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Internal Transposition on Constructing Quadratic Functions Based on Praxeological Theory

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Abstract: The sub-topic of constructing quadratic functions requires students to integrate various concepts. However, students still encounter various learning obstacles, one of which relates to the structure of knowledge in textbooks and teachers' lesson plans. This study aims to analyze the praxeological structure in textbooks and teachers' lesson plans, identify forms of internal transposition, and examine the didactic implications for potential obstacles to student learning in the sub-topic of constructing quadratic functions. This research employs a qualitative approach with a case study design. The participants in this research consist of one mathematics teacher and tenth-grade students at a private school in Bandung, selected by convenience sampling. Data were collected through document analysis, classroom observation, and semi-structured interviews. Analysis was conducted using praxeological-didactical analysis, as well as triangulation techniques, member checking, and audit trails to ensure the trustworthiness of the data. The results indicate that the praxeological structure in the textbooks is dominated by evaluative tasks (83.33%) with techniques that are not conceptually integrated, alongside a lack of explicit technology and theory. In the teacher's lesson plans, task types are reorganized. However, flexibility occurs in technical aspects without conceptual reinforcement. The internal transposition observed is adaptively limited, merely modifying tasks and techniques without developing conceptual aspects. The didactic implications of these conditions are reflected in the potential learning obstacles identified in the observed instructional context. The study concludes that modifying textbook tasks into open-ended, multi-representational activities is crucial for overcoming students' learning obstacles, with practical implications: the lesson plan must be reconstructed based on predictions of student responses to enhance teachers' adaptive instructional readiness.

Keywords: mathematics textbooks, quadratic functions, praxeology, teachers' lesson plans, potential learning obstacles.

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INTRODUCTION

Quadratic functions are an essential subject in mathematics education that students must master. This importance is reflected in the mathematics learning outcomes for Phase E (typically 10th-grade senior high school or equivalent) within the algebra domain, as well as its role as a prerequisite for advanced algebraic material in Phase F (typically 11th-grade senior high school or equivalent) (Setiyani et al., 2025). Beyond being a mandate of the national curriculum, quadratic functions are broadly relevant across various applied fields (Hidayanti et al., 2024), especially for modeling and analyzing contextual problems in everyday life.

One crucial subtopic in learning quadratic functions is constructing quadratic functions from various given conditions. This sub-topic requires students to integrate various concepts such as function representation, relationships between points, and algebraic modeling. Consequently, it can pose learning obstacles if not understood conceptually, and is one of the most complex parts of learning quadratic functions.

Conceptually, quadratic functions and quadratic equations are two interrelated representations, where the solutions of a quadratic equation represent the roots of the quadratic function or the x-intercepts of its graph. Understanding this connection is essential as a foundation for students to meaningfully integrate various mathematical representations. However, the reality of learning shows that students still encounter various obstacles to understanding quadratic functions, both conceptually and procedurally.

These obstacles are inseparable from the suboptimal mastery of the prerequisite topic, particularly the concept of functions, which can lead to epistemological obstacles (Utami et al., 2023). This is in accordance with the results of tests conducted on grade XI as a pre-research observation. Students who have studied the topic of quadratic functions in grade X are still not familiar with functions, especially quadratic functions, an cannot provide appropriate reasons.

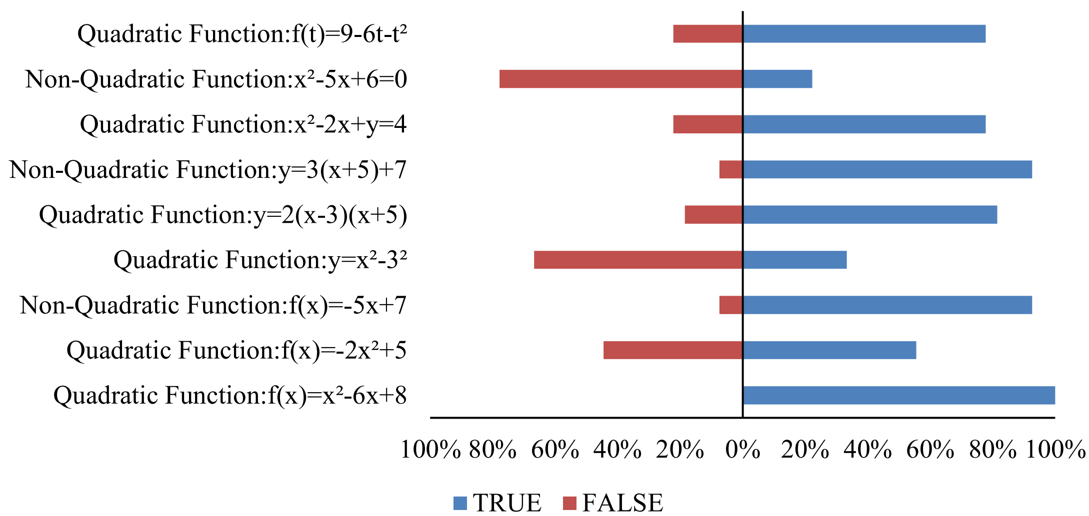

Figure 1. Percentage of student answers in differentiating quadratic functions and non-quadratic functions

Students tend to memorize procedures without understanding the underlying reasoning behind the steps performed (Biney et al., 2023), resulting in difficulties in interpreting functions within contexts that cannot be quantitatively generalized. Furthermore, students' understanding of quadratic equations, another prerequisite for quadratic functions, remains low (Zunianto & Gusmarini, 2023), thereby compounding the complexity of the obstacles encountered.

In line with these epistemological obstacles, students also encounter didactic obstacles that arise from the learning process. Students' attachment to understanding patterns of quadratic equations acquired at previous levels (Huang et al., 2012) and the tendency to view quadratic equations and quadratic functions as two separate topics (Ruli et al., 2019) indicate that conceptual connections have not been fully established. As an illustration, when given the function $y = 3x^2 - 9x + 6$, students simplify the x^2coefficient by dividing the right side by 3 to obtain. $Y = x^2 - 3x + 2$ Believing this to be correct due to their familiarity with quadratic equations. Furthermore, the limited variety of problem types, which generally focus only on the general form $y = ax^2 + bx + c, a \neq 0$, causes students to struggle when faced with specific forms such as $y = ax^2 + bx$ or $y = ax^2 + c$. This suggests that the instructional design has not fully supported the development of students' relational understanding.

