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Understanding Students' Learning Obstacles in Geometric Transformations and Their Implications for Didactical Design

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Abstract: Geometric transformations are one of the essential topics in mathematical learning, as they support students' spatial reasoning, spatial visualization, and understanding of geometric relationships. Despite its importance, many students still have difficulties in interpreting transformation concepts beyond routine procedural exercises, particularly problems that are presented in unfamiliar or contextual situations. Therefore, this study aimed to explore students' learning obstacles in learning geometric transformations and examine their implications for the development of didactical design. Qualitative approach with a didactical design research framework was employed using a phenomenological perspective. Participants involved were 36 ninth-grade students and one mathematics teacher from a junior high school in West Bandung Regency, Indonesia. Research data were collected through a written test on mathematical spatial ability consisting of four contextual problems, a semi-structured interview, and a praxeological analysis of a textbook. The written-test instrument was validated by two mathematics education experts and one experienced teacher before implementation. The data were analyzed using the Miles and Huberman interactive model, involving data reduction, display, and conclusion drawing, to categorize students' difficulties into epistemological, ontogenic, and didactical obstacles. The research revealed that students experienced interconnected categories of learning obstacles. Epistemological obstacles appeared in students' inability to concept generalization, misconceptions of the transformations' properties, and dependence of procedural examples. Ontogenic obstacles related to limited spatial relations ability, weak understanding of Cartesian coordinates, and difficult to interpret contextual problems. Lastly, didactical obstacles emerged from teacher-centered instructions, limited use of interactive visual media or technology, and emphasis on procedural completion compared to conceptual exploration. These obstacles interacted with one another and influenced students' understanding of geometric transformations. The study implies that effective didactical design should integrate conceptual understanding, spatial visualization, multiple representations, contextual learning, and dynamic technology to facilitate meaningful learning of geometric transformations.

Keywords: learning obstacles, geometric transformation, epistemology, ontogenic, didactic.

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INTRODUCTION

Geometric transformations are a fundamental topic in school mathematics, as they enable students to understand object movement, symmetry, similarity, congruence, and spatial relationships through the dynamic manipulation of geometric objects. It includes translations, reflections, rotations, and dilatations, which are crucial for developing spatial reasoning, visualization, and problem-solving abilities required in real-world data science, engineering, architecture, and other STEM disciplines (Battista, 2007; Lowrie et al., 2020; Ramful et al., 2017; Xie et al., 2025). Learning geometric transformations, which emphasize previously mentioned abilities, helps students to construct and manipulate psychological representations of multidimensional objects (Xie et al., 2025). Through transformation concepts, students are expected to interpret position, movement, orientation, and changes in size mathematically (Kyaw, 2025).

Learning geometric transformations is closely associated with mathematical spatial ability (Alvarez-Vargas et al., 2020; Bicer et al., 2024; Daker et al., 2022; Lu et al., 2026; Wei et al., 2024). Spatial ability enables students to mentally manipulate objects, recognize invariant geometric properties, coordinate multiple representations, and interpret relationships between original figures and their images as a result of transformations (Buckley, 2019; Cheng & Mix, 2014; Linn & Petersen, 1985; Lowrie et al., 2020; Maier, 1996). Students with well-developed spatial abilities generally demonstrate stronger conceptual understanding of geometry because they can visualize transformations mentally rather than relying exclusively on symbolic procedures or memorized formulas (Ramful et al., 2017; Sinclair & Bruce, 2015; Takeuchi & Shinno, 2020; Tay & Preciado Babb, 2023). Therefore, the development of spatial ability has become a major objective of geometry instruction in many mathematics curricula worldwide.

Despite its importance, many students struggle to understand geometric transformations meaningfully. Research revealed that most students have difficulty identifying contextual problems as they were confused in differentiating types of transformations, used procedural knowledge rather than conceptual understanding, and inaccurately coordinated algebraic and geometric representations (Ada & Kurtuluº, 2010; Asunda et al., 2026; Fujita et al., 2022; Ramful et al., 2017; Yang & Cho, 2024). Students often succeed in applying memorized coordinate transformation rules but fail to explain the underlying geometric meaning of the transformations or transfer their knowledge to unfamiliar contexts (Cesaria & Herman, 2019; Guven, 2012; Sinclair & Bruce, 2015; Xie et al., 2025). These findings indicate that procedural proficiency alone does not guarantee conceptual understanding.

From a didactical perspective, these difficulties cn be categorized as learning obstacles. Suryadi (2019) defined a learning obstacle as one of the learning difficulties caused by an external factor, which is didactical design. According to Brousseau's Theory of Didactical Situations, learning obstacles include epistemological, ontogenic, and didactical obstacles (Brousseau, 2002; Pauji & Juandi, 2023). Epistemological obstacles occur when students' prior knowledge is insufficient in new contexts; ontogenic obstacles relate to students' cognitive readiness; while didactical obstacles emerge from instructional approaches and learning situations provided by teachers (Brousseau, 2002; Isnawan, 2023; Suryadi, 2019b). Rather than occurring independently, these obstacles frequently interact and collectively influence students' conceptual development during mathematics learning (Can Cabrera et al., 2021).

The identification of learning obstacles is crucial for designing instructional situations that are responsive to how students think (Asunda et al., 2026; Daker et al., 2022; Lu et al., 2026; Meika et al., 2025; Priatna et al., 2026). Within the Didactical Design Research (DDR) framework, learning obstacles emphasis one of the principal foundations for developing didactical designs that anticipate students' misconceptions and support meaningful conceptual construction (Rosita et al., 2019; Suryadi, 2019a). Consequently, analyzing students' difficulties should not be limited to identifying errors in their answers. However, it should also investigate why those errors occur and how instructional design may contribute to or reduce them.

