Jurnal Pendidikan MIPA
e-ISSN: 2685-5488 | p-ISSN: 1411-2531
URL: https://jpmipa.fkip.unila.ac.id/index.php/jpmipa
Jounal's DOI: http://doi.org/10.23960/jpmipa

Vol. 27, No. 3, 2026
Article ID: 1468
Pages: 1555-1574

The Gauss-Jordan Method as a Representation Framework for HOTS in Rotational Dynamics

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Received: 07 July 2026 *Accepted: 10 August 2026* *Published: 29 August 2026*

Abstract: Developing higher-order thinking skills (HOTS) in physics requires learning experiences that encourage students to construct mathematical representations rather than rely solely on procedural problem solving. Although the Gauss–Jordan method has frequently been introduced as a computational technique for solving systems of linear equations, its potential as a mathematical representation framework has received little attention. This study investigated whether integrating the Gauss–Jordan elimination method into rotational dynamics instruction could support the development of students' higher-order thinking skills through mathematical representation. A quasi-experimental study involving 68 eleventh-grade students at MAN 1 Jember, Indonesia, used a nonequivalent control-class design. Students in the experimental class learned rotational dynamics through a representation-oriented Gauss–Jordan approach, whereas those in the control class received conventional instruction based on substitution and elimination methods. Students' mathematical thinking was assessed using five essay questions covering reasoning, generalizing, critical thinking, problem solving, and communicating. The results showed greater improvement in the experimental class (N-gain = 0.63) than in the control class (N-gain = 0.26). The difference between classes was statistically significant (Mann–Whitney, $p < 0.001$) with a large effect size ($r = 0.78$). The strongest gains were observed in generalizing, reasoning, and critical thinking. Analysis of students' written responses further indicated that learning through the Gauss–Jordan method helped students translate physical situations into mathematical models and recognize explicit relationships among variables. These findings suggest that the educational value of the Gauss–Jordan method extends beyond solving systems of linear equations, highlighting its role as a mathematical representation framework that supports higher-order thinking in physics learning.

Keywords: mathematical representation, Gauss–Jordan Method, mathematical thinking, Higher-order thinking skills, Rotational dynamics.

Article's DOI: https://doi.org/10.23960/jpmipa.v27i3.pp1555-1574

INTRODUCTION

Educational development in the twenty-first century emphasizes learning that nurtures students' potential through innovative, contextual, and future-oriented approaches. Beyond acquiring knowledge, students are expected to develop essential twenty-first-century competencies, including creativity, communication, critical thinking, and collaboration, as preparation for both everyday life and the workplace (González Pérez & Ramírez Montoya, 2022; Peña-Ayala, 2021). Meeting these expectations requires learning environments that foster students' motivation, active engagement, and meaningful learning outcomes (Ndraha et al., 2022; Wei et al., 2023; Xu et al., 2023). Within this context, developing higher-order thinking skills (HOTS) has become a major objective of contemporary education because such skills enable learners to analyze, evaluate, and respond to complex problems (Lu et al., 2021). Among the competencies underpinning HOTS, mathematical thinking is particularly important because it allows students to identify relationships among variables, construct mathematical models, and solve problems logically and systematically (Harahap et al., 2023; Szabo et al., 2020).

Mathematical thinking occupies a central position in physics because mathematics provides the language to describe natural phenomena and express relationships among physical quantities (Palmgren & Rasa, 2024; Serpe, 2023). From experimental analysis to the formulation of physical laws, physics relies on mathematical representations of varying complexity (Leng, 2021). Accordingly, students' success in learning physics depends not only on understanding physical concepts but also on their ability to connect those concepts with appropriate mathematical representations (Neumann et al., 2021; Woitkowski, 2020). Mathematical representations play an essential role in this process by enabling students to transform physical phenomena into visual, symbolic, and mathematical forms before arriving at a solution (Ainsworth, 2006; Duval, 2006). Conceptually, mathematical thinking involves logical, analytical, and quantitative reasoning to construct models, make generalizations, and solve mathematical problems (Devlin, 2021; Hartmann et al., 2021). These abilities allow students to interpret relationships among quantities, analyze data, and draw meaningful scientific conclusions, making mathematical thinking a fundamental basis for developing higher-order thinking skills in physics education (English, 2023; Zhao & Schuchardt, 2021).

