

Analysis of Students' Comprehension of Mathematical Concepts in the Indonesia-Timor Leste Border Region

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Abstract: The ability to understand mathematical concepts is one of the key skills that students must possess in mathematics learning; however, in reality, many students still struggle to understand and apply mathematical concepts correctly. This study aims to describe students' ability to understand mathematical concepts in the Indonesia–Timor-Leste border region. This study employed a qualitative descriptive approach and was conducted at SMK Negeri 2 Belu. The research subjects were 25 students in Year 11 of the Computer Science program, and three students representing the high, medium, and low categories of mathematical concept comprehension ability were purposively selected. Data collection techniques were carried out through a mathematical concept comprehension test, interviews, and documentation. Data analysis techniques included data reduction, data presentation, and drawing conclusions, whilst data validity was tested using triangulation. The research findings indicate that pupils' ability to understand mathematical concepts falls into three categories: high, moderate, and low. Pupils in the high category can understand problems, construct mathematical models, determine solution strategies, and systematically explain the reasoning behind each step. Pupils in the moderate category can generally solve problems correctly but still make some errors due to a lack of attention to detail and an insufficient understanding of the concepts. Meanwhile, students in the low category have difficulty understanding problems, constructing mathematical models, and determining solution steps. The research results also indicate that most students are better able to solve problems procedurally than to understand mathematical concepts in depth. Therefore, more innovative, contextual, and student-centered learning is required to improve students' mathematical conceptual understanding in border regions.

Keywords: concept, conceptual comprehension proficiency, border region.

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■ INTRODUCTION

Conceptual understanding is a key aspect of mathematics learning as it forms the basis for pupils to understand, develop, and apply mathematical concepts in various situations. Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. It is one of the interconnected strands of mathematical proficiency, alongside procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Pulles & Burns, 2022).

Pupils with a strong conceptual understanding will find it easier to link their knowledge to accurate solutions to mathematical problems. Conceptual understanding is also closely related to problem-solving skills, as pupils must understand the concepts involved before solving a mathematical problem. Furthermore, conceptual understanding can be defined as a student's ability to construct knowledge and restate the concepts they have learned in their own words, both orally and in writing, without altering the meaning of those

concepts, consistent with the distinction between conceptual and procedural knowledge in mathematics (Österman & Bråting, 2019). Thus, mathematical conceptual understanding is not merely about the ability to memorize formulas, but also the ability to grasp the meaning of concepts and apply them to solve mathematical problems.

Although mathematical conceptual understanding has been widely investigated, existing studies have predominantly focused on students in urban or mainstream educational settings, while evidence from border regions remains limited. Students living in border areas often experience distinctive educational conditions, including disparities in access to educational resources, learning facilities, and instructional support, which may influence their conceptual understanding of mathematics. Consequently, findings from previous studies cannot be readily generalized to students in the Indonesia–Timor Leste border region. Addressing this gap is important for providing empirical evidence that reflects the educational realities of border communities and for informing strategies to improve mathematics learning in similar contexts. Therefore, this study aims to analyze students' mathematical conceptual understanding in the Indonesia–Timor Leste border region based on established indicators of conceptual understanding. Specifically, this study seeks to answer the following research questions: (1) What is the level of students' mathematical conceptual understanding in the Indonesia–Timor Leste border region? and (2) How do students demonstrate their mathematical conceptual understanding across the indicators of conceptual understanding?

Understanding mathematical concepts plays a vital role in learning mathematics, as it helps students develop logical, systematic, and critical thinking. Students who understand concepts well tend to determine appropriate solution strategies

when faced with mathematical problems, as conceptual understanding, procedural knowledge, and problem-solving skills are closely interconnected components of mathematical ability (Al Mutawah et al., 2019). Furthermore, understanding mathematical concepts is a fundamental skill students must possess to construct meaning from the material they study, particularly because conceptual understanding underpins students' problem-solving skills across mathematical topics (Ferreder et al., 2024). The importance of mathematical conceptual understanding also aligns with the objectives of mathematics learning at primary and secondary levels: students should be able to understand mathematical concepts and apply them in various situations, reflecting the broader developmental trajectory of mathematics knowledge (Rittle-Johnson, 2017). Therefore, mathematical conceptual understanding is one of the skills that needs to be prioritized in mathematics education.

More broadly, mathematical conceptual understanding refers to students' ability to understand, absorb, master, and apply mathematical concepts in the learning process, consistent with the view that conceptual knowledge represents an integrated network of ideas rather than isolated facts (Österman & Bråting, 2019). Mathematical concept understanding can also be defined as students' cognitive ability to articulate ideas, process information, and rephrase mathematical concepts in their own words to solve problems in accordance with applicable rules and concepts, which aligns with findings that conceptual knowledge, procedural knowledge, and procedural flexibility jointly support students' mathematical problem-solving performance (Chan et al., 2023). This is further supported by frame-based analyses of mathematical understanding, which show how explicit information is combined with learners' background knowledge to shape their grasp of mathematical

concepts (Carl et al., 2021). Based on these views, mathematical conceptual understanding is a student's ability to master mathematical concepts in depth so they can explain, connect, and apply these concepts to solve mathematical problems.

Although understanding mathematical concepts plays a vital role, students' ability to grasp these concepts remains relatively low. This situation is also evident among Year 11 Information and Communication Technology (ICT) students at SMK Negeri 2 Belu, located in the Indonesia–Timor-Leste border region. Interviews with mathematics teachers found that students' understanding of mathematical concepts remains unsatisfactory. This is evident from the results of the students' mathematics tests, the majority of which still fall below the Minimum Passing Criteria (KKM) set by the school. A preliminary test administered to 25 students showed that only 5 students (20%) achieved scores at or above the school's Minimum Passing Criteria (KKM) of 80, while the remaining 20 students (80%) scored below this threshold, with an average score of 70.76. Interviews with mathematics teachers also found that students struggle to solve word problems and cannot explain the concepts they have learned. Some students also tend to follow the teacher's solution examples without understanding the reasoning behind the steps. Furthermore, students struggle to solve word problems and cannot explain the concepts they have learned. Some students also tend to follow the teacher's solution examples without understanding the reasoning behind the steps.