Recent studies related to students' difficulties in geometry, particularly in spatial reasoning, spatial visualization, conceptual understanding, and integration of technologies, have increased gradually in recent years (Bicer et al., 2024; Branoff et al., 2022; Crompton, 2024; Gebremeskel, 2025; Gunèaga, 2024; Osman et al., 2025). Nevertheless, they have predominantly focused on describing students' performance or evaluating instructional approaches. Meanwhile, limited attention has been given to exploring how students' learning obstacles can systematically inform the development of didactical design. Accordingly, empirical studies explicitly connecting learning obstacle analysis with the design of responsive learning situations in geometric transformations remain scarce.

Based on this research gap, the present study addresses the following research question: How do students' learning obstacles in geometric transformations inform the development of didactical design?

Accordingly, this study aims to explore students' learning obstacles in geometric transformations and examine how the findings can be more responsive to didactical design and more meaningful in improving students' experience of learning mathematics. The causes and forms of students' difficulties identified in this study will contribute to the development of instructional design that better supports students' understanding and spatial reasoning in learning geometric transformations.

■ METHOD

Research Design and Procedures

This study employed a qualitative approach with a didactical design research framework. The study was grounded in Brousseau's theory of learning obstacles, which classifies students' difficulties into epistemological, ontogenic, and didactical obstacles. The qualitative descriptive approach enabled the researchers to analyze students' written responses, interview data, and textbook analysis to identify the characteristics and underlying causes of learning obstacles and subsequently formulate implications for didactical design. From this perspective, the study not only focuses on students' errors but also on the proper understanding of the reasons behind those errors from students' point of view.

The study was conducted from October 2025 to April 2026 in a public junior high school in West Bandung Regency, Indonesia. The research consisted of four sequential stages: (1) administering a mathematical spatial ability-based written test, (2) identifying students' learning obstacles through analysis of their written responses, (3) conducting semi-structured interviews with representative students and the mathematics teacher to explore the underlying causes of the identified obstacles, and (4) analysing the findings together with the students' textbook to formulate implications for didactical design.

Participants

Participants of this study were 36 ninth-grade students and one mathematics teacher from a junior high school in West Bandung regency. The participants were purposively selected to obtain information-rich subjects relevant to the research objectives. The students considered in this study were those who had learned geometric transformations and had diverse mathematical abilities, to capture the various learning obstacles they experienced when solving contextual transformation problems. Prior achievement was determined by a diagnostic test administered before the study. The diagnostic test scores were categorized into three achievement levels using the mean and standard deviation method. Students whose scores were greater than M + 1SD were categorized as high-achieving, those whose scores fell between M " 1SD and M +

1SD were categorized as moderate-achieving, and those whose scores were below M " 1SD were categorized as low-achieving. Based on this categorization, six students were classified as high-achieving, twenty-five as moderate-achieving, and five as low-achieving. These categories were subsequently confirmed through classroom records and consultation with the mathematics teacher to ensure that the selected participants represented heterogeneous mathematical abilities. The students' characteristics shown in Table 1.

Table 1. Students' characteristics

Characteristics	Information
Grade	IX
Semester	First Semester
Number of students	36
Sampling	Purposive sampling
Diagnostic Test Mean (M)	59.027
Standard Deviation (SD)	11.55
High achievement	6 students
Moderate achievement	25 students
Low achievement	5 students
Basis of classification	Diagnostic test score, classroom performance, teacher recommendation

The mathematics teacher was selected for their direct involvement in teaching geometric transformations and their experiences in implementing instructional materials in the classroom. Prior to data collection, permission to conduct the study was obtained from the school administration. Permission for students' participation was coordinated through the school in accordance with the school's procedures for educational research involving minors. Students were informed that their participation was voluntary and that they could withdraw from the study at any stage without any academic consequences. To maintain confidentiality, all participant identities were anonymized, and no personally identifiable information is reported in this study.

Instruments

Data were collected through mathematical spatial ability-based written tests, semi-structured interviews, and students' textbooks to explore students' conceptual understanding and identify their difficulties with symbolic, visual, and procedural representations of geometric transformations. The written test consists of four contextual items that represent translation, reflection, rotation, and dilatation on the Cartesian plane. The items were developed based on the mathematical spatial ability domain, which includes spatial visualization (SV), mental rotation (MR), spatial orientation (SO), spatial relations (SR), and visualization transformation (TV) (Battista, 2007; Linn & Petersen, 1985; Maier, 1996). The written test underwent content validation by two mathematics education experts and one experienced mathematics teacher prior to the implementation. Several aspects were evaluated from the validation process, including: (1) relevance to research objectives, (2) suitability with transformation concepts, (3) clarity of contextual situations and linguistic appropriateness, and (4) items' potential to reveal learning obstacles faced by students. All suggestions from validators were considered to revise any ambiguous contextual representations and ensure the appropriateness of the problems before the test was administered to students. After the written test, five students that represents similar types of difficulties were selected for semi-structured interviews for in-depth analysis. The selection was based on similarities of error patterns and learning obstacles identified from the written test. Indicators used to classify students' difficulties include misunderstanding of transformation properties, errors in determining object and image relationships, inability to interpret contextual representations, difficulties with spatial visualization, and incorrect use of transformation procedures. Students who consistently presented similar characteristics in these indicators were classified in the same type of difficulty representation, resulting in five students chosen as representatives for the interview. The mathematics teacher was also interviewed to allow the researcher identified causes of the students' way of thinking during learning. The semi-structured interview protocol was developed based on Brousseau's framework of learning obstacles. The interview explored five main aspects: (1) students' understanding of transformation concepts, (2) reasoning used during problem solving, (3) interpretation of visual and symbolic representations, (4) learning experiences during classroom instruction, and (5) perceived difficulties when solving contextual transformation problems. Teacher interviews focused on instructional strategies, classroom practices, use of learning media, and teachers' perceptions of students' learning obstacles.