Despite its importance, students' mathematical thinking skills in Indonesia remain relatively weak. Evidence from PISA shows that only about 1% of Indonesian students can model complex situations mathematically while selecting and evaluating appropriate problem-solving strategies, compared with the global average of 11% (Wu et al., 2022). The 2022 PISA assessment also placed Indonesia 70th out of 81 participating countries in mathematical literacy (OECD, 2023). National studies have reported similar concerns. Tupulu et al. (2023) found that approximately 59% of students were still unable to solve problems comprehensively, whereas Fitriyah et al. (2021) reported persistently low performance in mathematical generalization and proof. Collectively, these findings suggest that limited mathematical thinking skills continue to hinder students' success in physics learning (Tong et al., 2025).

The challenges associated with mathematical thinking become particularly evident in rotational dynamics, where students must coordinate concepts such as force, torque, moment of inertia, and angular acceleration within an interconnected mathematical system (Ahlamy et al., 2022). Solving problems in this topic commonly requires systems of linear equations (SLEs), meaning that students are expected not only to manipulate equations but also to construct mathematical models that accurately represent relationships among variables (Tasarib et al., 2025). Classroom observations, however, indicate that many students continue to struggle to identify relevant variables, develop mathematical models, and link physical concepts with appropriate mathematical representations. As a result, students often obtain solutions through procedural manipulation rather than understanding the underlying physical relationships. These classroom observations are consistent with broader evidence showing that Indonesian students continue to experience difficulties in mathematical literacy (OECD, 2023), comprehensive problem solving (Tupulu et al., 2023), mathematical generalization and proof (Fitriyah et al., 2021), and, ultimately, physics learning achievement (Tong et al., 2025).

Taken together, these findings suggest that physics instruction continues to emphasize procedural problem solving, while the development of mathematical representations receives comparatively less attention (Pols & Dekkers, 2024). As a result, many students are able to follow computational procedures without fully understanding the conceptual meaning of the mathematical transformations they perform (Ghimire & Singh, 2025). This mismatch creates a gap between the objectives of Higher-Order Thinking Skills (HOTS)-oriented learning and classroom practice, particularly in supporting students as they develop mathematical representations that are systematic, conceptually meaningful, and closely connected to physical reasoning (Alvarez-Tinajero et al., 2026).

Several studies have attempted to address these challenges by incorporating SLE-based methods into physics instruction. Supriadi et al. (2025), for example, reported that Cramer's rule improved students' mathematical reasoning and learning outcomes through a structured algorithmic approach. Similar benefits have also been reported for the Gauss–Jordan method, particularly in improving students' mathematical thinking across different physics learning contexts (Supriadi et al., 2026). Collectively, these studies demonstrate the instructional potential of SLE-based methods. Nevertheless, existing work continues to treat the Gauss–Jordan method primarily as a technique for solving systems of linear equations, its potential as a mathematical representation framework for supporting knowledge construction largely unexplored.

These limitations point to the need for a learning approach that extends beyond procedural problem solving. Mathematical representations should function not merely as tools for obtaining solutions but also as cognitive resources that help students establish relationships among variables, construct mathematical models, and strengthen mathematical thinking. This approach is particularly relevant to rotational dynamics because understanding the topic requires students to coordinate visual, symbolic, and mathematical representations simultaneously.

The proposed framework differs from a sequence of well-organized instructional procedures because it is grounded in representation theory. Drawing on Duval's theory of registers of semiotic representation and Ainsworth's framework for multiple representations (Ainsworth, 2006; Duval, 2006), transforming a system of linear equations into matrix form is viewed as movement between representational registers. The process begins with verbal-symbolic descriptions of physical relationships. It progresses toward a structural-symbolic representation in the form of an augmented matrix, where relationships among variables become explicit and can be manipulated through elementary row operations. Rather than simply following a fixed computational procedure, students are expected to compare, reorganize, and translate relationships across different representational forms. Such representational transformations require higher-order cognitive processes, including abstraction and generalization. Accordingly, the five learning stages described in the Method section are designed around successive transformations between representational registers instead of procedural efficiency alone.