Complementing the previous description, ontogenetic obstacles also arise from students' limited experience connecting various mathematical representations. Selowa and Dhlamini (2023) demonstrate that students find it more difficult to form algebraic models from graphs than vice versa, indicating an imbalance in the ability to translate between representations (Selowa & Dhlamini, 2023). This condition suggests that instruction is still dominated by a one-way approach, leaving students with insufficient experience in fully integrating symbolic, visual, and contextual representations. This situation becomes increasingly crucial in the subtopic of constructing quadratic functions, as students are required to integrate multiple pieces of information into a single cohesive functional form.

These various obstacles are inseparable from the roles of teachers and instructional materials in the learning process. From the perspective of the Anthropological Theory of the Didactic (ATD), the topic of quadratic functions, specifically the sub-topic of constructing quadratic functions studied by students, is a product of the didactic transposition process (Bosch & Gascón, 2014), namely the transformation of knowledge from *scholarly knowledge* into *knowledge to be taught, taught knowledge,* and *learned knowledge* (Chevallard, 2019). The transformation from textbooks to teachers' lesson plans is a form of internal transposition that is contextual and subjective, depending on the teacher's interpretation of the content. (Castela, 2015). Findings by Shinno and Mizoguchi (2021) also indicate that teachers adapt textbooks when designing instruction, which potentially influences the quality of student learning experiences (Shinno & Mizoguchi, 2021). However, it remains unclear to what extent this adaptation process alters the presented knowledge structure, particularly the relationship between techniques and their conceptual justifications.

In line with this, several previous studies have examined mathematics textbooks and teaching practices using relevant theoretical frameworks. Several studies have used praxeology to analyze the mathematical organization of textbooks across various topics, including comparative studies between textbooks from different countries (Chandra et al., 2025; Hendriyanto et al., 2023). Other studies have applied the Anthropological Theory of the Didactic (ATD) to the analysis of lesson plans developed by prospective teachers and have integrated the Anthropological Theory of the Didactic (ATD) and the Theory of Didactic Situations (TDS) to simultaneously examine the mathematical content and didactic situations depicted in textbooks (Suryadi et al., 2023; Yunianta et al., 2023). Furthermore, studies have examined teachers' Pedagogical Content Knowledge (PCK) and their teaching strategies for quadratic functions in the classroom (Banjo, 2019; Huang et al., 2012). However, no research has yet integrated the transposition of knowledge from textbooks into teachers' lesson plans with direct observation of students' learning obstacles within a single research framework, particularly in the sub-chapter on Constructing Quadratic Functions.

Based on the analysis of these two artifacts, it is necessary to conduct a study using the didactic praxeology framework, which integrates the perspectives of ATD and the Theory of Didactical Situations (TDS) (Brousseau, 2002). The framework was adapted from Yunianta (2023), who synthesized the approaches proposed by Chevallard (2006) and Brousseau (2002). Within this framework, ATD was used to analyze the praxeological structure consisting of task types

(T), techniques (ô), technology (è), and theory (È). In contrast, TDS was used to examine didactic aspects such as student activities, the knowledge required by students, and teacher instructions. Through this integration, the study focused not only on the structure of mathematical knowledge but also on how it was organized and implemented in instructional practices.

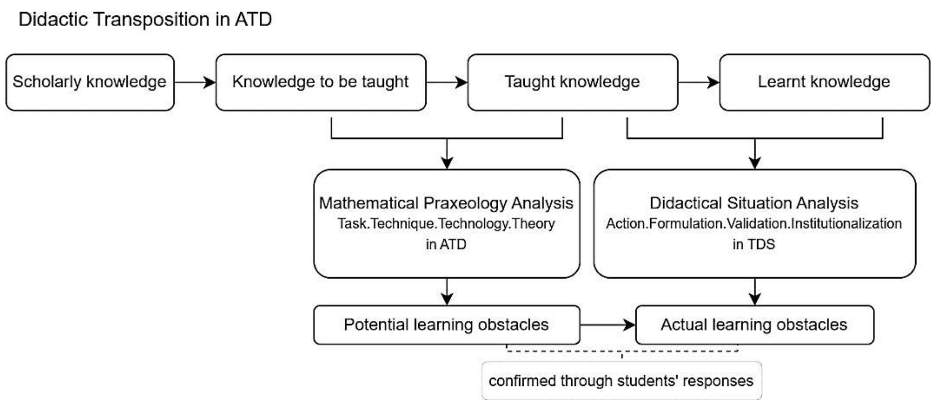

Figure 2. Conceptual map of the relation between ATD and TDS

Based on these gaps, this study addresses the following research questions: (1) What is the praxeological structure of the sub-topic of constructing quadratic functions as presented in textbooks and teachers' lesson plans; (2) What forms of internal praxeological transposition occur in the transition from textbooks to teachers' lesson plans; and (3) What are the didactic implications of the identified transposition for potential student learning obstacles. This research is expected to contribute to the development of instructional materials and more meaningful learning designs to minimize obstacles to student learning in the sub-topic of constructing quadratic functions.

■ METHOD

Research Design

This research is based on an interpretive paradigm using a qualitative approach and a case study design. This design was selected because the study focuses on an in-depth analysis of the phenomenon of internal praxeological transposition within a limited context, specifically the interaction between the textbook and the teacher's lesson plan for the sub-topic of constructing quadratic functions in a specific classroom. The case study enables the researcher to understand the phenomenon holistically within a real-life context by using multiple data sources (Baxter & Jack, 2015).

The case in this study is a bounded system, namely the instructional practice of the sub-topic of constructing quadratic functions, involving one teacher, one class, and one textbook as the primary learning resource. This approach enables an in-depth exploration of the internal transposition process, which is contextual and inseparable from the ongoing learning practices. This study employs a single instrumental case study to gain an in-depth understanding of internal praxeological transposition.

Participants Criteria

The participants consist of one mathematics teacher and tenth-grade students at a private school in Bandung, Jawa Barat. Participants' selection was conducted based on the following criteria: 1) the teacher was teaching the topic of quadratic functions at the time of the study; 2) the teacher utilized the textbook as the primary learning resource; 3) the teacher possessed relevant teaching experience; and 4) class selection was conducted conveniently based on the classes taught by the teachers who were the participants of the research. The selected classes were regular, meaning neither core nor advanced, with the understanding that the mathematical abilities of students in these classes were more varied and representative of general learning conditions. The total number of students was 30. However, due to some students being absent and others excused for school activities, only 17 students participated in classroom learning activities, comprising 8 females and 9 males.