Data Analysis

Qualitative data were analyzed using the interactive model of Miles et al. (2014), consisting of data reduction, display, and conclusion. Data analysis was conducted iteratively following a qualitative descriptive approach. Students' written responses, interview transcripts, teacher interview data, and textbook analysis were first organized and reviewed repeatedly to gain a comprehensive understanding of the data. Interview transcripts were then manually analyzed using an open coding procedure to identify meaningful units related to students' learning difficulties. Similar codes were subsequently grouped into broader categories and interpreted using Brousseau's framework of learning obstacles, namely epistemological, ontogenic, and didactical obstacles. Epistemological obstacles were identified as students' limited understanding of concepts across different contexts, and ontogenic obstacles were related to students' cognitive readiness and spatial reasoning ability. In contrast, didactical obstacles were associated with instructional approaches and learning situations experienced by students. The credibility of the data findings was ensured through triangulation of techniques. To enhance the trustworthiness of the analysis, coding results from the interview data were continuously compared with students' written responses, teacher interviews, and textbook analysis. Any discrepancies were discussed among the researchers until consensus was achieved regarding the classification of learning obstacles. Then, the findings of this study were used to formulate implications for didactical design in geometry transformation learning. These implications emphasized the importance of designing learning situations that encourage conceptual understanding, visualization, exploration, and meaningful mathematical reasoning.

RESULT AND DISCUSSION

The results of the study revealed that students experienced various learning obstacles in geometric transformation. The identified learning obstacles were closely related to students' mathematical spatial ability. Based on the analysis of students' written tests, item-by-item difficulties identification, interviews with five representative students (MRDR, TCH, JCP, NFA, and SNU), teacher interviews, and textbook analysis, findings revealed that three types of learning obstacles occurred: epistemological, ontogenic, and didactical obstacles. These obstacles did not occur independently, but they interacted and reinforced one another throughout the learning process.

Epistemological Obstacles

Research findings showed that the epistemological obstacles primarily emerged when students were required to apply transformation concepts in unfamiliar or non-routine situations rather than reproduce procedures previously demonstrated in class. Instead of transferring their conceptual understanding across different representations, many students relied on memorized procedures and became uncertain when the contextual features of the tasks changed. This pattern suggests that students' understanding remained highly context-dependent and had not yet developed into flexible conceptual knowledge that could be generalized across transformation problems. Representative evidence of this pattern is shown by Figure 1 and 2.

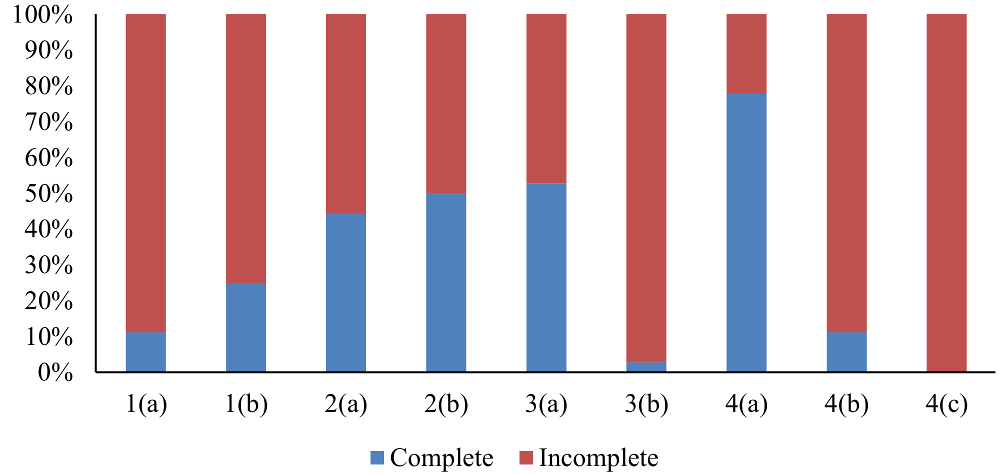


Figure 1. Proportion of complete and incomplete responses for each test item

Figure 1 illustrates the proportion of students who provided complete and incomplete responses for each test item. The visualization shows that students consistently encountered greater difficulties when solving non-routine transformation tasks, particularly those requiring conceptual generalization rather than direct procedural application. Items 3(b) and 4(c) exhibited the highest proportions of incomplete responses, indicating that unfamiliar problem contexts posed substantial challenges to students' conceptual understanding. An example of student answer showed by Figure 2.

Interview data strengthened these findings. When asked to explain the transformation concepts, MRDR stated,

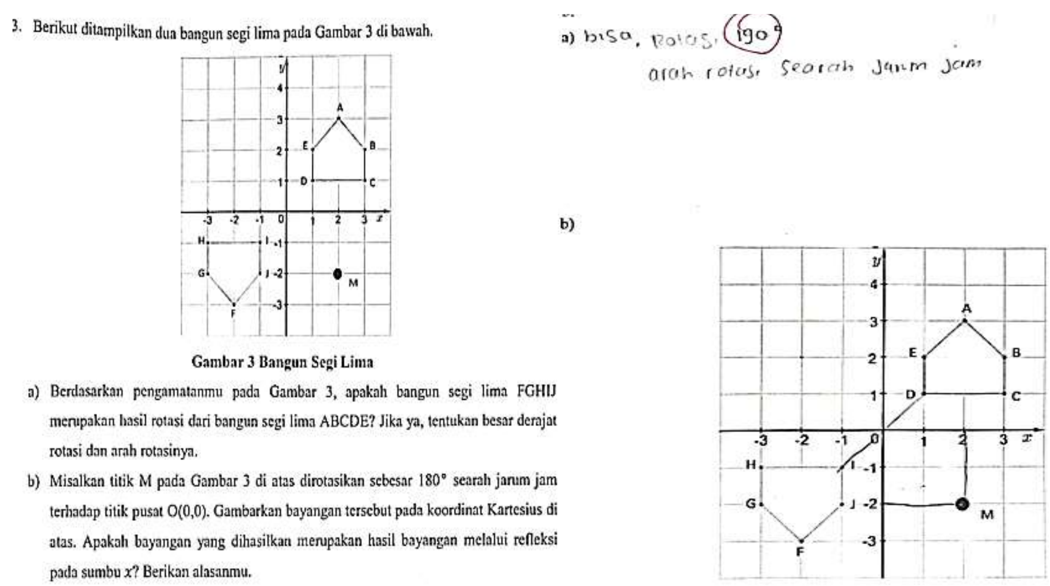


Figure 2. NFA answer for non-routine question 3

"Lupa lagi bu, cuma dua yang tau. Translasi, rotasi. Lupa lagi ibu."