Viewing the Gauss–Jordan method from the perspective of representation theory offers an opportunity to reconsider its educational role. Rather than treating it solely as a computational technique for solving systems of linear equations, the present study conceptualizes the method as a mathematical representation framework that supports mathematical modeling, the exploration of relationships among variables, and the generalization of physics concepts. Building on this perspective, the study investigates whether integrating the Gauss–Jordan elimination method into rotational dynamics learning as a mathematical representation framework enhances students' higher-order thinking skills. Its contribution lies not simply in applying the method to a different instructional context but in interpreting it as a representation-based framework for mathematical modeling and conceptual generalization rather than merely a sequence of procedural steps. This perspective extends the educational role of the Gauss–Jordan method beyond computation by positioning it as a mathematical representation framework for fostering higher-order thinking skills. Accordingly, this study addresses the following research question: Is integrating the Gauss–Jordan elimination method as a mathematical representation framework effective in enhancing students' higher-order thinking skills in learning rotational dynamics?

METHOD

Research Design

This research used a quantitative approach with a quasi-experimental design employing a non-equivalent control group. This design was chosen to evaluate the effectiveness of the Gauss-Jordan elimination method in developing students' mathematical thinking skills by comparing learning outcomes between the experimental and control classes. The research design involved two classes that were not selected through full randomization but had relatively equivalent initial ability characteristics (Fabrigar et al., 2024). Table 1 shows the research design.

Table 1. Research design

Class	Pre-Test	Treatment	Post Test
Experimental	O_1	X	O_2
Control	O_3	-	O_4

Source: (Creswell & Creswell, 2023).

Description:
O_1 and O_3 denote the pretest of mathematical thinking skills
O_2 and O_4 denote the posttest of mathematical thinking skills
X is the study of rotational dynamics through the integration of the Gauss-Jordan elimination method

Participants

The study was conducted at MAN 1 Jember during the odd semester of the 2025/2026 academic year. The research subjects consisted of two classes (an experimental class and a control class with 34 students in each class), selected using purposive sampling (Creswell & Creswell, 2023; Sugiyono, 2013) based on equivalent initial ability levels and the recommendations of the school's physics teacher. This technique was chosen because randomizing classes is not feasible in real-world school settings; the implications of this sampling approach for the generalizability of the findings are discussed further in the Limitations section. Both classes were taught by the same teacher and used the same instructional materials.

Learning Intervention

Before implementing this intervention, students had already received instruction on foundational matrix concepts and matrix algebra operations in eleventh-grade mathematics. This prerequisite competence allowed the three-session learning process to focus more specifically on applying and contextualizing matrix representation in rotational dynamics, rather than on introducing matrix concepts. Instruction in the experimental class applied the Gauss-Jordan elimination method as a mathematical representation framework for solving problems in rotational dynamics. This method is positioned not merely as a computational procedure for solving systems of linear equations, but as a mathematical representation framework that facilitates students' gradual transformation of physical phenomena into mathematical models (Ainsworth, 2006; Duval, 2006).

The learning process consists of five stages, namely: (1) analyzing physical phenomena and identifying variables; (2) formulating a mathematical model in the form of a system of linear equations; (3) transforming the system of linear equations into a matrix representation; (4) solving the matrix using Gauss–Jordan elementary row operations (ERO); and (5) interpreting the solution in a physics context.

Each stage is structured as a gradual transformation of representations, moving from physical phenomena through mathematical models and matrix representations to physical interpretations. Thus, the Gauss–Jordan method serves as a framework for mathematical representation that supports reasoning, generalization, critical thinking, problem-solving, and communication skills.