Data Collection

Data were collected through several techniques, including:

Document analysis

Analysis was conducted on documents such as a mathematics textbook published by a private publisher (Erlangga) and a teacher's lesson plan to identify the praxeological structure. Each praxeological component was identified based on operational indicators, namely: (1) task types (T) as the form of the problem provided; (2) techniques (τ) as the resolution procedures used; (3) technology (θ) as the explanation or reasoning underlying the technique; and (4) theory (Θ) as the broader conceptual framework. The distinction between technology (θ) and theory (Θ) was determined based on the level of generality. A concept was categorized as technology (θ) when it directly justified a specific technique (τ). In contrast, a concept was categorized as a theory (Θ) when it represented a broader mathematical organization that encompassed several technologies (θ) and techniques (τ). Specifically, the identification and justification of technologies and theories were guided by the mathematics learning outcomes established by the Ministry of Primary and Secondary Education (Kemendikdasmen). Therefore, the justification of the techniques, technologies, and theories should be considered contextual and provisional, as they are interpreted within the framework of school mathematics learning at the relevant educational level, namely Grade X (Phase E) (Setiyani et al., 2025).

Classroom observation

Non-participatory classroom observations were conducted directly during the learning process to observe the implementation of instruction and didactic interactions. The analytical focus is on the sub-topic of constructing quadratic functions, selected for its representation of the connections among various mathematical forms (verbal-to-algebraic and graphic-to-algebraic). This sub-topic was analyzed during a single learning session, totaling 2 x 45 minutes. This duration was considered sufficient because the study did not aim to examine students' learning development longitudinally, but rather to identify the emergence of learning obstacles and the manifestation of internal praxeological transposition within a bounded instructional situation. The components and indicators of observation in learning are as follows:

Table 1. Components and indicators of observation

Component	Teacher and Student Activity Indicators
TDS	
Action Situation	Teacher: Presents an open-ended problem (milieu) and does not immediately provide a formula or answer. Students: Explore independently, experiment, or develop a solution plan based on their own understanding.
Formulation Situation	Teacher: Facilitates group discussions, encouraging students to explain their ideas in their own words (informally).

	Students: Develop strategies together as a group and solve problems according to the planned strategy.
Validation Situation	Teacher: Don't immediately say whether an answer is right or wrong; ask students to validate their own answers first. Students: Prove the correctness of the answer based on reasoning, not just because the teacher says so.
Institutionalization	Teacher: Links students' informal findings to formal, standard scientific concepts or formulas. Students: Recognize the position of the new knowledge they discover.
Didactic Contract	Teacher-Student: Observing an unwritten agreement in the classroom. Students' reliance on teacher explanations and on teacher-provided notes as the primary learning sources.
ATD	
Task Type and Technique	Teacher: Presents several assignments with a variety of techniques. Students: Identify the type of assignment and apply appropriate and efficient completion techniques.
Technology and Theory	Teacher: Explain the logical reasoning behind the techniques and equivalent forms of quadratic functions used. Students: Understand the background of a formula, not just memorize it.
Didactic Transposition	Teacher: The teacher's way of packaging teaching materials to suit students' cognitive levels without compromising the authenticity of scholarly knowledge. Student: The level of ease with which students understand the material presented by the teacher.
Learning Obstacles	Teachers: Anticipate or respond to students' learning obstacles. Students: Demonstrate learning obstacles due to the teacher's teaching style or the student's level of thinking development.

Semi-structured interviews

Semi-structured interviews were conducted with the teacher and several students to gain a deeper understanding of the strategies used in completing tasks, the difficulties experienced by students, and the teacher's pedagogical considerations in planning the lesson. Teacher interviews were conducted before classroom observations. The following are guidelines for teacher interviews:

Table 2. Teacher interview guide

Theory	Key Indicators	Aspects Explored
ATD (Anthropological Theory of Didactics)	Didactic Transposition	The institutional relationship between teachers and mandatory mathematics textbooks
	Analysis of Praxeology Elements	A critique of the tasks, techniques, technologies, and theories presented in textbooks.
TDS (Theory of Didactical Situations)	Situating the Learning Environment (Milieu)	Teacher strategies for modifying tasks to create a didactic (self-discovery) situation.

Meanwhile, student interviews were conducted after the lesson. The following are the indicators for student interview guidelines:

Table 3. Students interview guide

Theory	Key Indicators	Aspects Explored
TDS (Theory of Didactical Situations)	Action Situation and Learning Obstacles	Students' initial reasoning process when faced with different problem characteristics.
	Didactic Contract	Students' reliance on teacher notes rather than textbooks during the problem-solving process.
	Formulation and Validation Situation	The process of exchanging information between groups to logically agree on the validity of concepts.
ATD (Anthropological Theory of Didactics)	Technique and Technology Selection	Students' considerations in selecting techniques based on the type of task.

Data Analysis

In qualitative research, data analysis involves organizing, categorizing, and interpreting data from various sources to identify patterns, develop interpretations, and formulate conclusions. This study employed the Miles and Huberman data analysis model, which consists of three main stages: data reduction, data display, and conclusion/verification (Miles et al., 2014). The analysis process was supported by the Anthropological Theory of Didactic (ATD), particularly mathematical praxeology for textbook and teachers' lesson plan analysis, and the Theory of Didactical Situations (TDS) for analyzing classroom interactions.

Data reduction

Data reduction involved selecting, focusing, coding, and categorizing relevant information from collected data sources, including textbook documents, teachers' lesson plans, classroom observations, and interview transcripts. The teacher interview transcripts were coded to identify information related to textbook preparation, adaptation, and utilization in mathematics learning.

Document analysis was conducted using the mathematical praxeology framework from ATD, which focuses on two aspects: mathematical objects (concepts, theorems, proofs, mathematical problems, and their solutions) as the objective of the mathematics learning process, and a series of mathematical tasks that shape the students' ways of thinking and understanding toward those mathematical objects (Harel, 2008; Suryadi et al., 2023). The identified praxeological elements, such as the mathematical tasks, techniques, technologies, and theories, were compared with scholarly knowledge from textbooks and scientific literature to examine the process of didactic transposition and identify possible simplifications, modifications, or distortions of mathematical knowledge.

Classroom observation data were analyzed by coding teacher actions, student responses, and interactions within the didactical situation. The variation in group composition was taken into account when interpreting classroom interactions. Rather than comparing groups by membership size, the analysis focused on the characteristics of students' interactions, engagement with the task, use of the milieu, and construction of mathematical knowledge within each group context. Field notes and observation transcripts were categorized based on the phases of action, formulation, validation, and institutionalization in TDS. Students' responses and learning difficulties were further analyzed identify possible ontogenic, didactical, and epistemological obstacles.

Post-lesson interview transcripts from selected students' interviews with Student 1 (Problem A), Student 2 (Problem B), and Student 3 (Problem C) were analyzed through qualitative coding to identify students' reasoning processes, misconceptions, and validation strategies related to the mathematical task they encountered.