This issue indicates that the mathematics education pupils receive still places greater emphasis on procedural skills than on deep conceptual understanding. A similar pattern has been documented in analyses of students' procedural knowledge, which show that persistent gaps between procedural execution

and conceptual reasoning often stem from insufficiently in-depth instruction (Dorner et al., 2026). Teaching often focuses on providing practice problems without adequately instilling concepts. Furthermore, students' low mathematical conceptual understanding is also influenced by several other factors, such as their still-low basic mathematical ability, a lack of interest in learning mathematics because it is perceived as a difficult subject, and limited learning facilities that support the teaching and learning process. Communication between teachers and students also needs to be addressed to ensure students understand the information conveyed (Ayuwanti et al., 2021).

Various previous studies have examined pupils' mathematical conceptual understanding. Research by Ningrum et al. (2022) shows that proficient/high-achieving pupils do not experience significant difficulties in solving mathematical problems based on indicators of conceptual understanding. In contrast, pupils with moderate and low achievement levels still encounter various difficulties. Research by Al Mutawah et al. (2019) also showed that students in the high category had very good conceptual understanding, whilst students in the moderate and low categories had fairly good conceptual understanding. Furthermore, Kholid et al. (2021) found that students' mathematical conceptual understanding varies across ability categories, with high-ability students solving problems correctly, moderate-ability students meeting only some indicators, and low-ability students unable to solve problems precisely. However, these previous studies primarily relied on quantitative categorization of achievement levels, with limited exploration of the underlying reasoning processes that lead students to succeed or struggle with specific indicators of conceptual understanding. Moreover, none of these studies were conducted in a border-region vocational school context, where students' learning conditions may differ from those in the

more extensively studied urban or non-border settings. This study addresses this gap by employing a descriptive qualitative approach that combines a conceptual understanding test with in-depth, semi-structured interviews, allowing for triangulated, in-depth exploration of how and why students at different achievement levels understand or fail to understand mathematical concepts based on four specific indicators: restating a concept, classifying objects according to a concept's properties, identifying the properties of operations or concepts, and providing examples and non-examples of a concept.

Pupils' ability to understand mathematical concepts is assessed using several indicators of conceptual understanding (Sengkey et al., 2023), where indicators of conceptual understanding include: (1) restating a concept that has been learned; (2) classifying objects based on whether or not they meet the criteria that define that concept; (3) identifying the properties of operations or concepts; (4) providing examples and non-examples of the concept that has been learned; (5) presenting the concept in various mathematical representations; (6) relating various concepts within and outside mathematics; and (7) developing the necessary conditions of a concept. Therefore, the indicators of conceptual understanding used in this study are: (1) restating a concept that has been studied; (2) classifying objects based on whether or not the conditions forming the concept are met; (3) identifying the properties of operations or concepts; and (4) providing examples and non-examples of the concept studied. These indicators were selected because they illustrate students' ability to understand, explain, and apply mathematical concepts in greater depth.

This study selected Systems of Linear Equations in Three Variables (SPLTV) as the instructional content for the conceptual understanding test. SPLTV was chosen because it is a core topic in the eleventh-grade

mathematics curriculum that inherently requires integrating multiple cognitive processes: translating verbal information into mathematical models, applying systematic solution procedures, and justifying each step, making it well-suited to assess all four indicators of conceptual understanding simultaneously. Moreover, prior studies have consistently identified SPLTV as a topic in which students frequently exhibit procedural rather than conceptual understanding (Kuswanti et al., 2018), a pattern also observed more broadly in students' conceptual understanding of linear equation systems (Dewi et al., 2020), and in students' conceptual and procedural knowledge of linear equations more broadly (Qetrani et al., 2021; Sandoval et al., 2023). The mathematical connections preservice teachers construct when solving linear equation tasks (Del Carmen et al., 2024), and the specific difficulty students face when translating word problems into mathematical models (Jupri & Drijvers, 2016), further justify the relevance of investigating the research problem in this study.

Consequently, this study aims to describe the mathematical concept comprehension abilities of Year 11 TKJ students at SMK Negeri 2 Belu in the Indonesia–Timor-Leste border region, based on the predetermined indicators of mathematical concept comprehension.

■ **METHOD**

Research Design

This study employed a descriptive qualitative approach, designed as a multiple case study (Yin, 2018), to investigate students' mathematical conceptual understanding based on test and interview data. The qualitative approach enabled an in-depth exploration of students' understanding of mathematical concepts and provided a comprehensive description of their conceptual understanding. The study was conducted in an eleventh-grade Computer and Network Engineering (TKJ) class at SMK

Negeri 2 Belu, a vocational high school located in the Indonesia–Timor Leste border region.

Within this design, each selected student was treated as an individual, information-rich case, consistent with the rationale for multiple case study designs in applied educational research (Adams et al., 2022), rather than as a statistically representative unit of their respective achievement group. Case study designs are strengthened, not weakened, when the rationale for case selection is made explicit and methodologically justified; studies that adopt the case study label without such justification have been shown to lack methodological rigor (Turnbull et al., 2021).

Participants

The participants comprised 25 eleventh-grade students who completed a mathematical conceptual understanding test. The research subjects were selected using purposive sampling, a technique in which participants are chosen based on specific criteria aligned with the study's objectives. After all students completed the mathematical conceptual understanding test, the test results were analyzed, and the students were categorized into high, medium, and low levels of mathematical conceptual understanding.

Subsequently, one student from each category was selected as the main research subject. The selection was based on the following criteria: (1) representing the high, medium, or low category according to the test results; (2) being able to clearly articulate their thought processes during the interview; (3) providing responses that allowed for further exploration of their reasoning and problem-solving processes; and (4) demonstrating willingness to participate in all stages of the study. Based on these criteria, three students were selected as the main subjects, representing the high, medium, and low categories of mathematical conceptual understanding.

The purpose of selecting one subject per achievement category was not to statistically or

thematically represent the broader population within that category, but to enable an in-depth, idiographic exploration of each case (Yin, 2018). This aligns with the concept of information-rich cases in case selection, in which a case's analytical value lies in the depth and richness of the information it provides, rather than in the number of cases per group (Adams et al., 2022). Accordingly, the three subjects in this study (ST, SS, and SR) are positioned as three individual cases within a multiple case study design, each analyzed in depth (within-case analysis) before patterns were compared across cases (cross-case analysis) (Fàbregues & Fetters, 2019).