In contrast, the control class received conventional instruction using the elimination–substitution method to solve systems of linear equations, without utilizing matrix representations as part of the learning process. This instruction followed a teacher-centered format: a lecture-based explanation of the elimination–substitution procedure, followed by individual practice exercises in which students solved systems of linear equations using the same steps demonstrated by the teacher. Unlike the experimental class, the control class was not guided through an explicit representational transformation process (from physical description to matrix form); instead, students worked directly with the algebraic equations throughout the problem-solving process.

Instruments

The research instrument consisted of an essay test comprising five items developed to assess five mathematical thinking indicators, namely reasoning, generalizing, critical thinking, problem solving, and communicating. Each essay item was assessed based on all five indicators, with each indicator assigned a score ranging from 0 to 4. Thus, each item had a maximum score of 20, and the maximum total score for the five items was 100. Each item is designed to measure students' ability to integrate concepts of rotational dynamics with mathematical representations by formulating and solving multivariable word problems. The instrument was developed by considering the characteristics of rotational dynamics material, which involves relationships between physical quantities such as torque, moment of inertia, and angular acceleration, and thus requires systematic mathematical modeling skills. In addition, the questions were designed to accommodate higher-order thinking skills, particularly in analysis, evaluation, and conceptual generalization.

Table 2 below presents the indicators of mathematical thinking skills used in this study.

Table 2. Indicators of mathematical thinking skills

No	Indicators	Description
1	Reasoning	Be able to identify known and unknown information, and be able to establish logical relationships between physical quantities in problems involving rotational dynamics.
2	Generalizing	Able to represent physics problems in mathematical models, such as formulating SPLs from the relationships between variables.

3	Critical Thinking	Be able to evaluate solution steps, choose the appropriate strategy, and verify the consistency of calculation results.
4	Problem Solving	Be able to systematically solve a system of linear equations using the Gauss-Jordan elimination method to obtain the correct solution.
5	Communicating	Be able to interpret and communicate solutions clearly and systematically, both in mathematical form and in a physics context.

Source: (Shidqiya & Sukestiyarno, 2022)

Scoring Rubric

The assessment of mathematical thinking skills was conducted using an analytical rubric with a 0–4 scale for each indicator: reasoning, generalizing, critical thinking, problem solving, and communicating. A score of 0 indicates the absence of a relevant response or an answer that does not address the problem. In contrast, a score of 4 indicates excellent ability to meet the measured indicators in a comprehensive, logical, and systematic manner. Table 3 presents the assessment rubric. The rubric assesses several key aspects: accuracy in constructing mathematical models, clarity and a systematic approach in the solution steps, consistency of logic in the thought process, and the ability to interpret and communicate results in the context of physics and mathematics.

Each of the five essay items was scored across the five mathematical thinking indicators: reasoning, generalizing, critical thinking, problem solving, and communicating. Each indicator was scored from 0 to 4 for each item. Therefore, the maximum score for each item was 20, while the maximum total score across the five items was 100. The total score was obtained by summing the scores assigned to all five indicators across the five essay items. Consequently, the pre-test and post-test scores reported in the Results section were directly expressed on a 0–100 scale, with no further score conversion.

Table 3. Assessment rubric for mathematical thinking skills

Score	Criteria
4	Complete answers, an accurate systematic model, systematic and logical steps, and results communicated clearly
3	The answer is fairly comprehensive; there are a few minor errors, the steps are fairly systematic, and the results are still understandable.
2	The answer is partially correct; the model or procedure is not quite right, and there are some conceptual errors.
1	Incomplete answer, inaccurate mathematical model, unsystematic steps
0	No answer or irrelevant

Validity and Reliability

Two physics education faculty members and one physics teacher validated the research instruments for content, construct, and linguistic validity. Content validity referred to the alignment of the items with the material on rotational dynamics and mathematical concepts. In contrast, construct validity referred to the alignment of the items with indicators of mathematical thinking skills, which include reasoning, generalizing, critical thinking, problem solving, and communicating.