Data display

After the data reduction process, the categorized data were organized and presented systematically to facilitate interpretation. The data were presented in tables, figures, and narrative descriptions that illustrated the relationships among findings from textbook analysis, classroom interactions, and students' learning processes. These displays enabled the researcher to identify patterns and establish connections among the analyzed data sources.

Conclusion drawing/verification

The final stage of data analysis involved synthesizing all qualitative data to ensure the validity and reliability of the conclusions. The researcher conducted case-by-case theoretical triangulation by cross-referencing the practical intent of the documents, classroom interactions, and the three students' interview reflections. To comply with the principle of data correspondence and avoid bias, the draft analysis was returned to the teacher (member checking) to ensure alignment with field events. This inference process was guided by the principle of coherence to produce a coherent narrative of how the teacher's textbook adaptation influenced the design of the TDS situation and shaped the students' learning trajectories.

Ethical Considerations

Permission to conduct the study was obtained from the school administration and the mathematics teacher before data collection. Participants were informed about the purpose of the study, the voluntary nature of participation, and the confidentiality of their responses. Informed consent was obtained from the participants and/or their guardians before classroom observations and interviews. Participants had the right to withdraw from the research without penalty. To protect participants' identities, pseudonyms were used in reporting the findings.

Research Limitations

This study is limited by its scope of cases, which includes only one teacher, one class, and one textbook. Therefore, the research findings are not intended for statistical generalization. Rather, they aim to provide an in-depth understanding (analytical generalization) of the phenomenon of internal praxeological transposition. In addition, classroom observation was conducted in a single instructional session. Therefore, the findings should be interpreted as a contextual representation of the identified learning obstacles within a specific learning situation rather than as a longitudinal description of students' learning development.

RESULT AND DISCUSSION

The results and discussion of this research include narratives and documentation from textbook analysis, teachers' lesson plans, observations, and interviews with students on their perceptions, as follows.

Praxeological Structure in Textbooks and Teachers' Lesson Plans

Praxeological structure in textbooks

Based on the praxeological analysis of the textbook, the following table shows the results.

Table 4. Praxeological structure in textbooks

Sequence of Task	Task (T)	Technique (τ)	Technology (θ)	Theory (Θ)
E_1: $P_1 \to P_2 \to P_6 \to P_{10} \to P_{15}$	Constructing quadratic functions from the extreme point and one other point, involving translations from verbal to symbolic or graphical to symbolic representations	τ_1: (generational) using model selection $y - k = a(x-h)^2$, $a \neq 0$	θ_1: the concept of completing the square	Θ_1: Quadratic Function
			θ_2: The general form of a quadratic function or its equivalents	
			θ_3: the concept of extreme points (extreme values and the axis of symmetry)	
		τ_2: (transformational) Value substitution and algebraic simplification	θ_4: properties of algebraic operations	
E_2: $P_3 \to P_5 \to P_7 \to P_{15}$	Constructing quadratic functions from x-intercepts and one other point, involving translations from verbal to symbolic or graphical to symbolic representations	τ_3: (generational) using model selection $y = a(x-p)(x-q)$, $a \neq 0$	θ_5: the concept of factorization	Θ_1: Quadratic Function
			θ_2: the general form of a quadratic function or its equivalents	
			θ_6: the concept of x-intercepts	
		τ_2: (transformational) Value substitution and algebraic simplification	θ_5: properties of algebraic operations	
E_3: $P_3 \to P_4 \to P_8 \to P_{11} \to P_{12}$	Constructing quadratic functions from three arbitrary points, involving translations from verbal to symbolic or graphical to symbolic representations	τ_4: (generational) using model selection $y = ax^2 + bx + c$, $a \neq 0$	θ_2: the general form of a quadratic function or its equivalents	Θ_1: Quadratic Function
		τ_2: (transformational) Value substitution and algebraic simplification	θ_7: using system of linear equations in three variables (SPLTV)	
P_{14}	Constructing quadratic functions tangent to the x-axis and intersecting the y-axis, involving a translation from verbal to symbolic representation	τ_5: (generational) using model selection $y = a(x-x_1)^2$, $a \neq 0$	θ_1: the concept of completing the square	Θ_1: Quadratic Function
			θ_2: The general form of a quadratic function or its equivalents	
			θ_3: the concept of extreme points (extreme values and the axis of symmetry)	
		τ_2: (transformational) Value substitution and algebraic simplification	θ_5: properties of algebraic operations	
P_9	Determining the parabola's equation when its axis of symmetry is	τ_6: (generational) using model selection $x = ay^2 + by + c$, $a \neq 0$	θ_8: A quadratic equation with variable y results in a horizontal parabola	Θ_2: Conic Sections (Parabola) / Analytical

	parallel to the x-axis, involving a translation from verbal to symbolic representation	τ_2: (transformational) Value substitution and algebraic simplification	θ_7: using system of linear equations in three variables (SPLTV)	Geometry
P_{13}	Determining x-intercepts and the extreme point, involving a translation from verbal to symbolic representation	τ_7: (generational) Formula selection and determining the values of each coefficient and constant τ_2: (transformational) Value substitution and algebraic simplification	θ_3: the concept of extreme points (extreme values and the axis of symmetry) θ_6: the concept of x-intercepts	Θ_1: Quadratic Function

Praxis Block (Type of Task (T))

The analysis results show that there are 18 tasks in the sub-topic of constructing quadratic functions, with 3 tasks as examples (E) and 15 as practice exercises (P). The majority of tasks in the textbook are evaluative. It is about 83,33%.

Table 5. Type of task (T)

Type of Task (T)	Type of Task Based on Use
T_1: to construct a quadratic function of a point or a graph	E: for example, (number of tasks (n=3)) as explorative tasks
	P: for a test of comprehension (number of tasks (n=15)) as evaluative tasks, 5 multiple-choice tasks, 10 essay tasks.

This dominance of evaluative tasks indicates that learning activities are more procedure-oriented than concept-exploration-oriented, which may limit the development of students' deep understanding. Furthermore, most of the mathematical tasks in the textbook are 'closed tasks,' consequently encouraging students to engage in imitative reasoning (Lithner, 2017; Watson & Mason, 2006).