(I forgot, I only know two: translation and rotation. I forgot the others.)

This statement suggests that the participant's understanding was limited to familiar procedural knowledge rather than a comprehensive conceptual understanding of geometric transformations. Similarly, JCP was able to mention all four transformation types but still relied primarily on procedural explanations without understanding the conceptual relationships among them. NFA explained that "translation is like subtraction… reflection is the same shape but reversed… rotation is like something without air, dilation is like having to enlarge; it can be reduced. It has to go through several boxes. If you enlarge the box, it increases; if you reduce the box, it decreases," revealing misconceptions regarding the properties and meanings of transformations, especially rotation and dilation.

These findings suggest that students' understanding of geometric transformations remained largely procedural and context-dependent. Although most students were able to recall transformation formulas and manipulate coordinates, they encountered considerable difficulties when the same concepts were presented in unfamiliar representations or contextual situations. The teacher interview further confirmed this condition. During class sessions, the teacher stated that students often seem to understand the concept but become confused when similar concepts are presented in a different numerical form or in story-based problems. This showed that their knowledge had not yet developed into flexible conceptual understanding that could be transferred across different mathematical contexts.

These findings were in line with previous research that students in geometric transformations frequently rely on procedural strategies without fully understanding the geometric meaning of transformations. For example, Ada & Kurtuluº (2010) found that secondary school students could often apply transformation algorithms correctly but had difficulty explaining the underlying geometric concepts. Similarly, Ramful et al. (2017) reported that students' limited ability to coordinate visual and symbolic representations hindered their conceptual understanding of transformations. Sinclair & Bruce (2015) further argued that meaningful geometry learning requires students to actively construct relationships among multiple representations rather than memorize isolated procedures.

Another significant epistemological obstacle appeared in students' misconceptions regarding rotation and dilation. Students frequently misunderstood the direction of rotation, orientation changes, and the role of scale factors in dilation. Figure 3 shows that students misunderstood the role of the scale factor of 2 in the area of the object and image of the triangle after the dilation process. However, they were able to draw the image correctly. The student also mistakenly marked the points B' and C'. This indicates that students failed to understand transformation as geometric movement. Instead, they interpreted it merely as symbolic coordinate operations. This finding supports previous studies showing that geometry learning requires students to coordinate visual and analytical reasoning simultaneously (Ramful et al., 2017; Sinclair & Bruce, 2015). Without meaningful visualization and the ability to make spatial reasoning of the transformation process, students tend to memorize procedures without understanding the underlying concepts.

While these international studies primarily explain students' misconceptions from cognitive and representational perspectives, the present study extends this body of knowledge by interpreting those misconceptions through the framework of learning obstacles. Specifically, our findings indicate that students' procedural dependence cannot be understood solely as a conceptual deficiency but should also be viewed as the result f interactions among epistemological, ontogenic, and didactical obstacles. This interpretation provides a more comprehensive explanation of why misconceptions persist and offers a stronger empirical basis for developing responsive didactical designs.

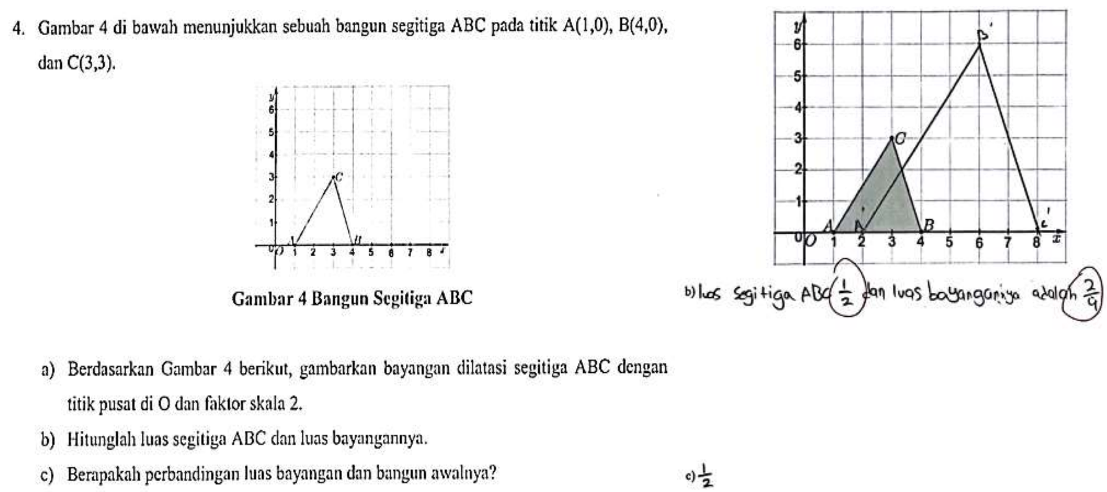


Figure 3. Student answer for question 4

Consequently, the three main epistemological obstacles perceived in this study were: 1) difficulties in generalizing transformation concepts, 2) misconceptions on transformations' characteristics (particularly rotation and dilation), and 3) dependency on conventional examples and imitation of procedures.

Ontogenic Obstacles

The study also revealed the ontogenic obstacles associated with students' cognitive readiness, spatial ability, and mathematical literacy.