The empirical validity of the test items was analyzed using Pearson's product-moment correlation, while the reliability of the instrument was analyzed using Cronbach's alpha. The results showed that all test items had validity coefficients above the minimum criteria, and the instrument had a Cronbach's alpha of 0.80, indicating that it was both valid and reliable.

Since the instrument consists of essay questions, assessment is conducted using an analytical rubric developed based on indicators of mathematical thinking skills. Using the same rubric for all student responses is intended to ensure consistency and objectivity in the assessment.

Data Collection Procedure

Data collection was conducted by administering tests at the beginning and end of the learning process (pre-test–post-test) to measure students' mathematical thinking skills before and after the intervention. The learning process took place over three sessions, each lasting 2 x 45 minutes. All assessment procedures were carried out systematically, from the preparation stage through implementation to evaluation.

Data Analysis

The data were analyzed using descriptive and inferential statistics. Improvements in mathematical thinking skills were analyzed using the following statistical techniques:

1. The classification of normalized N-Gain values refers to the criteria proposed by Syaiful et al. (2025). The N-Gain value measures the level of increase in high-level thinking abilities. This increase is categorized as high if the value of g e" 0.70, medium if 0.30 d" $g < 0.70$, and low if the value is $g < 0.30$.
2. Assumptions were tested using the Shapiro–Wilk test for normality. The data were first tested for normality using the Shapiro–Wilk test. If the data were normally distributed, the parametric independent-samples t-test was used; if the assumption of normality was not met, the Mann–Whitney test was used.
3. The interpretation of effect size values refers to the criteria proposed by Dick et al. (2025). Effect size values calculated using the nonparametric (r) statistic are categorized as small if $r = 0.1$, medium if $r = 0.3$, and large if $r \geq 0.5$

The results were interpreted based on the N - gain categories (high, medium, low) and the effect size categories (small, medium, large), so that the research findings demonstrate not only statistical significance but also the strength of the treatment's effect.

■ RESULT AND DISCUSSION

Initial Competency Description

Students' initial mathematical thinking abilities were analyzed using pre-test results before the instructional intervention was implemented. This analysis aimed to ensure equivalence in initial ability between the experimental and control classes, so that any small differences observed at the final stage could be validly attributed to the instructional intervention implemented (Creswell, 2014; Fraenkel et al., 2012). The pre-test instrument was developed based on indicators of mathematical thinking skills, including reasoning, generalizing, critical thinking, problem solving, and communicating.

Table 4. Pretest descriptive statistics and results of the n-gain calculation

Class	Pretest					Posttest		
	N	Mean	SD	Min Score	Max Score	Mean	N-Gain	Category
Experiment	34	26.76	9.43	7	47	73.26	0.63	Medium
Control	34	26.29	10.13	9	53	45.35	0.26	Low

Based on Table 4, the average pre-test scores for the experimental class (M = 26.76, SD = 9.43) and the control class (M = 26.29, SD = 10.13) were relatively similar, with a very small difference. This indicates that the student's initial mathematical thinking abilities in both classes were at comparable levels.

The results of the normality test using the Shapiro-Wilk test showed that the pre-test data for both classes were normally distributed ($p >$ 0.05). In addition, the test for homogeneity of variances using Levene's test showed that the variances of the two classes were homogeneous ($p > 0.05$). With the assumptions of normality and homogeneity met, the subsequent analysis used an independent-samples t-test The results of the independent-samples t-test showed a significance value (2 - *tailed*) of 0.861 ($p > 0.05$), indicating that there was no significant difference between the initial abilities of the two classes. These findings confirm that the experimental and control classes had equivalent baseline conditions before the treatment was administered.

The equivalence in initial ability observed in this study is an important prerequisite for quasi-experimental research because it minimizes selection bias and strengthens internal validity (Shadish et al., 2002). Therefore, the differences in posttest outcomes can be interpreted as being primarily associated with the instructional treatment.