Furthermore, two tasks were found to be inconsistent with the subtopic of constructing quadratic functions. First, the task P_9: "*The axis of symmetry of a parabola is parallel to the x-axis. If the parabola passes through the points (1,-2), (5,6), and (2,-6), determine the equation of the parabola.*"

This task presents a parabola with an axis of symmetry parallel to the x-axis, which mathematically leads to the equation $x = ay^2 + by + c$ If students are not careful, the task P_9 will be solved using the technique τ_4 with justification θ_3, because three points are provided in the task. Procedurally, a quadratic function would be formed in the general form y = ax2 + bx + c namely $y = 2x^2 - 10x + 6$. However, given the information that the parabola's axis of symmetry is parallel to the x-axis, the equation should be $x = \frac{1}{16}y^2 + \frac{1}{4}y + \frac{5}{4}$

This conceptual inconsistency can lead to epistemological learning obstacles when material design or learning activities do not support the development of epistemic knowledge, thereby failing to help students build understanding aligned with the structure and logic of mathematics (Utami et al., 2023). Consequently, students may encounter difficulties in extending this understanding to other mathematical concepts, potentially leading to ontogenetic obstacles stemming from an individual's cognitive development or learning experience (Abou-Hayt, 2024). According to Chevallard, this can be understood as a consequence of didactic transposition that does not fully maintain epistemological consistency between scientific knowledge and school knowledge.

Based on the interview with the teacher, the book's tasks were used when appropriate or modified by the teacher. This means that if there were any inconsistencies, such as a task , it was not given to students. However, the task would also confuse students when they were studying independently and encountered the task in the textbook.

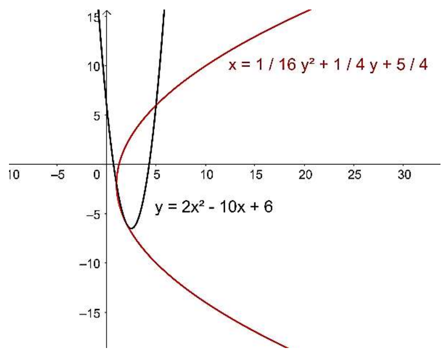

Figure 3. Graph of a quadratic function $y = 2x^2 - 10x + 6$ dan $x = \frac{1}{16}y^2 + \frac{1}{4}y + \frac{5}{4}$ Source: Researcher's GeoGebra application)

Second, the task P_{13} (see Table 4) does not demonstrate a clear function as part of the practice exercises within the sub-topic of constructing quadratic functions. This lack of clarity stems from its placement within the task sequence, which prevents it from serving as a prerequisite recall task (apperception) that should ideally appear at the beginning of the learning phase.

Praxis block (technique(τ))

The impact of task quality is directly reflected in the techniques employed, with analysis adopting Kieran's categorization of algebraic activities: generational, transformational, and global-meta-level techniques. These components focus on meaning-making, equivalence rules, and broader problem-solving through an analysis of praxis and logos blocks (Kieran, 2004). The provided text outlines the conceptual framework for this analysis.

Analysis of Table 4 indicates that the task is the only task allowing for solution technique selection with justifications at the task E_2 or E_3. Furthermore, the alternative solution presented in task E2 is more efficient.

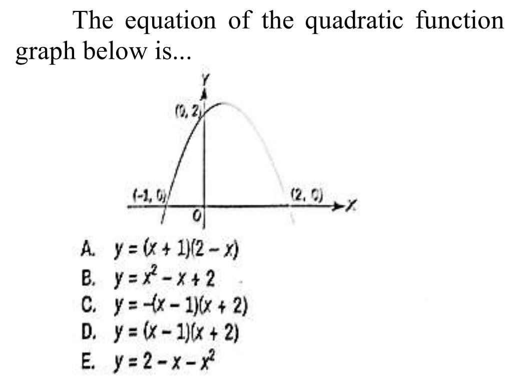

Figure 4. Task $P_{3,}$ (Source: Mathematics textbook for 10th-grade students by erlangga publisher)

Look at the task , the known point information is the x-instercept point, namely (-1, 0) and (2, 0), so that if using the technique in task E_2, substitute the coordinates of the x-intercept point into the equivalent form of the quadratic function y = $a(x - p)(x - q)$, a ≠ 0 to become y = $a(x - (-1))(x - 2)$. The other point that can be used in the point (0, 2) on the graph, which is the y-intercept point. Substitute it into the quadratic function y = $a(x - (-1))(x - 2)$ to become 2 = $a(0 + 1)(0-2)$so that a = -1 is obtained. So, the equation of the quadratic function graph is y = -$(x + 1)(x - 2)$ or if looking at the answer choices, it is option A Y = $(x + 1)(2 - x)$.

Another alternative can be solved using the technique in E_3. Substitute the known coordinates of the points from the graph into the general form of the quadratic function, y = $ax^2 + bx + c$,. $a \neq 0$. Three linear equations with three variables are obtained as follows: Point (-1, 0)→ 0 = $a(-1)^2 + b(-1) + c \rightarrow a - b + c = 0$… (1)

Point (2, 0)→ 0 = $a(2)^2 + b(2) + c \rightarrow 4a + 2b + c = 0$ … (2)

Point (0, 2)→2 = $a(0)^2 + b(0) + c$ → $c = 2$… (3) Next, substituting the value of into (1) and (2), we obtain:

$A - b + 2 = 0$ → $a - b = -2$ … (4) $4a + 2b + 2 = 0$ → $4a + 2b = -2$… (5) Next, eliminate equations (4) and (5), so that the values of , , and are obtained. Substitute the values , , and into the general form of the quadratic function so that .Looking at the answer choices, it is option A $Y = (x + 1)(2 - x)$.

Based on that task, students have many technical options for completing assignments. Nevertheless, overall, the presented tasks tend to position techniques primarily at the transformational stage. Each task relies on techniques with fragmented justifications. Consequently, the praxis block (T–τ) is not optimally integrated, leading to a potential situation where students perceive each technique as an isolated procedure.

Logos block (technology (è))

One of the technologies is not explicitly reintroduced in the textbook, such as technology. Moreover, technology θ_8 is similarto θ_2, so it is highly vulnerable becoming epistemological obstacle.

Table 6. Description of the relationship between techniques and technologies

Technique (τ)	**Technology (θ)**
τ_1: (generational) using model selection $y - k = a(x-h)^2, a \neq 0$	θ_1: the general form of a quadratic $y = ax^2 + bx + c, a \neq 0$, can be transformed into the following form using the method of completing the square: $y = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$ $y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a}\right)$ $y = a((x + \frac{b}{2a})^2 - \frac{b^2}{4a^2} + \frac{c}{a})$ $y = a(x + \frac{b}{2a})^2 - (\frac{b^2 - 4ac}{4a})$ $y = a(x - (-\frac{b}{2a}))^2 - (\frac{b^2 - 4ac}{4a})$ $y - \left(-\left(\frac{b^2 - 4ac}{4a}\right)\right) = a\left(x - \left(-\frac{b}{2a}\right)\right)^2, a \neq 0$ suppose that $h = -\frac{b}{2a}$ dan $k = -\left(\frac{b^2-4ac}{4a}\right)$
	θ_2: the equivalent form of the quadratic function is $y - k = a(x-h)^2, a \neq 0$
	θ_3: the coordinate point (h, k) represents the extreme point of the quadratic function, where the abscissa is the axis of symmetry and the ordinate is the extreme value.
τ_2: (transformational) Value substitution and algebraic simplification	θ_4: the properties of algebraic operations are used to solve the equations obtained after algebraic substitution. θ_7: the system of linear equations in three variables (SPLTV) is used specifically to solve a system of three algebraically substituted equations involving three variables.
τ_3: (generational) using model selection $y = a(x-p)(x-q)$, $a \neq 0$	θ_5: the general form of a quadratic function $y = ax^2 + bx + c, a \neq 0$, can be transformed into the following form using the factoring method: $y = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$ If p and q represent the roots of a quadratic equation, then

