' interest in learning. In contrast, this research integrates APOS theory into PBL learning. Exploring this novelty is crucial, especially in the teaching of probability theory. Meanwhile, Ahdhianto et al. (2020) and Sahyar et al. (2017) also found the effects of the PBL model on problem-solving, but they did not specifically involve the APOS theory in the learning process.

The implementation of the PBL model based on the APOS theory emphasizes students engaging in mental mechanisms designed in the preliminary genetic decomposition so that students can build a mental structure of probability material. Considering its various advantages, this research aims to explore the effectiveness of the APOS-based PBL model in enhancing students' problem-solving skills.

▪ METHOD

Design

This quantitative research applied a quasi-experimental approach with non-equivalent pretest-only control group design. Here is the experimental research illustration.

Experimental group : X --------- O
Control group : --------- O
with:
O: the problem-solving skill test (posttest)
X: the given treatment – APOS-based PBL model

Table 1 shows the APOS-based PBL model for experimental group and direct learning for control group within 7 meetings.

Table 1. The APOS-based PBL material divisions

Meetings	Materials
1	Sample and events
2	Rules of multiplication, permutation, and combination
3	Probability of events
4	Probability of independent events

5	Conditional probability
6	Total probability
7	Bayesian rule
8	Problem-solving skill test

After completing the learning sessions, the students received a mathematical problem-solving test. Once the test results were obtained, an analysis of the students' answers was conducted. Quantitative analysis to test the effectiveness of PBL learning was based on the APOS theory in achieving students' problem-solving skills.

Participants

The research population consists of students from Universitas Negeri Semarang's Mathematics Department, Faculty of Mathematics and Natural Sciences (FMIPA), who were taking a probability theory course. The researchers randomly selected the research sample from the existing population. One experimental class learned with a modified PBL model based on the APOS theory, while one control class received direct instruction. The researchers selected Class A of the Mathematics Study Program as the experimental group to receive learning with the modified PBL model based on the APOS theory, and Class B of the Mathematics Study Program as the control group to receive direct instruction.

Instruments

This research used a test questionnaire on probability theory problem solving as its instrument. The question consisted of descriptive questions, 4 items. The problem-solving skills used in this study are based on the NCTM (2000) mathematics learning standards. These standards include (1) learning new math through problem-solving, (2) solving math and non-mathematical problems, (3) using and adapting a variety of good strategies to solve problems, and (4) keeping track of and thinking about the math problem-solving process. Seven experts validated the questions before their use, achieving an average validation score of 3.48, a score meeting the strong criteria. The researchers empirically examined the questions, yielding a high reskill score of 0.611 for the problem-solving skill test items.

Data Analysis

The data analysis includes tests for normality, homogeneity, right-sided one-sample mean difference, right-sided proportion difference, right-sided two mean difference, and tests for enhancing problem-solving skills using N-gain. Using the N-Gain formula from the pretest and post-test scores of the control class allows the researchers calculate the N-Gain value for both the control class and the experimental class. Table 2 shows the N-gain result interpretation classification.

$$N.Gain = \frac{Posttest\ score - Pretest\ score}{Ideal\ Maximum\ Score - Pretest\ score}$$

Table 2. N-Gain criteria

N.Gain Scores	Criteria
$-100 \leq \text{GT} < 0.00$	Decrease

$\mathrm{GT} = 0.00$	Stable
$0.00 < \mathrm{GT} < 0.30$	Low
$0.30 \leq \mathrm{GT} < 0.70$	Moderate
$0.70 \leq \mathrm{GT} \leq 1.00$	High

(Source: Lestari&Yudhanegara, 2018)

▪ RESULT AND DISSCUSSION

The Implementation of APOS-based PBL Model for Probability Theory Course

Step I: Problem Orientation

At this stage, the lecturer explained the learning objectives and then presented a problem for the students to solve. The students attempted to connect between the problems in the probability material and everyday life pique students' curiosity. The lecturer presented a problem-solving question on the student assignment sheet (LTM) to elicit their solutions.

During the first meeting, the students were surprised by the presentation of problems at the beginning of the lesson. Students were still accustomed to the presentation of material at the beginning of the lecture, followed by examples and practice questions. However, after the third meeting and onwards, students were more prepared at the start of the learning process. Some students already studied the probability theory textbook before attending the lessons, making them more prepared to tackle the problems.

After receiving the problem, teachers prepare students to act by instructing them to read, comprehend, and individually represent the problem. At this stage, some students are able to quickly grasp the problem. However, there are also students who still struggle to visualize the situation presented in the problem.