Improving Mathematical Thinking Skills (*N-gain*)

Improvements in mathematical thinking skills were analyzed using normalized gain (*N*-*gain*) to measure the effectiveness of learning while taking into account the students' initial levels (Hake, 1998; Sirianansopa, 2024). The results of the *N* - gain calculations for the students' mathematical thinking skills are presented in Table 4. The results indicate that the experimental class demonstrated greater improvement in mathematical thinking skills than the control class. The average score in the experimental class increased from 26.76 to 73.26, with an *N-gain* of 0.63, which falls into the moderate category. In contrast, the control class increased from 26.29 to 45.35, with an N-gain of 0.26, which falls into the low category.

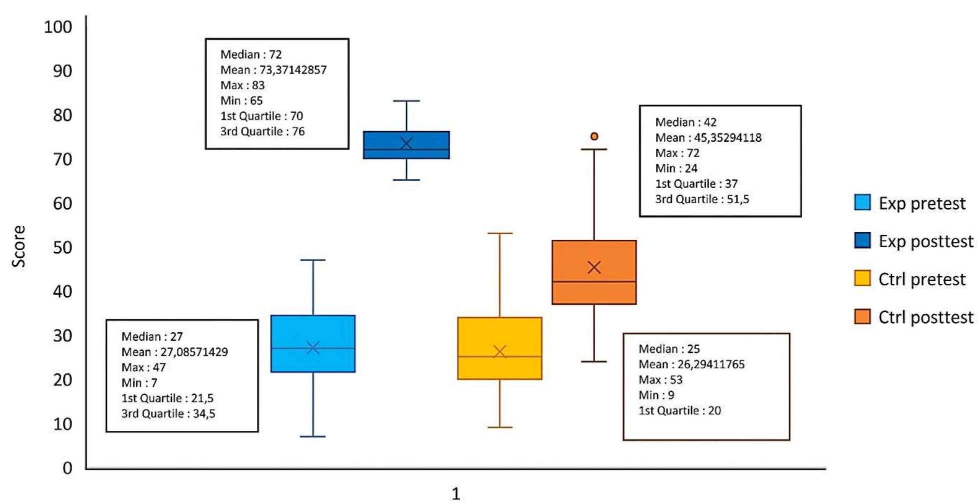


Figure 1. Visualization of the score distribution for the experimental class and the control class

Figure 1 shows that the experimental class improved more from pretest to posttest than the control class. This result is consistent with the N-gain analysis, indicating that instruction integrating the Gauss–Jordan method was more effective in improving students' mathematical thinking skills than conventional instruction. The greater improvement observed in the experimental class suggests that using the Gauss-Jordan method as a framework for mathematical representation not only helps students solve problems procedurally but also facilitates mathematical modeling and a more systematic understanding of the relationships between variables.

Thus, these findings suggest that integrating the Gauss-Jordan method into the teaching of rotational dynamics is more effective in enhancing students' mathematical thinking skills than conventional teaching methods. This improvement indicates that using the method as a framework for mathematical representation can support the development of higher-order thinking skills. However, further analysis of each indicator is needed to identify specific contributions to aspects of HOTS.

Differences and the Power of Learning's Influence

To test the difference in the improvement of mathematical thinking skills between the experimental and control classes, an inferential analysis was conducted using the Mann-Whitney test. The choice of this nonparametric test was based on the results of the Shapiro-Wilk normality test, which indicated that the post-test data were not fully normally distributed $(p < 0.05)$, making parametric analysis unsuitable. In addition, to determine the magnitude of the instructional treatment effect, an effect size calculation was performed.

The results of the Mann-Whitney test showed a significance value ($p < 0.001$), indicating that there is a significant difference between the mathematical thinking abilities of students in the experimental class and those in the control class. This suggested that the instructional intervention provided to the experimental class resulted in a statistically significant improvement compared to conventional instruction.

Furthermore, the effect size calculation yielded a value of $r = 0.78$, which falls into the "large" category. This value indicates that the effect of using the Gauss-Jordan method on improving students' mathematical thinking skills is not only statistically significant but also has a high practical impact.

Thus, these findings indicate that integrating the Gauss-Jordan method into the teaching of rotational dynamics makes a substantial contribution to improving students' mathematical thinking skills. This suggests that using the Gauss-Jordan method not only improves learning outcomes but also strengthens mathematical thinking structures by systematically representing relationships among variables.