	$p+q=-\frac{b}{a}$ $pq=\frac{c}{a}$ $y=a(x^2-(p+q)x+pq)$ $y=a(x^2-px-qx+pq)$ $y=a(x(x-p)-q(x-p))$ $y=a(x-p)(x-q), a\neq 0$
	θ_2: the equivalent form of the quadratic function is $y=a(x-p)(x-q), a\neq 0$
	θ_6: the x-intercepts of the quadratic function graph are $(p,0)$ and $(q,0)$.
τ_4: (generational) using model selection $y=ax^2+bx+c, a\neq 0$	θ_2: the general form of a quadratic function $y=ax^2+bx+c, a\neq 0$
τ_5: (generational) using model selection $y=a(x-x_1)^2, a\neq 0$	θ_1: As previously explained θ_2: the equivalent form of the quadratic function is $y=a(x-x_1)^2, a\neq 0$ θ_3: when the parabola is tangent to the x-axis, it intersects the x-axis at only one point, namely $(x_1,0)$. This point can also be referred to as the extreme point, where the axis of symmetry is x_1 and the extreme value is zero.
τ_6: (generational) using model selection $x=ay^2+by+c, a\neq 0$	θ_8: A quadratic equation in terms of the variable y produces a horizontal parabola, therefore, it is not considered a quadratic function.
τ_7: (generational) Formula selection and determining the values of each coefficient and constant	θ_3: As previously explained θ_6: As previously explained

Logos block (theory (Θ))

There are three theories built from a series of tasks, techniques, and technologies in the textbook, including: 1) Quadratic Function; and 2) Conic Sections (Parabola)/Analytical Geometry. Theory Θ_3 should have been introduced at Grade XII or within Phase F of Advanced Mathematics. However, based on the learning outcomes issued by Kemendikdasmen in 2022, conic sections/analytical geometry topics, particularly parabola and hyperbola, are no longer included in the Indonesian high school mathematics curriculum. The remaining topics are limited to circle equations, ellipse equations, and their tangent line equations. Therefore, Theory Θ_3 should not have been presented in the textbook, particularly in Grade X, because it is not part of the current high school mathematics curriculum, and its inclusion disrupts the intended praxeological progression, in which students should first develop the prerequisite knowledge before constructing more advanced mathematical structures.

Praxeological structure in teachers' lesson plans

The study found that teachers modified the textbook task because it was deemed too concise and rigid. Based on the interviews, the teacher explained that the textbook presents only one solution technique for each task type. Therefore, the teacher modified the instrument to include open-ended tasks that allow exploration of various solution techniques. However, teachers selectively retained some tasks with a single technique to strengthen students' mastery of basic procedures and align with learning objectives.

Table 7. Several tasks were assigned by the teacher in the classroom

Task (T)
Problem A: Given three points that are located on the curve of a quadratic relation. Those three points are (0, -6), (2, -6), and (3, 0). Determine the form of the mentioned quadratic relation.
Problem B: Given the parabola of a quadratic equation that passes through point (3, 0) and (-1, 0). If the inflection point of the parabola is (1, -8), determine the form of the quadratic relation.
Problem C: Given three points that are located on the curve of a quadratic equation. Those three points are (3, 0), (-1, 0), and (4, 10). Determine the form of the mentioned quadratic relation.

Learning objectives are directed not only toward students' ability to determine the general form of quadratic functions based on given points but also toward developing an understanding of the interconnectedness among praxeological components: technique, technology, and theory. The teacher appears to strive to encourage students to realize that various equivalent general forms of quadratic functions are obtained through techniques that possess interconnected technological and theoretical foundations. This is further evidenced by the task instructions and requirements in the teacher's lesson plan:

"The teacher will make the class work in groups consisting of two or three students maximum. Each group will receive one problem related to quadratic equations. Students are given time to work on the problem and discuss their solution. If students encounter difficulties, the teacher encourages them to collaborate with other groups that have similar problems. Students are then asked to present their final answers and discuss the solution strategies."

The lesson plan also indicates that students were engaged with several types of quadratic equation problems: "Three different types of problems related to creating quadratic equations with the same final answer are provided: Problem A related to Type 1 of quadratic equation, Problem B related to the union of Type 2 and Type 3 of quadratic equation, and Problem C related to Type 3 of quadratic equation."

Consequently, students have the opportunity to select and use the techniques they deem most efficient, based on their own conceptual understanding. The teacher's expectations regarding the techniques students will employ to construct quadratic functions include the following:

In Problem A, students can utilize the techniques, technologies, and theories associated with E_3, using the general form of a quadratic function derived from three arbitrary points on the parabola with the equation. $y = ax^2 + bx + c, a \neq 0$ Procedurally, this technique is the most common but also involves a more extensive solution process.

For Problem B, students may choose techniques, technologies, and theories from E_1 and E_2. These involve the general forms derived from the parabola's extreme point $y - k = 0a(x - h)^2, a \neq 0$ and those derived from the x-intercepts $y = a(x - p)(x - q), a \neq 0$.

Similarly, Problem C allows for the use of techniques, technologies, and theories from E_2, using the x-intercept form $y = a(x - p)(x - q)$, a 0. As with Problem B, the x-intercepts are not explicitly provided, and students are expected to recognize them. However, unlike Problem B, Problem C does not provide information regarding the extreme point. Therefore, the techniques, technologies, and theories associated with E_1 $y - k = a(x - h)^2$ $(a \neq 0)$ cannot be applied.

Meanwhile, based on the textbook's praxeological analysis, the technique presented in Task E3 can be applied to all types of tasks modified by the teacher. However, for some problems, this technique is not as effective as the techniques presented in other tasks. Nevertheless, the general technique introduced in Task E3 should still be presented, as it offers a more general approach applicable to a range of problems.