Data Collection Instruments and Procedures

The mathematical conceptual understanding test consisted of four items, each targeting one indicator of conceptual understanding, based on the topic of Systems of Linear Equations in Three Variables (SPLTV). Table 1 presents the test blueprint (instrument specifications).

Table 1. Test blueprint (instrument specifications)

Topic	Indicator	Question Type
Understanding SPLTV	Restating concepts	Essay
Mathematical models of word problems	Identifying properties of operations/concepts	Essay
Solving SPLTV	Identifying properties of operations/concepts	Essay
Classification of PLTV/non-PLTV	Classification of objects; Examples and non-examples	Essay

The research instruments consisted of a mathematical conceptual understanding test and a semi-structured interview guide. The test was developed based on four indicators of mathematical conceptual understanding: (1) restating a concept, (2) classifying objects

according to the properties of a concept, (3) identifying the properties of operations or concepts, and (4) providing examples and non-examples of a concept. These indicators were used to assess students' understanding of mathematical concepts comprehensively. The semi-structured interview guide was employed to gain deeper insights into students' thought processes while solving the test items and to confirm the responses provided in the written test. The interviews also enabled the researcher to explore students' reasoning and conceptual understanding in greater depth.

Prior to data collection, the mathematical conceptual understanding test and the semi-structured interview guide underwent content validation by two experts in mathematics education. The validation process aimed to ensure the appropriateness and quality of the research instruments. The aspects evaluated included the instruments' alignment with the research objectives, the representativeness of the indicators of mathematical conceptual understanding, the construction of the test items, the clarity of the language used, and the instruments' overall readability. Based on the experts' feedback and recommendations, the researchers revised the test items and refined the interview questions. After the revision process, the instruments were deemed appropriate and suitable for use in data collection.

Before data collection, two experts in mathematics education validated the instruments using a content validity assessment sheet covering five aspects: alignment with research objectives, representativeness of indicators, item construction, language clarity, and readability. Each aspect was rated on a 4-point scale (1 = not appropriate to 4 = highly appropriate). The content validity index was calculated using Aiken's V formula. The results showed V values ranging from 0.75 to 0.92 (mean $V = 0.85$), indicating that the instruments had high content validity ($V_e > 0.60$

as the minimum threshold. Based on this quantitative validation and the experts' qualitative feedback, minor revisions were made to item wording before the instruments were used for data collection.

Data collection was conducted in two stages. The first stage involved administering a mathematical conceptual understanding test to all participating students. The test results were used to determine the students' levels of mathematical conceptual understanding and to select the research subjects for further investigation. The second stage consisted of conducting in-depth interviews with the three selected subjects. These interviews were carried out to obtain more detailed information about the students' thought processes and reasoning while solving the test problems, as well as to gain a deeper understanding of their mathematical conceptual understanding.

Data Analysis

The data were analyzed using the Miles and Huberman model, which consists of three stages: data reduction, data display, and conclusion drawing. Data reduction was carried out through the following operational steps. First, each student's written test was examined item by item, and each answer segment was assigned a code representing the corresponding indicator of mathematical conceptual understanding. Second, the interview transcripts were segmented utterance by utterance and coded using the same indicator scheme, combined with speaker codes: T for the interviewer (Teacher), ST for the high-achieving subject, SS for the moderate-achieving subject, and SR for the low-achieving subject.

Third, the coded test data and interview data sharing the same subject and indicator code were compared to examine the consistency between students' written answers and their verbal explanations, in accordance with the

methodological triangulation procedure described earlier.

The final stage involved drawing conclusions based on the interpretation of the analyzed data to provide a comprehensive understanding of students' conceptual understanding.

The trustworthiness of the data was established through methodological triangulation by comparing each subject's written test responses with their interview responses rather than through source triangulation across multiple students within the same achievement category. This choice is methodologically appropriate for a multiple case study design, in which each subject constitutes an independent case rather than a representative unit of a comparative group; case study methodology recognizes multiple legitimate approaches to establishing trustworthiness depending on the data sources available and the study's underlying purpose (Fàbregues & Fetters, 2019; Miller et al., 2023). This design and justification approach aligns with methodological recommendations for reporting qualitative case study research, which emphasize making case selection and triangulation strategies explicit and aligned with the study's underlying purpose rather than following a fixed, one-size-fits-all standard (Mtisi, 2022).

. If a student's oral explanation differs from their written answer, the researcher asks the student to demonstrate their thought process again, either in writing or orally (think-aloud), to ascertain which reflects their actual understanding. In practice, the researcher first analyzed students' responses based on each indicator of mathematical conceptual understanding. The results of this analysis then formed the basis for developing interview questions aimed at clarifying and exploring students' thought processes in greater depth. Subsequently, the interview data were transcribed and compared with the students' written responses. If the information obtained from both data collection techniques was

consistent, the data were considered valid. Conversely, when discrepancies were identified between the test results and interview findings, further clarification was sought through follow-up questions until consistent information was obtained. This triangulation process enhanced the credibility of the data and ensured that the interpretation of students' mathematical conceptual understanding was supported by valid and trustworthy evidence.

■ RESULT AND DISCUSSION

This study aims to describe the mathematical conceptual understanding of Year 11 students in the Computer Science program at SMK Negeri 2 Belu, located in the Indonesia–Timor-Leste border region. Research data were obtained through a mathematical conceptual understanding test and in-depth interviews with research subjects representing high-, medium-, and low-ability categories. The analysis of students' mathematical concept comprehension was based on four indicators, namely: (1) restating concepts that have been learned, (2) classifying objects based on properties consistent with the concept, (3) identifying the properties of operations or concepts, and (4) providing examples and non-examples of a concept.