Step II: Organization of the Students

At the student organization stage, the lecturer divided the students into several heterogeneous groups based on cognitive styles. The lecturer determined the students' cognitive style by administering the Matching Familiar Figure Test (MFFT). Based on the test results, the researchers categorized students into four types of cognitive styles: fast accurate, impulsive, reflective, and slow inaccurate. Then, the researchers carried out the heterogeneous grouping to minimize the gap in characteristics among students with different cognitive styles.

After grouping, the students observed and identified the problem's elements. At this stage, students are trying to understand the purpose of the problem and the steps to solve it. The lecturer assisted the students in defining and organizing their answers based on the problem-solving steps. Then, the students organized the tasks that were the responsibility of each group member, so that every student played a role in completing the group assignment.

Stage III: The Individual and Group Investigations

At the stage of individual and group investigation, students independently and collaboratively conducted inquiries and experiments to solve problem-solving tasks. Students collected information and data from the questions at this stage. They strived to identify the appropriate strategies for solving the problems.

In meetings 1 and 2, students had difficulty remembering the concepts of sample space and events, which were topics covered in high school. After meeting 3, students began to actively engage in investigations compared to before. Students were looking for information on various problem-solving strategies in probability theory textbooks. The students were very enthusiastic about solving the problems. After conducting individual investigations, the activity continued with a group discussion to share the solutions that had been found. Discussions do not always go smoothly. There were several students who had different strategies, so it took longer discussion time to reach a decision.

The lecturer's task during the individual and group investigation stage was to monitor and evaluate the students' work. If students kept making mistakes, the lecturer would guide them by providing direction. The lecturer's role was to facilitate students connect probability concepts together. Some students had ideas or strategies for problem-solving, but they were insecure. Therefore, the lecturer strived to encourage students to represent their ideas in tables, images, symbols, and various other representations.

Stage IV: Presentation

The students presented their problem-solving results to the class after completing their individual and group investigation activities. The lecturer asked a representative from each group to present their findings (answers to the given problem), and provided an opportunity for other groups to respond and give their opinions on the group's presentation. Students had different problem-solving strategies to present their ideas for confirmation. During the presentation stage, the lecturer evaluates the students' work.

At the presentation stage, students paid attention to the answers from other groups. If the answers between groups differ, students could transform the process through reversal, reversing the process they internalized previously.

Stage V: Analysis and Evaluation

In the analysis and evaluation stage, students along with the lecturer reviewed the process and final answers obtained so that students would be confident in their results. The lecturer checked each step of the problem-solving process carried out by the students. At this stage, the lecturer's role was to provide confirmation and reinforcement of the students' answers. Once the students found the answers, they drew conclusions about the concepts they learned.

After completing the concluding activities, students moved on to problem-solving exercises. Students could work on the exercise both individually and in groups outside of class. The practice questions consisted of problem-solving items designed to reinforce learning activities in the classroom. Following seven sessions of learning using a modified PBL model based on APOS theory, students took a stage 2 problem-solving skill test at the eighth meeting.

The analysis of the test results revealed that the experimental class outperformed the control class, with an average problem-solving skill of 76.06 and a completeness rate of 79.17%, compared to an average of 62.27 and a classical completeness rate of 27.27%. The highest score in the experimental class was 96.11, while the highest score in the control class was 87.22. Table 3 is a recap of the problem-solving skill test results for the experimental class and the control class.

Table 3. The recapitulated of problem-solving test skills of experimental and control groups

Remarks	Experimental	Control
Mean	76.06	62.27
Maximum	96.11	87.22
Minimum	47.22	43.89
Scores higher than the B-criteria (71)	19 (79.17%)	6 (27.27%)
Scores lower than B-criteria (71)	5	16

The Effectiveness of APOS-based PBL Model Modification toward the Student Mathematics Problem Solving Skills

Perquisite Test

A prerequisite test, including a normality and homogeneity test, must be conducted before a hypothesis test can be held. The data used in this quantitative research stage is based on problem-solving ability. Based on the SPSS calculation, the experimental class sig value is obtained = $0.463 \geq 0.05$, and the control class sig value = $0.323 \geq 0.05$, so it can be concluded that the problem-solving ability data for the experimental and control classes come from a normally distributed population. The next prerequisite test is the homogeneity test. Based on the calculation assisted by SPSS, the sig value is obtained = $0.9 \geq 0.05$, so it can be concluded that both the experimental and control class variances are homogeneous.