Table 8. Adaptation of the praxis block in the teacher's lesson plan from the textbook

Tasks in Teachers' Lesson Plans (T_R)	Expectation Teacher	Tasks in Textbook (T_B)
Problem A	Problem A can only be solved through the technique identified in Task E_3	E_3
Problem B	Problem B can only be solved through the technique identified in Task E_1 and Task E_2.	E_1. E_2. and E_3
Problem C	Problem C can only be solved through the technique identified in Task E_2	E_2 and E_3

Furthermore, the lesson plan designed by the teacher indicates the characteristics of 'procedures with connections' (level 3) within the cognitive demand framework developed by Margaret Schwan Smith and colleagues at the University of Pittsburgh (Hsu & Yao, 2023). This is evident in the opportunities provided for students to generalize various forms of quadratic functions by examining the relationships among the concepts involved.

Forms of Internal Transposition

The comparative results between the textbook and the teacher's lesson plan reveal a shift in the praxeological structure during internal transposition. This change primarily occurs within the praxis block (T–ô), while the logos block (è–È) remains relatively unchanged. Within the praxis block (T–ô), the task types in both the textbook and the teachers' lesson plans tend to be similar, classifying tasks such as those based on the extreme point, x-intercepts, and three arbitrary points. However, the teacher modified or reorganized the tasks by designing scenarios in which various given conditions lead to equivalent forms of quadratic functions. This reorganization indicates an effort to simplify the task structure for more controlled implementation during instruction.

On the other hand, there was a reduction in mathematical representation. Tasks in the textbook that originally included transformations from graphical to algebraic representations no longer appear in the teacher's lesson plan. The omission of graphical-to-algebraic tasks from the teacher's lesson plan may limit students' opportunities to develop representational translation skills. In the context of quadratic functions, the ability to coordinate graphical, symbolic, and verbal representations is essential for building relational understanding rather than merely procedural competence. Consequently, the reduction in representational variety may contribute to epistemological and ontogenetic obstacles, particularly when students encounter unfamiliar forms of quadratic representation outside routine symbolic procedures.

A more significant difference is observed in the technical aspect (ô), despite sharing the same categorical levels of algebraic activity according to Kieran (Kieran, 2004). Procedurally, however, these tasks differ as they involve mathematical models that refer to different technologies. The textbook tends to present resolution techniques separately for each task type, so the relationships between techniques are not made explicit. In contrast, the teacher's lesson plan provides students with the flexibility to utilize various techniques to solve a single type of task.

At first glance, the availability of multiple techniques appears to increase the cognitive demand of the tasks by allowing students to select strategies based on given conditions. However, because these alternative techniques lacked explicit technological and theoretical reinforcement, the flexibility tended to remain procedural rather than conceptual. As a result, students may perceive different quadratic forms as merely interchangeable formulas rather than interconnected representations grounded in the same mathematical structure.

The gap between the developing praxis block and the stagnant logos block indicates a praxis that is operational yet epistemologically fragile. Students can apply procedures adaptively without understanding their conceptual justification. This aligns with previous research that procedural adaptation without theoretical underpinning hinders conceptual coherence and relational understanding (Jatisunda et al., 2025).

Table 9. Internal praxeological transposition from the textbook to the teacher's lesson plan

Components	Textbooks	Teachers' Lesson Plans	Forms of Change
Task (T)	Separate	Reorganization	Changed
Technique (τ)	Separate	Flexible (multi-technique)	Adaptation
Technology (θ)	Implicit	Implicit	Unchanged
Theory (Θ)	Not Explicit	Not Explicit	Unchanged

According to Didactic Anthropology Theory (ATD), this fragmentation leads mathematical activities to focus more on technical execution than on the reconstruction of meaning. An excessive emphasis on operational aspects without conceptual support in teaching materials risks limiting epistemic access and hindering comprehensive understanding (Sholikhakh et al., 2025).

This dialectic between technical and conceptual dimensions is also examined in studies on the development of 'instrumentation' in technology-assisted mathematics learning (Computer-Assisted Systems) (Artigue, 2002). While technology can expand students' technical capabilities, the challenge of building deep knowledge remains. Consequently, teacher intervention to emphasize concepts through reflective activities and reasoning tasks becomes crucial.

A critical analysis of mathematics pedagogy across various regions reinforces these findings. In the context of instructional technology use, the focus tends to be on task completion and the application of techniques, while the underlying rationale is rarely articulated (Mensah et al., 2024). This phenomenon suggests that without deliberate efforts to build explicit logos through instructional design, the resulting praxeology, for both trainee teachers and pupils, tends to reproduce the same structural imbalance.

Empirical evidence of this imbalance is evident in a case study of GeoGebra's use in Ghana. Although the learning process improved procedural fluency and student participation, the practices consistently applied were still dominated by the praxis block rather than the logos block (Mensah, 2025). Consequently, students' mathematical knowledge remained incomplete. As emphasized by the NCTM, procedural instruction must be accompanied by conceptual understanding to be effective. Without a strong foundation of understanding, students are prone to making errors and failing to recognize illogical solutions (National Council of Teachers of Mathematics, 2023).

This situation ultimately creates learning barriers, particularly in students' ability to connect representations of quadratic functions and determine reasoning-based solution strategies. Although students can achieve operational proficiency, they often lack the reflective capacity to explain the reasoning behind procedures or to identify their contextual applications (Sadiah et al., 2024).

In light of these patterns of institutional and pedagogical tension, the concept of 'limited adaptive transposition' becomes relevant. This differs from constructive transposition, which seeks to link praxis and logos coherently. Limited adaptive transposition prioritizes operational effectiveness without epistemological reconstruction. Therefore, making implicit reasoning explicit and reconstructing fragmented conceptual explanations is not merely an additional improvement but a key prerequisite for developing mathematical praxis that is both operationally effective and epistemologically robust.

Didactic Implications for Potential Student Learning Obstacles

Analysis reveals that the textbook's praxeological structure and the lesson plan's form of internal transposition have different implications for potential learning obstacles in both teacher-led and independent learning contexts.

Epistemological obstacle

Regarding instructional materials, several tasks in the textbook on constructing quadratic functions lack adequate praxeological support. These tasks were not accommodated in the lesson plan and did not appear in classroom practice, indicating the teacher's selective role in anticipating material weaknesses (Fuadiah et al., 2019). However, the didactic implications remain significant, as these tasks could trigger learning obstacles when students use the textbook independently. Furthermore, classroom tasks are still dominated by verbal-to-algebraic expressions, excluding the transformation of graphical representations into algebraic ones. This limited representational variety implies the emergence of epistemological obstacles, particularly in connecting different representations of quadratic functions.