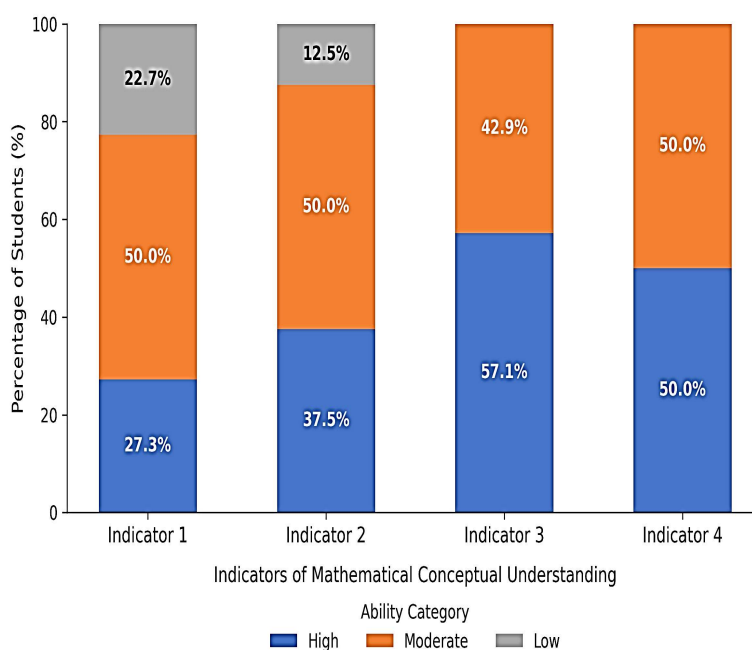
A test of mathematical conceptual understanding was administered to 25 Year 11 students in the Computer Science class at SMK Negeri 2 Belu. Based on the test results, the students were then grouped into three ability categories: high, medium, and low. Subsequently, three research subjects were selected using purposive sampling for in-depth analysis through interviews. The selection of subjects was based on test results and students' communication skills to ensure the data obtained was more in-depth and aligned with the research objectives. The results of the grouping of students' mathematical concept comprehension abilities are presented in Table 2 below.

Table 2. Categories of mathematical concept understanding ability

Category	Total	Percentage
High	6	24%
Moderate	11	44%
Low	8	32%

In addition to categorizing students' abilities, this study also analyzed students' mastery of indicators of mathematical conceptual understanding. Based on the analysis of students' answer sheets, the indicator of restating concepts was the one most frequently mastered, as shown in Figure 1. Most students were able to restate the SPLTV concept in their own words based

on previously studied material. Regarding the indicator of classifying objects based on conceptual properties, some students were able to determine a mathematical model based on the information provided in the questions. However, some students were still unable to link the information in the questions accurately to the SPLTV concept. Meanwhile, relatively few pupils met the indicator for identifying the properties of operations or concepts and providing examples and non-examples of the concepts studied. Some pupils still struggled to determine problem-solving strategies and explain the reasons for using specific steps to solve problems. The overall picture can be seen in Figure 1 below.

**Figure 1.** Distribution of students' mathematical concept understanding ability

Subject ST (High Category)

Based on the results of the mathematical conceptual understanding test, six pupils were classified in the high category. One pupil from this category was then selected as the research subject and assigned the code ST. The subject

was selected based on achieving the highest test score and having good communication skills to ensure the interview process ran smoothly. The following are the results of the high-performing subject's work in completing the provided test questions.

Nama : Daniel messiah
 kelas : XI TKJ

1. Jelaskan apa yg dimaksud dengan Sistem Persamaan linear Tiga Variabel (SPLTV)?
 Suatu Persamaan matematika yg terdiri atas 3 Persamaan linear yg masing-masing Persamaan bervariabel tiga/misal x, y dan z .

2. Peny
 Mis x = buku tulis
 y = Pensil
 z = Penggaris

Paket 1
 $2x + 2y + z = 5.200$

Paket 2
 $2x + 3y + 2z = 6.450$

Paket 3
 $3x + 2y + z = 6.800$

3. Peny
 $x + y + z = 3 \dots (1)$
 $3x - y + 2z = 4 \dots (2)$
 $x + y - z = 1 \dots (3)$
 Eliminasi y dari Pers (1) dan (2)

$x + y + z = 3$
 $3x - y + 2z = 4$
 $4x + 3z = 7 \dots (4)$

Eliminasi y dari Pers (4) dan (3)

$3x - y + 2z = 4$
 $x + y - z = 1$
 $4x + 3z = 5 \dots (5)$

Eliminasi x dari pers (4) dan (5)

$4x + 3z = 7$
 $4x + z = 5$
 $2z = 2$
 $z = 2/2$
 $z = 1$

Substitusi nilai z ke dalam Persamaan (5)

$4x + z = 5$
 $4x + 1 = 5$
 $4x = 5 - 1$
 $4x = 4$
 $x = 4/4$
 $x = 1$

Substitusi nilai x dan z ke dalam Persamaan (1)

$x + y + z = 3$
 $1 + y + 1 = 3$
 $2 + y = 3$
 $y = 3 - 2$
 $y = 1$

Figure 2. Results of high-academic-ability students' work (ST)

Based on the analysis of the answer sheet, subject ST solved all questions effectively and systematically. The subject wrote down the given

and required information in full, constructed mathematical models correctly, determined the appropriate solution strategy, and provided

Table 3. Transcription and english translation of subject st written work (companion to figure 2)

No	Handwritten Text (Transcription)	English Translation
1	Jelaskan apa yg dimaksud dengan Sistem Persamaan Linear Tiga Variabel (SPLTV)? Suatu persamaan matematika yang terdiri atas 3 persamaan linier yang masing-masing persamaan bervariabel tiga (misal x, y dan z)	Explain what is meant by a System of Linear Equations in Three Variables (SPLTV)? A mathematical equation consisting of 3 linear equations, each equation having three variables (e.g., x, y , and z)
2	Peny: Mis x = buku tulis, y = pensil, z = penggaris Paket I: $2x + 2y + z = 5.200$ Paket II: $2x + 3y + 2z = 6.450$ Paket III: $3x + 2y + z = 6.800$	Solution: Let x = exercise book, y = pencil, z = ruler Package I: $2x + 2y + z = 5,200$ Package II: $2x + 3y + 2z = 6,450$ Package III: $3x + 2y + z = 6,800$
3	Peny: $x + y + z = 3 \dots (1)$ $3x - y + 2z = 4 \dots (2)$ $x + y - z = 1 \dots (3)$ Eliminasi y dari Pers (1) dan (2), Pers (2) dan (3)	Solution: $x + y + z = 3 \dots (1)$ $3x - y + 2z = 4 \dots (2)$ $x + y - z = 1 \dots (3)$ Eliminating y from Equations (1) and (2), and Equations (2) and (3)

reasons for the steps taken. Regarding the concept restatement indicator, subject ST explained the concept of a system of linear equations with three variables (SPLTV) coherently in their own words. The subject explained that SPLTV is a system of linear equations with three variables and can be solved using elimination, substitution, or mixed methods.