Hypothesis Test 1: The Test of Problem-Solving Skill Accomplishment Test

To test the learning completeness, the researcher used a one-sample t-test. The test used to analyse the results of the problem-solving test of students who participated in PBL learning can achieve a minimum criterion of 71. The test results were assisted by SPSS calculations. Based on the calculation results obtained, the t-count value = 2.331. At a significance level of 5% and dk = 23, the t-table value = 1.7138 was obtained, so t-count > t-table means that the average problem-solving ability of students who participated in modified PBL learning based on APOS theory, namely 76.06, reached a minimum value of criterion B (71). This means that after a lesson using the PBL learning model based on APOS theory, the average problem-solving ability of students can achieve learning completeness.

Hypothesis Test 2: The Proportional Differences of Problem-Solving Skill Test

The comparison test of two proportion samples was tested using the z test. In this study, the proportion of students who achieved a minimum score of 71 in the experimental class and the control class will be tested. After the calculation, the results showed that the number of students who achieved a minimum score of 71 in the experimental class and the control class were 19 people and 6 people. Based on the calculation results obtained, the z_{count} value = 3.2204 and the z_{table} value = 1.64, so $z_{count} > z_{table}$. The conclusion that can be obtained is that the proportion of completeness of problem-solving abilities of students in the experimental class is 79.17% more than the proportion of completeness of problem-solving abilities of students in the control class of 27.27%.

The Hypothesis Test 3: the Two Sample T-Test of Problem-Solving Skills

Test the difference between the two averages of the experimental class and the control class using the t-independent sample t-test. Based on the calculation results, the t-value is 4.106. At a significance level of 5% and dk = 44, the t-table value is 1.68, so t-count > t-table means that the average problem-solving ability of students taught with the modified PBL model based on APOS theory is 76.06, better than the average class taught with the direct learning model of 62.27.

The Hypothesis Test 4: The Improvement Differences of the Problem-Solving Skills

The steps for testing the differences in the improvement of problem-solving abilities using Mann-Whitney U. assisted by SPSS calculations, obtained a sig value = 0.000 <5%, meaning that the improvement in mathematical problem-solving skills between students taught with the modified PBL model based on APOS theory is better than the improvement in mathematical problem-solving skills of students taught with the direct learning model.

The modified PBL model learning based on the APOS theory was deemed effective because it meets three criteria, namely: (1) the average problem-solving skill in the experimental class reached a score of 71; (2) the proportion of students achieving a score of 71 in the experimental class was higher than the control class; (3) the average problem-solving skill in the experimental class was higher than in the control class; (4) the improvement in mathematical problem-solving skill among students taught with the modified PBL model based on APOS theory was higher than the improvement in mathematical problem-solving skill of students taught with the direct learning model.

The results of this study align with the research by Ahdhianto et al. (2020) and Harisantoso et al. (2020). Those researchers found that students who learn using the PBL model had higher average scores than those with direct instruction. Thus, the PBL model in mathematics education was more effective and had a significant impact. The PBL model can enhance students' mathematical problem-solving skills (McConnell et al., 2019). Argaw et al. (2017) also found that direct learning only made students acquire information or lessons from the teacher, reinforces this opinion.

The research results indicate that the proportion of students who meet the criteria for a B in classes taught using a modified PBL model based on the APOS theory is higher than in classes taught with a direct instruction model. These findings align with previous research, which concluded that the proportion of students who succeed in PBL learning is higher compared to the proportion of students in the control class who succeed (Maretasani, 2018; Sholihat & Amalia, 2019).

Theory-based learning using APOS leads to better mathematical skills in students compared to conventional learning. Brown et al. found that students in the APOS group performed better on mathematical tasks than students in the traditional learning group (Arnon et al., 2014). This result is also consistent with Arnawa (2009). The results indicate that students who received abstract algebra instruction based on the APOS theory have significantly excellent skills than students who received abstract algebra instruction conventionally. Anwar et al. (2013) also found a significant difference in the learning outcomes of differential equations taught using the APOS theory approach compared to conventional methods.

One factor that can lead to an increase in the number of students with mathematical problem-solving skills is the practice of facing problem-solving questions in real-life situations on students' assignment sheets (LTM) (Sahyar et al., 2017). (LTM). The modified PBL learning based on the APOS theory begins with the presentation of problems related to other fields in real life, specifically in the problem orientation step. This is in line with previous research, in which efforts to facilitate mathematical problem-solving skills involve getting students accustomed to solving contextual problems by applying problem-solving steps (Adifta et al., 2022). Problems serve as research material for students, allowing them to construct their own knowledge, develop higher-order skills through inquiry, foster independence, and enhance their self-confidence (Sastrawati et al., 2011).