Figure 4 presents the distribution of students' average scores across the five mathematical spatial ability domains using violin plots combined with jittered scatter plots. The violin plots illustrate the density and distribution of students' scores. At the same time, the jittered points represent the actual scores obtained by each participant, allowing individual variations to be observed across domains.

The visualization shows that Spatial Visualization (SV) exhibited the highest overall performance, with most students obtaining scores between 3 and 4 and relatively low variability. Mental Rotation (MR) also demonstrated relatively high performance, although the distribution was more dispersed, indicating greater variation in students' abilities. Spatial Orientation (SO) showed moderate performance, with scores spanning nearly the entire scale, suggesting considerable heterogeneity among participants. Transformation Visualization (TV) showed substantial variability, with scores ranging from 0 to 4, indicating that while some students successfully performed transformation tasks, others experienced considerable difficulty. In contrast, Spatial Relation (SR) recorded the lowest scores overall, with most students clustered between 1 and 2, suggesting that identifying spatial relationships remained the most challenging aspect for many participants.

Based on Figure 4, most students experienced difficulties in mentally visualizing geometric movement, which required their spatial relation ability, especially in rotation and dilatation tasks. Although MRDR claimed to be able to "imagine" the transformation mentally, other students such as TCH and SNU stated that they could not determine transformation results without first drawing the object manually. TCH explicitly explained, "I cannot imagine it… I need to draw it first." This tendency was also consistently reflected in students' written responses, particularly in tasks involving rotation and dilation, where many students relied on sketches before attempting formal solutions. The interview excerpts therefore represent a broader pattern identified across the dataset rather than isolated individual experiences.

The relatively low median score in Spatial Relations (SR), together with evidence from students' written responses and interview data, indicates that many students experienced difficulties in identifying and coordinating spatial relationships during geometric transformation tasks. It shows that students' spatial visualization ability was still predominantly concrete and had not yet developed into abstract mental representation. Students struggled to visualize geometric objects mentally, thus depending on physical or drawn representations frequently. These challenges or limitations affected their ability to dynamically understand transformations process.

Other difficulties also appeared in understanding Cartesian coordinates. Several students were confused about stating the x-axis and y-axis or reversed coordinate pairs during problem solving (Figure 5). NFA admitted confusion in identifying the point value of x and y coordinates, while the teacher confirmed that students often reversed ordered pairs such as (4,6) into incorrect positions. This indicates weaknesses in prerequisite knowledge, particularly regarding coordinate systems and spatial orientation.

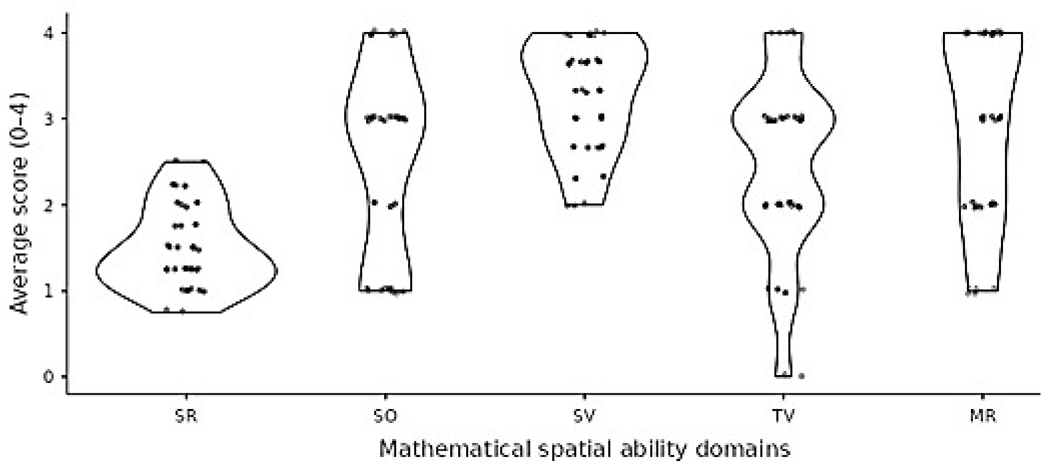


Figure 4. Distribution of students' average scores across the five mathematical spatial ability domains (SR = Spatial Relation; SO = Spatial Orientation; SV = Spatial Visualization; TV = Transformation Visualization; MR = Mental Rotation)

These findings are in line with previous studies showing that spatial reasoning has an important role in geometry learning, particularly in helping students visualize object movement and understand spatial relationships mentally (Lowrie et al., 2020; Ramful et al., 2017). Students with limited spatial ability often experience difficulties when interpreting transformations, especially tasks that require mental visualization and coordination between visual and symbolic representations (Ramful et al., 2017).

In addition, mathematical literacy emerged as another ontogenical obstacle. According to the teacher, students had difficulties understanding