In the indicator concerning the classification of objects based on conceptual properties, ST students can identify the information in the question and accurately translate it into a mathematical model. Students can also determine the relationships between the variables in the question to construct a mathematical model correctly. Furthermore, in the indicator concerning the identification of the properties of operations or concepts, ST students are able to determine the appropriate solution method and explain the rationale for using that method. The students are able to explain the steps of elimination and substitution systematically and demonstrate the relationships between the steps of the solution. In the indicator of providing examples and non-examples of the concepts studied, ST students can provide examples of SPLTV and distinguish them from forms of equations that do not fall under SPLTV.

Based on the interview results, the subject ST clearly and coherently explained the process of solving the problem. The subject also provided reasons for the steps used to solve the problem. Furthermore, the subject appeared more confident when explaining their work and was able to check their answers. The following is an excerpt from the interview conducted with subject ST.

T: Right, then. How did you determine the variables used in this question? Why did you choose to make those assumptions?

ST: Right, sir... In the problem, there are three sets of books and stationery. From these three sets, I categorized which were exercise books, which were pencils, and which were

rulers. Then I let the exercise books equal x , the pencils equal y , and the rulers equal z .

T: What method did you use to solve this system of equations (elimination, substitution, or a combination)? Why did you choose that method?

ST: The elimination method, sir. Because I think it's easier to use.

In addition, the ST subject also demonstrated the ability to reflect on their work. The subject reviewed the steps taken to reach a solution and ensured that the answer obtained was consistent with the concepts used.

Subject SS (Moderate Category)

Based on the results of the mathematical conceptual understanding test, eleven pupils were classified in the 'moderate' category. One pupil was then selected as the research subject in the 'moderate' category and assigned the code SS. The subject was selected based on their score within the 'moderate' range and their good communication skills. The following are the results of the 'moderate' subject's work in completing the test questions provided.

$$\begin{aligned} \text{(1)} \quad & x + y + z = 5 \\ \text{(2)} \quad & 2x - y + 2z = 9 \\ \text{(3)} \quad & x + y - z = 1 \end{aligned}$$

Eliminasi: (1) dan (3)

$$\begin{aligned} & x + y + z = 5 \\ & 3x - y + 2z = 9 \\ & -2x - 2z = -4 \quad \dots (4) \end{aligned}$$

Eliminasi: (2) dan (3)

$$\begin{aligned} & 3x - y + 2z = 9 \\ & 2x - 2z = -4 \quad \dots (5) \end{aligned}$$

Eliminasi: (4) dan (5)

$$\begin{aligned} & -2x - 2z = -4 \quad \times 1 \\ & 2x - 2z = -4 \quad \times 3 \\ & -2x - 2z = -4 \\ & 14x - 2z = -10 \\ & + 16x = 9 \\ & x = \frac{9}{16} \end{aligned}$$

$$z = 4$$

$$y = -2$$

Figure 3. Results of the work of the subject with moderate academic ability (SS)

Based on the analysis of the answer sheets, subject SS answered most questions correctly but still made some errors in the problem-solving process. The errors were generally due to inaccurate arithmetic and errors in determining the steps to solve the problems. Regarding the indicator of concept restatement, subject SS was able to restate the SPLTV concept in their own words, although the explanation provided was not entirely complete. The subject understood that SPLTV consists of three equations and three variables, but was not yet able to explain in detail

the relationships between the variables in the system of equations.

T : What method did you use to solve this system of equations (elimination, substitution, or a combination)? Why did you choose that method?

SS: *The elimination method, sir. Because it's easy.*

T : How did you make sure that the result you obtained was correct?

SS : *I'm just sure of it, sir... because I'm not entirely convinced it's correct either.*

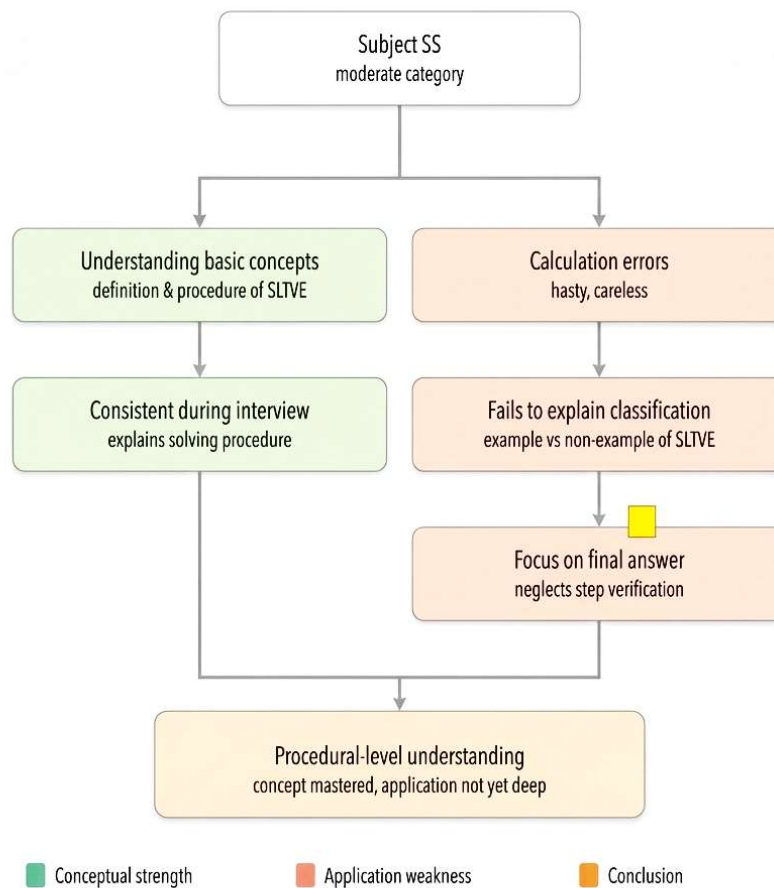


Figure 4. Concept map of mathematical understanding in the subject of SS

Figure 4 presents a concept map summarizing SS's mathematical conceptual understanding in SS, based on the analysis of test responses and interview results. As illustrated, SS demonstrated a solid grasp of the basic concept

and general procedure of SPLTV, which the interview consistently confirmed. However, this conceptual foundation was undermined by three recurring issues: careless calculation due to rushing, an inability to justify why a given equation

is or is not classified as SPLTV, and a tendency to prioritize obtaining the final answer over verifying each solution step.