The things that students did during the APOS-based PBL learning process about probability in the mental structure of action show that they could change the things that are involved into outside activities . Students could complete the LTM according to the given instructions. In the mental structure of the process, students could think about the same action without external stimuli. The students were giving a presentation in front of the class about the LTM they completed. The mental structure of objects enabled students to grasp the complete process and appreciate the possible transformations. After the presentation, students determined the results. Students could identify a collection of interconnected actions, processes, and objects in the mental structure of a schema and use them to solve problems and answer the problem-solving questions given. The results of this study indicate that the problem-solving skills of students taught using the APOS theory-based PBL model are excellent compared to those taught with the direct instruction model. Eliza et al. (2022) and Syam (2021) concluded that the mathematical problem-solving skills of students using the APOS learning model are superior to those using the conventional learning model.

The stages of modifying the PBL model based on the APOS theory emphasize active learning. This is due to the design of student-centered learning, which aims to boost students' active engagement and participation in the learning process. Problem-based learning encourages students to become more active and accustomed to problem-solving, enabling them to enhance their problem-solving skills and acquire new knowledge (Yuhani et al., 2018). Students' cognition inspires the idea of finding a solution to a problem. Active learning can be used to motivate problem-solving (Argaw et al., 2017; Saputra et al., 2018).

Each stage of PBL effectively contributes to the enhancement of students' mathematical problem-solving skills. During the first stage of problem orientation, the lecturer presents a problem for the students to solve. The relevance of the problems in the probability material to everyday life piques students' curiosity. The lecturer presents issues that include problem-solving questions and solutions not formally studied in class. The lecturer develops the problem-solving questions based on the pre-prepared genetic decomposition. Students construct a mental structure within the genetic decomposition through these problem-solving questions.

Students received training in this first stage to read, understand, and represent problems individually. Problem solvers use reading as a tool to help conceptualize mathematical problems (Capraro et al., 2012). Reading also allows them to discover suitable problem-solving strategies for addressing issues. Students heavily rely on their good reading and reasoning skills to solve written problems (Pakarinen & Kikas, 2019). After reading the problem, the next step for the students is to understand the issue presented (English, 1997; Sari et al., 2021).

During the problem orientation stage, the lecturer expects students to build a mental structure of action through the application of specific rules and the explicit expression of the presented problems. The questions' focus is more on enhancing reflective abstraction or mental mechanisms than obtaining correct answers from the students.

Students were accustomed to solving problems using five steps of problem-solving. Therefore, students' mathematical problem-solving skills will gradually improve when they frequently apply the APOS theory-based PBL model modifications. If students frequently practice problem-solving exercises and word problems, their problem-solving skills can improve (Fadella et al., 2018). When working on problem-solving questions, students must be able to represent the situation in the question and determine the appropriate problem-solving strategy. Students engage in the APOS-based modified PBL learning, expressing all descriptions of the problem situation and their ideas for problem-solving through various representations.

Stage II of the PBL model modification based on the APOS theory is student organization. At this stage, the lecturer divides the students into several heterogeneous groups based on cognitive styles. Student interaction within groups allows them to better understand difficult concepts through group discussions that address the problems they encounter (Subudi et al., 2018).

During the student organisation stage, students constructed the mental structure of action, marked by their beginning to calculate specific matters. Then, the students organised the tasks that were the responsibility of each group member. Each learning the modified PBL model based on the APOS theory, each student had to take part in completing group assignments. Given roles in group work, students enthusiastically participated in classroom learning. Kartono & Shora (2020) and Suarsana et al. (2019) assert that when given roles in group work, students enthusiastically participate in classroom learning. A systematic group or team work process truly optimises students' problem-solving skills, enabling them to empower, hone, test, and continuously develop their thinking skills. The researchers categorized problem orientation and organisation (stages I and II) as a mental structure of action. When students encountered stages I and II, they continued to apply the action material to the object.

Stage III of the PBL model modification based on the APOS theory is an individual and group investigation. At this stage, students independently and in groups conduct investigations and experiments to solve problem-solving tasks. At this point, the process of mental interiorization existed. When students attempted to collect facts, make preliminary estimates, and investigate, they utilized pre-existing schemas as prerequisites for executing subsequent mental mechanisms (Maharaj, 2010).