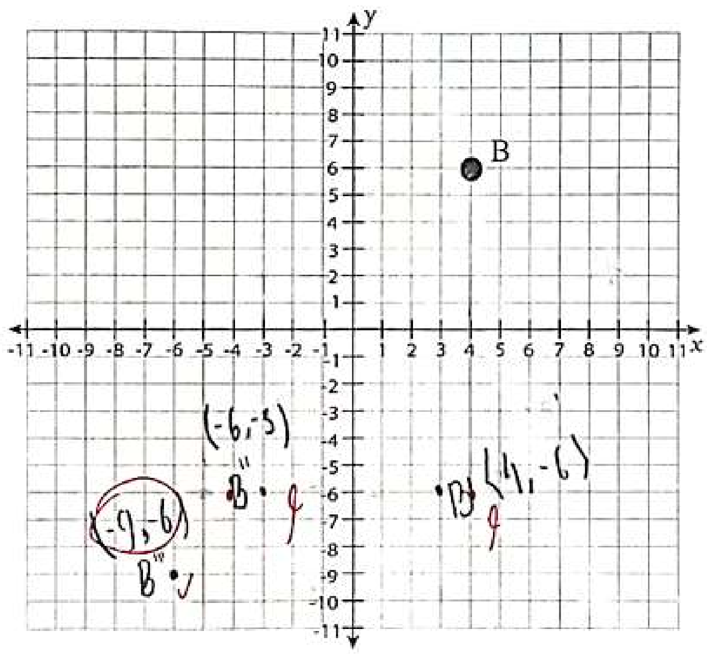
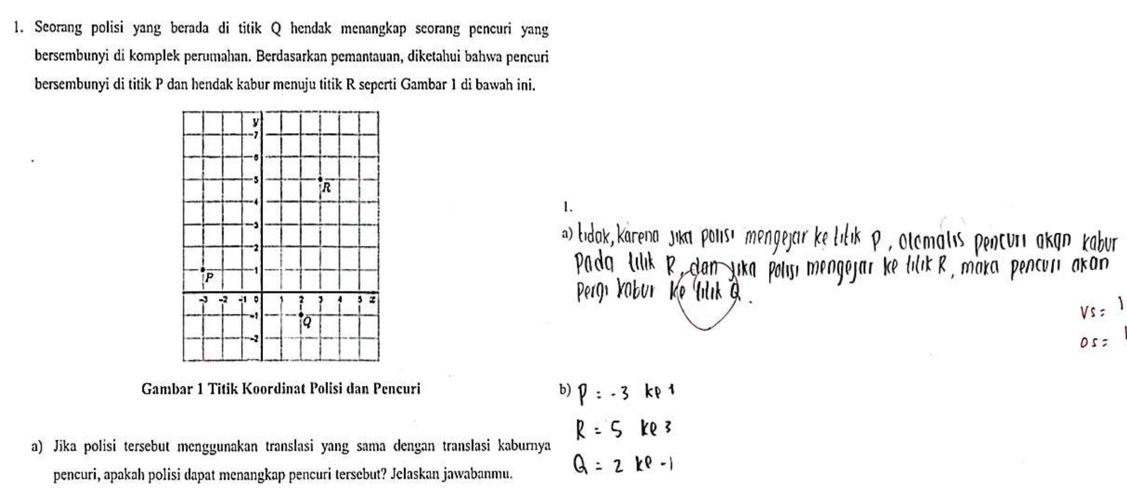


Figure 5. AAH confused between the value of x-Axis and y-Axis

story-based problems regardless of their mathematical complexity. A substantial proportion of students experienced similar challenges in identifying relevant mathematical information embedded in contextual transformation problems before selecting an appropriate solution strategy. As an example, only four (11%) of the students were able to answer question 1 completely. Figure 6 presents JCP's response, as it clearly illustrates this recurring pattern observed across the dataset. This suggests that students were not only challenged conceptually but also struggled to interpret mathematical information presented in a long paragraph.

Overall, the ontogenical obstacles identified in this study included: (1) limited spatial relation ability, (2) weak understanding of Cartesian coordinates, and (3) low mathematical literacy, particularly in interpreting contextual problems.


1. Seorang polisi yang berada di titik Q hendak menangkap seorang pencuri yang bersembunyi di komplek perumahan. Berdasarkan pemantauan, diketahui bahwa pencuri bersembunyi di titik P dan hendak kabur menuju titik R seperti Gambar 1 di bawah ini.

Gambar 1 Titik Koordinat Polisi dan Pencuri

a) Jika polisi tersebut menggunakan translasi yang sama dengan translasi kaburnya pencuri, apakah polisi dapat menangkap pencuri tersebut? Jelaskan jawabanmu.

1\.
a) tidak, karena jika polisi mengejar ke titik P, otomatis pencuri akan kabur pada titik R, dan jika polisi mengejar ke titik R, maka pencuri akan pergi kabur ke titik Q.

VS: 1
OS: 1

b) P = -3 ke 1
R = 5 ke 3
Q = 2 ke -1

Figure 6. JCP answer for story-based problem

Didactical Obstacles

The findings further revealed didactical obstacles related to instructional approaches, learning strategies, and the use of learning media. Teacher interviews showed that transformation geometry lessons were predominantly conducted using expository and teacher-centered methods. The teacher explained that instruction mainly involved explaining formulas directly in front of the class without using interactive learning media. As the teacher stated:

"I usually teach directly. I explain the material in front of the class, write on the board, and use the textbook. I do not use learning media such as videos." (Teacher interview, translated by the authors)

As a consequence, students tended to receive information passively rather than construct concepts independently through exploration and visualization activities. The absence of meaningful learning experiences limited students' opportunities to understand transformation as dynamic geometric movement. This finding supports previous studies indicating that teacher-centered instruction often contributes to students' weak conceptual understanding, low engagement in geometry learning, and difficulties in mathematical problem solving (Sutarni & Aryuana, 2023). Learning that mainly focuses on direct explanation and procedural exercises tends to limit students' opportunities to explore concepts meaningfully and develop their own reasoning processes (Can Cabrera et al., 2021; Gebremeskel, 2025).

Another reason for these learning barriers was the limited use of interactive visual media during instruction. As the teacher explained:

"I only use the textbook. Sometimes I look for additional explanations on YouTube to find easier examples, but I do not use learning media during the lesson." (Teacher interview, translated by the authors)

Consequently, students had limited opportunities to observe geometric transformations dynamically through interactive visualization.

Textbook analysis further indicated that learning materials focused primarily on formula application and coordinate procedures. As shown in Figure 7, the exercises primarily emphasized routine symbolic manipulation, with limited encouragement for conceptual exploration or multiple representations. Textbook analysis revealed that most learning activities emphasized routine coordinate manipulation and procedural exercises, with limited opportunities for students to interpret transformations conceptually or through multiple representations. This pattern closely corresponded to students' written responses, in which many students relied on memorized procedures, had difficulty interpreting contextual problems, and frequently made errors in identifying transformation directions. Table 2 summarizes the relationships among textbook characteristics, students' dominant error patterns, and the corresponding learning obstacles identified in this study.