Furthermore, regarding the indicator for identifying the properties of operations or concepts, the SS subject was able to use the correct method of solution. However, the subject still made errors in the calculation process. The subject tends to rush through problems, resulting in imprecise calculations. Regarding the indicator on providing examples and non-examples of the concepts studied, the subject was able to provide examples of SPLTV, but was not yet able to explain why a particular form of equation is included in or excluded from SPLTV. Based on the interview results, Subject SS admitted to rushing through the questions, which led to errors in the calculation process. Furthermore, the

subject stated that they were more focused on obtaining the final answer than on checking the steps they took to reach it. Nevertheless, Subject SS was still able to explain the basic concepts of SPLTV and the general procedures used to solve the problem.

Subject SR (Low Category)

Based on the results of the mathematical conceptual understanding test, eight pupils were classified in the low category. One pupil from this category was then selected as the research subject and assigned the code SR. The subject was selected because they had the lowest test score and reasonably good communication skills. The following are the results of the low-performing subject's work in completing the provided test questions.

1. Jelaskan apa yang dimaksud dengan Sistem Persamaan Linear Tiga Variabel (SPLTV).
Jawab: Konsep dan matematika yang digunakan untuk memecahkan masalah yang berkaitan dengan persamaan linear satu atau dua variabel.

2. Rumus:
Misal $x = \text{buku}$
 $y = \text{pensil}$
 $z = \text{penghapus}$
Rumus 1
 $2x + 2y + z = 5.200$
Rumus 2
 $2x + 3y + 2z = 6.400$
Rumus 3
 $3x + 2y + z = 6.800$

3. $2x + 2y + z = 5.200$
 $2x + 3y + 2z = 6.400$
 $3x + 2y + z = 6.800$
 $x + x + z = \dots 3$
 $3x - x + z = \dots 4$
 $x + y - z = \dots 1$

Figure 5. Results of the subject's work of the subject with low academic ability

Based on the analysis of the answer sheets, the SR subject answered only a small proportion of the questions correctly. The subject could write down the basic information provided in the questions but struggled to identify the appropriate mathematical model and solution steps. Regarding

the indicator of restating concepts, the SR subject could explain the concept of SPLTV only in simple terms. The explanation provided was limited and did not demonstrate a deep understanding of the SPLTV concept. Regarding the indicator of classifying objects based on

Table 4. Transcription and english translation of subject SR's written work (companion to figure 5)

No	Handwritten Text (Transcription)	English Translation
1	<i>Jelaskan apa yang dimaksud dengan Sistem Persamaan Linear Tiga Variabel (SPLTV)?</i>	Explain what is meant by a System of Linear Equations in Three Variables (SPLTV)?
	<i>Jawab: konsep dan matematika yang digunakan untuk menyelesaikan kasus yang tidak dapat disederhanakan dengan Persamaan Linear satu atau dua Variabel?</i>	Answer: a mathematical concept used to solve cases that cannot be simplified using linear equations with one or two variables.
2	<i>Peny:</i> <i>Mis x = buku tulis, y = pensil, z = penggaris</i> <i>Paket I: $2x+2y+z = 5.200$</i> <i>Paket II: $2x+3y+2z = 6.450$</i> <i>Paket III: $3x+2y+z = 6.800$</i>	Solution: Let x = exercise book, y = pencil, z = ruler Package I: $2x+2y+z = 5,200$ Package II: $2x+3y+2z = 6,450$ Package III: $3x+2y+z = 6,800$
3	$x + x + z = \dots(3)$ $3x - x + 2z = \dots(4)$ $x + y - z = \dots(1)$ [not continued]	$x + x + z = \dots(3)$ $3x - x + 2z = \dots(4)$ $x + y - z = \dots(1)$ [not continued]

conceptual properties, the SR subject struggled to determine the mathematical model based on the information provided in the questions. The subject was not yet able to link the information in the questions accurately to the SPLTV concept.

The following are the results of the interview conducted with the subject:

T: Try to explain again, in your own words, what is meant by SPLTV based on your answer here.

SR: *A mathematical concept used to solve problems that cannot be simplified using linear equations with one or two variables.*

T: Right, OK. How did you decide on the variables used in this problem? Why did you choose to set them up that way?

SR: *Well, there were some stationery items, so I let the book be x, the pencil y, and the ruler z.*

T: What method did you use to solve this system of equations (elimination, substitution, or a combination)? Why did you choose that method?

SR: *I don't know, sir.*

Furthermore, regarding the indicator for identifying the properties of operations or concepts, the SR subjects were unable to determine the appropriate solution method. In some questions, the subjects were only able to write down the initial steps without being able to continue the solution process to obtain the final answer. In the indicator regarding providing examples and non-examples of the concepts studied, the SR subjects experienced difficulty in distinguishing between forms of equations that are SPLTV and those that are not. Based on the interview results, the SR subjects admitted difficulty understanding the steps to solve SPLTV problems and often felt confused about which method to use. Furthermore, the subjects were also unable to explain the reasons for using certain steps in solving the problems.

The research findings indicate that the mathematical conceptual understanding of Year 11 Information and Communication Technology (ICT) students at SMK Negeri 2 Belu falls into varying categories: high, moderate, and low. Of

all the students who took the test, the majority fell into the moderate category.