The problem-solving questions stimulated the activation of mental mechanisms (internalization and coordination), fostering the development of mental structures (processes and objects) that underpin the cognitive formation of mathematical concepts. Students, both independently and in groups, worked on a student assignment sheet containing tasks to construct a mental structure of action and internalize it into a process. The researchers categorized this step as internalisation because it leads to a comprehension of the students' underlying issues. Lecturers monitored and evaluated students' work. If students continued to make mistakes, the lecturer would guide them by providing direction. The lecturer's role was to facilitate students connect probability concepts together. The lecturer guided students to coordinate the internalized processes after they experienced the mental mechanism of internalization.

Group discussion activities helped students reflect on their work during the individual investigation stage. Collaborative work motivated students in groups to not only make decisions and solve problems, but also to build and expand their developed and maintained ideas, processes, concepts, conditions, and more (Aljarrah, 2020). Group interaction can enhance student motivation (Seifert, 2016). The input and arguments provided by group members will contribute to problem-solving, allowing for a quicker resolution with more accurate answers. In PBL learning, the investigative process develops students' skill to process the information and knowledge they acquire. This is supported by studies conducted by Belland et al. (2010) and Suntusia & Hobri (2019).

The PBL model presents students with problems via group work worksheets and individual evaluation sheets, improving their mathematical problem-solving skills (Musliha & Revita, 2021). The responsibility for solving problems rests with the students in group work (Imam et al., 2018). The provided LTM aims to guide students more effectively in solving problem-solving questions through problem-based learning stages. Thus, the application of the PBL learning model based on the APOS theory can provide students with the freedom to learn, receive information, transfer knowledge, and express opinions in the group problem-solving process. Additionally, the PBL learning model trains students to think more broadly and find unconventional solutions. Problem-based learning can motivate students to spark their curiosity, thereby increasing their knowledge and new experiences.

Stage IV in the modification of the APOS-based PBL model involves presenting the findings. The lecturer asked the students to synthesize the results of their investigation into a general form. Presenting various problem-solving solutions can enhance students' problem-solving skills (Gregory et al., 2013). During the presentation stage, students paid attention to the answers from other groups, then responded and gave their opinions on the group presentations (Maryati, 2018). If groups' answers differed, students could transform the process through reversal, involving reversing the previously internalized process. Reviewing the steps of problem-solving to address the specified problem and drawing conclusions from alternative solutions constituted a reversal step. When the conclusion was unacceptable, a reversal occurred, returning to the mental structure of action and process.

The APOS-based PBL model was modified at Stage V, analysis and evaluation. At this stage, the lecturer and students reviewed the process and final answers to the problem. Once the students refined and agreed upon the answers, they would proceed to the encapsulation step, the alternative approach to solving the problem. This encapsulation step yields a problem-solving product, eventually transforming into an object. Illogical answers after testing would trigger de-encapsulation.

After the students found the answers, they drew conclusions about the concepts they learned, ultimately forming a schema. The thematicization mental mechanism is expected to occur, characterized by: (1) students identifying a component and its relations as a schema; and (2) students examining various properties of the schema (Arnon et al., 2014). After completing the concluding activity, students moved on to problem-solving exercises. Students could work on the exercise both individually and in groups outside of class. The practice questions consisted of problem-solving questions designed to strengthen learning activities within the classroom. Problem-solving practice questions assisted in the further development of students' mental constructs, as suggested by genetic decomposition. The goal of this exercise was to reinforce the foundation of the mathematical theory under study. In the training stage, a mental generalization mechanism occurred, where students applied existing schemas to a broader context.

The exercise facilitated students to work both individually and in groups outside of the classroom. As stated in genetic decomposition, problem-solving exercises facilitated the development of students' mental constructs. The goal of this exercise was to reinforce the foundation of the mathematical theory under study. In the training stage, a mental generalization mechanism occurs, where students applied existing schemas to a broader context. Practice problems are an effective way to learn mathematics (Gallian, 2017). The discussion of the practice questions would take place in the next meeting, and focused on the questions that the students could not solve.

▪ CONCLUSION

The APOS theory-based PBL model is effective in improving students' mathematical problem-solving skills on probability. The evidence include (1) the problem-solving skills of students reaching a learning completeness of 71; (2) a difference in the proportion of learning completeness between students taught with the modified PBL model based on the APOS theory, higher than the class average taught with the direct learning model; (3) the average problem-solving skills of students taught with the modified PBL model based on the APOS theory were higher than the class average taught with the direct learning model; (4) the improvement in problem-solving skills of students taught with the modified PBL model based on the APOS theory was higher than the improvement of those taught with the direct learning model

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