6. Diketahui titik $A(3,-2)$ dan $B(9,11)$. Arah translasi titik A ke titik B adalah ...
   a. $\begin{pmatrix} 6 \\ 13 \end{pmatrix}$ c. $\begin{pmatrix} 13 \\ -6 \end{pmatrix}$
   b. $\begin{pmatrix} 6 \\ -13 \end{pmatrix}$ d. $\begin{pmatrix} -13 \\ 6 \end{pmatrix}$
7. Sebuah layang-layang $KLMN$ memiliki koordinat $K(8,6)$, $L(10,8)$, $M(12,6)$ dan $N(10,2)$. Jika layang-layang tersebut ditranslasikan oleh $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$, tentukan koordinat bayangannya dan gambarkan bayangannya pada bidang koordinat!
8. Segitiga PQR ditranslasikan sehingga menghasilkan bayangan STU. Diketahui $P(-1,2)$, $Q(3,-2)$, $S(2,-3)$, dan $U(3,4)$, tentukan koordinat T dan R. Tentukan pula translasinya.
9. Diketahui segitiga ABC dengan $A(0,-3)$, $B(2,2)$ dan $C(4,5)$ ditranslasikan sehingga bayangannya adalah $A'(-2,1)$. Tentukan translasinya serta koordinat titik B' dan C'.

Figure 7. Translation exercises discussed in students' textbook

Students' solution strategies also reflected the impact of didactical obstacles. MRDR tended to write answers immediately without understanding the process, TCH could read the

Table 2. Relationship between textbook characteristics and students' learning obstacles

Textbook characteristics	Evidence from the textbook	Dominant student errors	Learning obstacle
Exercises emphasize coordinate procedures.	Students are asked to substitute coordinates into formulas	Students memorized procedures but failed to explain the meaning of transformation	Epistemological
Few contextual problems	Most tasks are routine and symbolic	Students struggled to interpret story-based transformation problems	Ontogenical
Limited visual exploration	Static diagrams without dynamic visualization	Students confused the transformation direction and spatial orientation	Ontogenical
Lack of multiple representations	Algebraic procedures dominate over visual reasoning	Students relied on formulas instead of geometric reasoning	Didactical

problem but did not know how to begin solving it, while JCP relied solely on drawing without conceptual reasoning. These patterns suggest that instructional practices had not yet supported the development of systematic mathematical thinking and problem-solving strategies. The teacher also acknowledged adapting the instruction to students' learning readiness. As teacher explained:

"I always adjust my teaching to the students' abilities. I prefer that they really understand one topic rather than simply finishing all the planned chapters." (Teacher interview, translated by the authors)

However, the teacher also noted that although students generally understood the concepts during classroom explanations, they became confused when solving problems with different numerical values or contextual wording:

"They understand during the explanation, but when I change the numbers or the wording, they become confused. They understand the concept, but they cannot apply it to the problems." (Teacher interview, translated by the authors)

These findings indicate that instruction tended to prioritize procedural familiarity over conceptual transfer, which may have contributed to a fragmented understanding of geometric transformations.

Therefore, the didactical obstacles identified in this study included: (1) dominance of teacher-centered instruction, (2) limited use of visual and interactive media, (3) lack of representational variation, and (4) underdeveloped mathematical problem-solving strategies.

Interconnection among Learning Obstacles and Implications for Didactical Design

An important finding of this study is that epistemological, ontogenical, and didactical obstacles were strongly interconnected. Didactical obstacles, such as limited visualization activities and teacher-centered instruction, contributed to students' weak spatial visualization abilities. In turn, these ontogenical limitations reinforced epistemological obstacles, particularly students' inability to construct conceptual understanding of geometric transformations. To provide a holistic visualization of the causal relationships identified through triangulation, Figure 8 presents a Sankey diagram illustrating how instructional conditions contributed to students' misconceptions and subsequently to the emergence of learning obstacles.

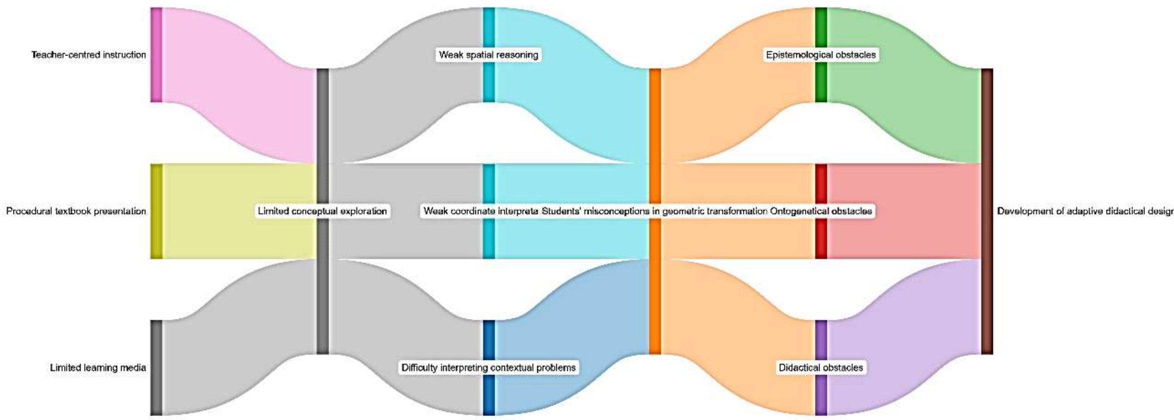


Figure 8. Sankey diagram illustrating the qualitative causal pathways from instructional conditions to students' misconceptions and the emergence of learning obstacles

For example, students' difficulties in determining the direction of translation or rotation were not solely caused by conceptual misunderstanding. These difficulties also emerged because students lacked sufficient visual learning experiences, had limited ability to mentally visualize geometric movement, and failed to generalize transformation concepts across different contexts.