Subject ST (High Category)

This indicates that the subject has a good understanding of the basic concepts of SPLTV and the characteristics of the SPLTV concept. These findings characterize the reasoning pattern observed in this specific case (ST) and should not be interpreted as representative of all six pupils in the high-achievement category, given the single-case design adopted for in-depth analysis (Yin, 2018), and are consistent with the multiple case study design adopted in this study, in which ST was analyzed as an individual, information-rich case representing the high-achievement category, rather than as a statistical representative of all six pupils within that category (Adams et al., 2022). The abilities demonstrated by subject ST indicate that high-achieving students have a good conceptual and procedural understanding. The subject was not only able to solve problems mechanically but also understood the concepts underlying problem-solving.

The findings of this study are consistent with Al Mutawah et al. (2019), who state that high-achieving students can identify known and requested information, construct mathematical models, and determine appropriate problem-solving strategies. Furthermore, Ningrum et al. (2022) also state that high-achieving students can restate concepts and classify objects based on specific properties accurately and correctly. This alignment with prior literature supports the analytic generalizability of the pattern observed in this case; that is, the pattern's consistency with existing theory, rather than claiming statistical generalizability across all pupils in the high category (Yin, 2018).

High-achieving students' ability to understand mathematical concepts indicates that they can integrate conceptual and procedural knowledge simultaneously. This is evident in their

ability to explain why they use specific procedures and to link the concepts studied to solving mathematical problems.

This ability indicates that the student has developed metacognitive awareness in solving mathematical problems, consistent with research showing that metacognitive monitoring plays a central role in both routine and nonroutine mathematical problem solving (Braithwaite & Sprague, 2021).

Subject SS (Moderate Category)

Taken together, these patterns indicate that SS's conceptual understanding remains at the procedural stage-competent in applying known steps, but not yet extending toward deeper conceptual reasoning. This indicates that the subject's conceptual understanding remains at the procedural stage. This indicates that students in the intermediate category already possess a basic understanding of mathematical concepts, but are not yet able to develop that understanding in greater depth.

These findings are consistent with analyses of students' procedural knowledge deficiencies, which show that students with moderate mathematical ability tend to be less careful and rush through problem-solving, resulting in errors in applying problem-solving procedures (Dorner et al., 2026). Furthermore, the same point is supported by research on the relations among conceptual knowledge, procedural knowledge, and procedural flexibility, which notes that shifts in conceptual knowledge, procedural knowledge, and procedural flexibility do not always occur uniformly, such that not all students with moderate mathematical ability are able to understand mathematical concepts well, as they have not yet met all the indicators of conceptual understanding (Chan et al., 2023). Similarly, an analysis of students' error patterns (Lin et al., 2025) also noted that students in the moderate category can meet only some of the indicators of conceptual

understanding. The research findings indicate that students in the intermediate category tend to use memorized problem-solving procedures without a deep understanding of the rationale behind them. Consequently, when faced with problems that differ from the examples previously provided by the teacher, students struggle to determine the appropriate problem-solving strategy. This situation suggests that mathematics teaching still needs to focus on developing students' critical thinking and mathematical reasoning skills. Teachers need to give students opportunities to explain the rationale behind a procedure and to practice solving non-routine problems.

Subject SR (Low Category)

This indicates that the subject has not yet fully understood the characteristics of the SPLTV concept. These results indicate that students in the low category do not yet possess a good understanding of mathematical concepts. Students still experience difficulties in understanding basic concepts, connecting information in the problems, and applying problem-solving procedures.

The findings of this study are consistent with the systematic review of error patterns among students with mathematics difficulty (Lin et al., 2025), which states that students with low ability are not yet able to understand concepts properly and therefore have difficulty determining the steps to solve problems. Furthermore, research on students' difficulties in mathematizing word problems in algebra (Jupri & Drijvers, 2016) also states that students in the low-ability category are not yet able to translate verbal information into mathematical models or apply concepts and algorithms to solve mathematical problems. Research on word-problem intervention (Powell et al., 2021) adds that students with low ability tend to have difficulty using procedures and solution steps because they do not yet understand the concepts underlying the problem-solving process. Several factors can influence students'

ability to understand mathematical concepts, such as poor mastery of basic mathematical concepts, a lack of varied practice questions, and low mathematical reasoning ability. Furthermore, limited learning resources and facilities in border regions can also affect students' learning processes.

General Discussion

These results indicate that students' ability to understand mathematical concepts remains at the procedural stage. Students are more accustomed to using the solution steps demonstrated by the teacher than to understanding mathematical concepts in depth. Consequently, when students face problems that differ slightly from the examples previously provided, they struggle to determine the appropriate solution strategy.

This suggests that students have basic abilities in understanding mathematical concepts but are not yet fully capable of developing a deep conceptual understanding. This finding is consistent with Fererde et al. (2024), which states that students tend to solve routine problems but still experience difficulties understanding and applying mathematical concepts in depth. Differences in ability between student categories are evident in their ability to understand concepts, construct mathematical models, determine solution strategies, and provide reasons for the steps used in solving mathematical problems. High-achieving students met all indicators of mathematical conceptual understanding well because they had stronger analytical and reasoning skills than students in the medium and low categories. This finding is consistent with Ningrum et al. (2022), which states that high-achieving students can understand the information in the problem, determine a mathematical model, and solve problems systematically.

Differences in students' levels of mathematical conceptual understanding may also

relate to broader sociocultural and educational characteristics of the Indonesia–Timor-Leste border region. Border communities in Belu Regency are characterized by diverse linguistic backgrounds, where Indonesian is often used alongside Tetum and other local languages in daily communication. While the present study did not specifically record or examine the language used by students during test completion or interviews, this multilingual context represents a plausible contextual factor that may warrant further investigation, given that language plays an important role in conceptual construction—students must first interpret verbal information before translating it into mathematical representations. This interpretation is consistent with Palla et al. (2026), who identified communication and learning environment as contextual determinants of numeracy achievement in the same border region. However, their study likewise did not directly measure students' language use during assessment. Future research employing direct observation or language-use documentation during data collection would be needed to substantiate this hypothesis empirically.

Furthermore, the sociocultural context of the border region provides valuable opportunities for contextual mathematics learning. Communities in the Indonesia–Timor-Leste border area possess rich cultural practices, such as traditional weaving activities and local games, which contain numerous mathematical concepts. However, these cultural resources are not always integrated into classroom instruction. When mathematics is taught without connecting concepts to students' everyday experiences, learners may perceive mathematical knowledge as abstract and disconnected from their social environment. Dominikus et al. (2023) demonstrated that weaving activities on Timor Island contain mathematical ideas that can be utilized as meaningful learning contexts. Similarly, Deda & Disnawati (2024) found that traditional games

practiced in border communities embody mathematical concepts and have the potential to support students' conceptual understanding through ethnomathematics-based learning.