In line with the case from students assigned to Problem C, they initially struggled to interpret the information provided in the task. This difficulty can be interpreted as an epistemological obstacle because students initially relied on explicit mathematical labels to identify important features of quadratic functions rather than interpreting the underlying meaning of the given representations. In the task, the x-intercepts were not directly labeled. Instead, they were presented through coordinate representations in the form of $(a, 0)$ and $(b, 0)$. Although students had previously encountered this representation, they were unable to immediately recognize that coordinates with a y-coordinate of zero represented the points where the quadratic function intersected the x-axis. Therefore, students referred back to their previous notes and found that the coordinate forms $(a, 0)$ and $(b, 0)$ corresponded to the x-intercepts. This indicates that students' understanding was still dependent on explicit terminology and previously provided representations rather than on a flexible connection between symbolic and conceptual representations of quadratic functions. The obstacle was resolved when students reconstructed the relationship between the coordinate representation and the characteristics of quadratic functions, enabling them to complete the task. Ideally, teachers should anticipate this, since multiple representations foster meaningful understanding (Zhu et al., 2025).

Didactical obstacle

Anticipation involved adapting technical aspects in the lesson plan. However, observations show that student strategy exploration has not developed optimally. Students tend to use techniques matching the provided examples, even though the teacher facilitated alternative strategies. This indicates a didactic obstacle, in which technical flexibility at the planning level is not fully realized in practice. This is reinforced by student statements: *"We usually follow the examples. If it's different, we get confused"* (Interview with student S3), showing dependence on modeled patterns. While temporary assistance or scaffolding through simplification is a valid didactic anticipation (Mariyani et al., 2021), over-scaffolding occurred, resulting in passive learners who were unable to develop strategies or draw sound conclusions (Aridanthy et al., 2026; Zakiah et al., 2025).

Observations also revealed performance differences across task types. The group assigned to Problem B completed the task faster because, based on the praxeological analysis, Problem B offers more alternative techniques. Although students appeared able to complete the task efficiently, interview data suggest that their success stemmed from first identifying the extreme points and then following established procedures to resolve the problem. This suggests that students' procedural fluency may not always reflect their relational understanding of the concept, which is consistent with Skemp's notion of pseudo-understanding (Skemp, 1976). In this context, the flexibility intended in the lesson plan was not fully transformed into reflective mathematical reasoning during classroom practice. However, following an information exchange with the group assigned to Problem C, students identified the x-intercepts and recognized that Problem B could be resolved using alternative techniques. This outcome indicates that the pedagogical scenario aligned with the teacher's expectations, particularly for the students in the Problem B group.

A didactical obstacle also arose regarding the learning organization. Group discussion is effective only when well managed (Gagulu, 2023). Previously, the teacher's lesson plan specified a maximum of three students per group. However, due to several practical considerations during classroom implementation, the groups were ultimately reorganized into four to five members, with some consisting of only two. The two-member group was formed because two students arrived late to the class. To avoid disrupting the existing groups, the teacher decided to create a separate group for these students. The group composition appeared to limit opportunities for equitable participation in mathematics during discussion activities. From a didactical perspective, active student engagement is essential across the phases of action, formulation, validation, and institutionalization. When participation is dominated by one or two students, the didactic milieu becomes less accessible to other group members, potentially limiting their opportunities to construct mathematical meaning through interaction.

Ontogenic obstacle

Regarding technical complexity, students assigned Problem A struggled with tasks involving three points because of the lengthy, complex system of linear equations in three variables. This is further evidenced by student testimonials regarding the arduous procedural requirements, such as the elimination of three-variable linear equations, and the occurrence of computational errors. Based on the observation results, several students repeated the solving process and re-examined the steps that contained errors. Some members of other groups only observed the process, while others gave up and decided not to review the errors. Before repeating the solving process, the students attempted to identify the source of their errors but were unable to do so, prompting them to seek assistance from the teacher. The teacher then asked the students to recheck their solution steps. However, they still could not identify the mistake. Finally, the teacher pointed out that the error occurred during the simplification of the quadratic function, where the students divided the left side by the coefficient of x squared, but x did not apply the same operation to the right side. After understanding the source of the error, the students repeated the solving process and successfully constructed the correct quadratic function.Top of Form

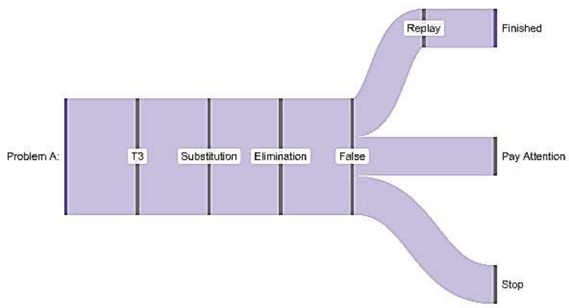

Figure 5. Stages of completing problem a

These challenges highlight a conceptual ontogenic obstacle (Suryadi, 2023) involving the misconception that simplifying quadratic functions reduces them to quadratic equations (Huang et al., 2012). The students' initial lack of error recognition suggests that such obstacles impede the construction of subsequent mathematical knowledge.

CONCLUSION

This research demonstrates that the praxeological structure of constructing quadratic functions in both the textbook and the teacher's lesson plan shares similar characteristics in terms of task types (T), but differs in the technical aspect (ô). Nevertheless, in both sources, the technological (è) and theoretical (È) aspects are not explicitly developed, resulting in a praxeological structure dominated by a procedural approach.

The internal transposition occurring from the textbook to the lesson plan is categorized as limited adaptive transposition, characterized by task modifications and technical adaptations without strengthening the conceptual aspects.

The didactic implications of these conditions are reflected in the potential learning obstacles identified in the observed instructional context. Epistemological obstacles arise from limitations in connecting various representations of quadratic functions. Didactic obstacles arise from the dominance of techniques that lack conceptual explanation, leading students to rely on modeled examples and to engage in ineffective group work. In contrast, ontogenic obstacles stem from misconceptions about learning experiences and psychological factors such as resilience.

Based on ATD theory, teachers are recommended to independently analyze mathematical concepts, especially the topic of quadratic functions in textbooks, and modify rigid tasks into open-ended ones. Learning should also incorporate multiple representations to make it more meaningful. To minimize learning obstacles, the lesson plan format needs to be reconstructed using the TDS framework by integrating predictions of student responses and activities. Through this a priori analytical readiness, teachers will be more adaptive in teaching students in the classroom.

DECLARATION OF GENERATIVE AI USAGE IN THE WRITING PROCESS

During the writing of this manuscript, the authors employed ChatGPT and Gemini to assist with language refinement/proofreading. The authors have reviewed and edited the content generated by this tool and assume full responsibility for the content of the published article.

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