This finding confirms that learning obstacles in geometric transformations should not be viewed as isolated cognitive errors. Instead, they emerge from the interaction between students' cognitive development, prior knowledge, instructional design, and classroom learning experiences. Similar conclusions were reported by Osman et al. (2025) and Sinclair & Bruce (2015), who emphasized that meaningful geometry learning requires dynamic visual exploration, representational flexibility, and active conceptual construction.

Therefore, the findings imply the need for a didactical design that integrates conceptual understanding, spatial visualization, and exploratory learning simultaneously. Learning activities should provide opportunities for students to manipulate geometric objects visually, discuss their reasoning, and connect symbolic procedures with geometric meaning. The integration of interactive media such as GeoGebra, contextual problems, and multiple representations may help students overcome the identified obstacles and develop a more meaningful understanding of geometric transformations.

The three categories of learning obstacles identified in this study should not be interpreted as isolated phenomena. Instead, they interacted to shape students' difficulties in learning geometric transformations. Epistemological obstacles emerged from fragmented conceptual understanding, ontogenical obstacles reflected students' developmental limitations in spatial reasoning and contextual interpretation, whereas didactical obstacles were associated with instructional practices and learning resources. These interconnected findings provided the primary basis for developing the hypothetical didactical design proposed in this study. While Figure 8 emphasizes the causal pathways leading to learning obstacles, Figure 9 synthesizes these findings into a conceptual framework that guided the development of the proposed adaptive didactical design.

As illustrated in Figure 9, the interaction among epistemological, ontogenical, and didactical obstacles indicates that improving students' understanding requires more than procedural instruction. Consequently, the

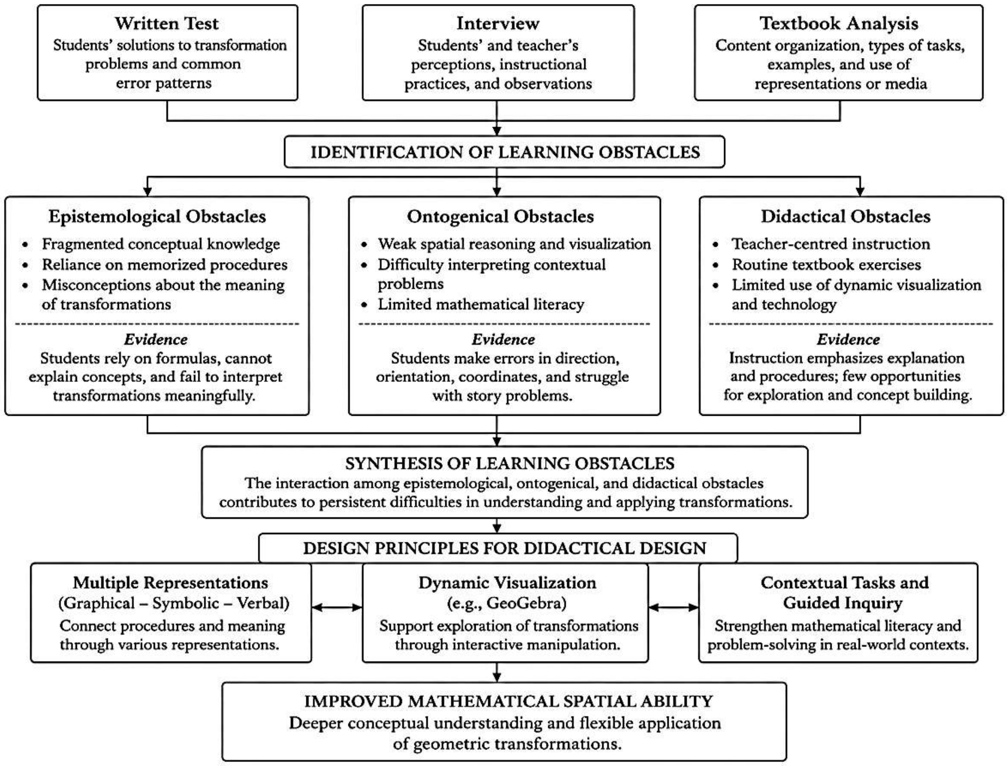


Figure 9. Conceptual flow of the development of learning obstacles in geometric transformation learning

proposed didactical design emphasizes conceptual exploration, dynamic visualization, contextual learning activities, and guided institutionalization to simultaneously address the identified learning obstacles.

CONCLUSION

This study identified three interconnected categories of learning obstacles encountered by junior high school students in learning geometric transformations: epistemological, ontogenical, and didactical obstacles. The findings revealed that students exhibited fragmented conceptual understanding, limited spatial reasoning, difficulty interpreting contextual transformation problems, and challenges stemming from instructional practices that emphasized procedural rather than conceptual learning. By integrating evidence from written tests, interviews, and textbook analysis, this study provides a comprehensive understanding of how these learning obstacles interact and offers an empirical foundation for developing adaptive didactical designs to improve students' mathematical spatial ability.

These findings have important implications for mathematics education. Teachers are encouraged to design learning experiences that promote conceptual understanding through multiple representations, contextual transformation tasks, dynamic visualization, and guided mathematical discussions rather than relying primarily on routine procedural exercises. Such instructional approaches may help students develop stronger spatial reasoning and transfer their understanding to unfamiliar situations. Nevertheless, this study was conducted in a single school involving one mathematics teacher; therefore, the findings should be interpreted within this particular context. Future studies are recommended to implement and evaluate the proposed didactical design in different educational settings to examine its broader applicability and effectiveness.

DECLARATION OF GENERATIVE AI USAGE IN THE WRITING PROCESS

During the writing of this manuscript, the author(s) employed ChatGPT to assist with article flow, sentence clarity, and language refinement. The author(s) have reviewed and edited the content generated by this tool and assume full responsibility for the content of the published article.

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