These findings resonate with broader ethnomathematics research demonstrating that traditional games can enhance students' mathematical engagement (Kamid et al., 2022), and that ethnomodeling approaches can reveal mathematical reasoning embedded in indigenous cultural practices (Umbara et al., 2021).

The predominance of students in the moderate category may be attributed to contextual challenges commonly faced in border-region schools, such as limited access to varied learning resources, infrequent exposure to problem-solving tasks requiring conceptual reasoning, and instructional practices that emphasize procedural mastery over conceptual depth. These factors align with prior findings in similar geographically remote settings (Palla et al., 2026).

Therefore, the findings of this study suggest that efforts to improve students' mathematical conceptual understanding in border areas should focus not only on cognitive aspects but also on contextual factors such as language diversity, access to educational resources, and integrating local culture into mathematics instruction. Contextual and culturally responsive learning approaches may help bridge the gap between procedural knowledge and conceptual understanding, enabling students to develop deeper mathematical reasoning skills (Deda & Disnawati, 2024; Dominikus et al., 2023).

Based on the analysis of students' answers, the indicator relating to restating concepts was the one most frequently mastered by students. Most students could restate the SPLTV concept in their own words. However, regarding the indicators of identifying the properties of conceptual operations and providing examples and non-examples, some students still

experienced difficulties. This indicates that students' understanding remains procedural rather than conceptual. Students find it easier to follow problem-solving procedures than to understand the relationships between mathematical concepts in depth. This finding is consistent with Österman & Bråting (2019), which states that students tend to solve problems procedurally but still struggle to comprehend mathematical concepts comprehensively.

Based on the interview results, students were able to explain the information in the problems, construct mathematical models, and justify the steps they took to solve them. This indicates that high-achieving students possess a good conceptual understanding. This finding is consistent with Ningrum et al. (2022), which states that high-ability students are able to explain mathematical concepts logically and relate them to problem-solving. Meanwhile, students in the intermediate category can generally solve problems using the correct procedures but still make mistakes due to a lack of attention to detail and a limited understanding of the relationships between mathematical concepts. Furthermore, students were unable to explain the rationale behind using specific procedures, meaning their conceptual understanding remained limited to the examples provided by the teacher. This finding is consistent with the research (Chan et al., 2023), which states that students in the intermediate category still face difficulties in understanding mathematical concepts in depth.

In the low-achieving category, pupils struggled to understand the questions, construct mathematical models, and determine the steps required to solve the problems. Pupils were only able to solve problems similar to the examples provided by the teacher and became confused when the question format was modified. Furthermore, pupils were unable to explain the reasoning behind the solution steps they employed. These findings are consistent with the

systematic review of mathematical error patterns among students with mathematics difficulty (Lin et al., 2025), which states that students with low ability still experience difficulties in understanding and applying mathematical concepts.

The findings indicate that some students still struggle with non-routine problems that require mathematical analysis and reasoning. This situation suggests that mathematics teaching still places greater emphasis on procedural skills than on conceptual understanding. These findings are consistent with research on students' difficulties in mathematizing word problems in algebra (Jupri & Drijvers, 2016), which states that students still struggle to understand mathematical concepts when solving algebraic problems. Therefore, more innovative, student-centered learning is needed so students' understanding of mathematical concepts can develop more effectively. Overall, the results of this study indicate that students' ability to understand mathematical concepts in the Indonesia–Timor-Leste border region still needs improvement, particularly in conceptual analysis and applying concepts to solve mathematical problems. Consequently, more innovative, contextual, and student-centered learning is required so that students cannot only memorize solution procedures but also understand the meaning of mathematical concepts in depth.

■ LIMITATIONS

This study acknowledges an important limitation in selecting a single subject per achievement category. While this design is methodologically justified within a multiple case study framework (Yin, 2018), which prioritizes in-depth, information-rich cases over statistical representativeness (Adams et al., 2022), it inherently limits opportunities for source triangulation within each achievement group. This is a deliberate trade-off common to case-oriented qualitative research, where analytic

generalizability (transferability of patterns to existing theory) is prioritized over statistical generalizability (representativeness of a population) (Flyvbjerg, 2006). Future research involving multiple subjects per category and cross-case comparisons would help determine whether the reasoning patterns observed in this study are shared broadly within each achievement group.

■ CONCLUSION

Based on the results of the data analysis and discussion, it can be concluded that the mathematical conceptual understanding of Year 11 Information and Communication Technology (ICT) students at SMK Negeri 2 Belu in the Indonesia–Timor-Leste border region falls into the high, moderate, and low categories, with the majority of students falling into the moderate category. This indicates that the students' mathematical conceptual understanding has not yet developed optimally across all indicators of conceptual understanding. Students in the high category can understand problems, construct mathematical models, determine solution strategies, and explain the reasons for using steps systematically. Meanwhile, students in the moderate category can generally solve problems using the correct procedures but still make some mistakes due to a lack of attention to detail and a lack of deep conceptual understanding. Students in the low category still experience difficulties understanding concepts, constructing mathematical models, determining solution steps, and explaining the rationale for using procedures to solve mathematical problems.

Although most students were classified in the moderate category rather than the low category, this finding should not be interpreted as indicating that students' conceptual understanding is adequate. As explained in the discussion, students in the moderate category

generally followed correct procedures but frequently made errors due to insufficient attention to detail and a lack of deep conceptual grounding, consistent with the low academic performance and below-KKM scores that motivated this study. This suggests that procedural competence without strong conceptual understanding remains a persistent issue among students in this border region, reinforcing rather than contradicting the initial research problem.

Overall, the findings indicate that pupils' understanding of mathematical concepts in border regions still needs improvement, particularly in their ability to analyze concepts and apply them to mathematical problem-solving. Therefore, more innovative, contextual, and pupil-centered teaching is needed so pupils cannot only solve problems procedurally but also understand and apply mathematical concepts in greater depth.

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