Analysis Based on HOTS Indicators

To better understand the effectiveness of learning, improvements in mathematical thinking skills are analyzed not only as a whole but also by individual indicator. This analysis aims to identify which indicators have shown the most significant improvement and to describe the profile of students' development in mathematical thinking skills across each aspect: reasoning, generalizing, critical thinking, problem solving, and communicating.

Table 5. N-Gain and effect size calculation results for each indicator

Indicators	Experiment Class	Control Class	r
Reasoning	0.86	0.37	0.60
Generalizing	0.91	0.38	0.80
Critical Thinking	0.82	0.27	0.77
Problem Solving	0.55	0.24	0.66
Communicating	0.23	0.12	0.13

As presented in Table 5, all indicators of mathematical thinking skills improved more in the experimental class than in the control class. Generalizing and reasoning showed the largest gains, while communicating showed the smallest gain. This pattern reflects the nature of the Gauss-Jordan method. Solving a problem with this method requires students to first translate a physical situation into a system of equations and then reorganize that system into matrix form. This process directly exercises the ability to identify relationships between variables and to reason about them systematically, which explains why generalizing and reasoning show the largest improvement. In contrast, communicating requires students to explain a representation to others rather than simply construct or manipulate it. The Gauss-Jordan method does not directly train this skill, which likely explains its comparatively small improvement. The control class showed the same ordering of indicators but at consistently lower magnitudes. This supported the interpretation that conventional instruction was less effective in comprehensively developing mathematical thinking skills.

The visualization in Figure 2 showed that the experimental class consistently had higher *N-gain* values than the control class across all indicators. The most striking differences were observed in the generalizing and critical thinking indicators, suggesting that learning using the Gauss-Jordan method significantly contributed to developing students' abilities to construct mathematical models and conduct in-depth analyses of problems.

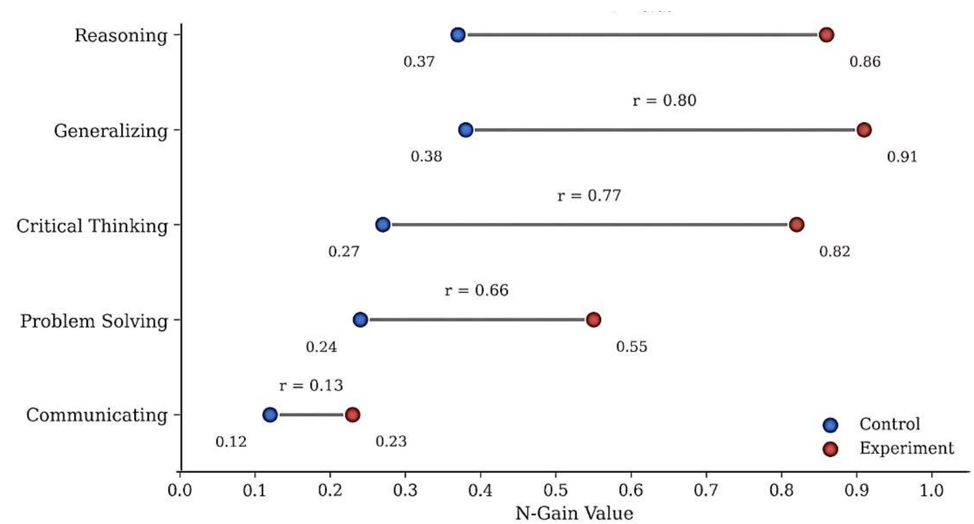


Figure 2. Average n-gain value for each indicator

In addition, the significant improvement in the reasoning indicator suggested that this method helped students understand the logical relationships between variables more systematically. In contrast, the communicating indicator showed a relatively small improvement, suggesting that students' ability to present and communicate results mathematically still requires further strengthening.

Based on Table 5, the effect size values indicate that the impact of the Gauss-Jordan method on the improvement of mathematical thinking skills varies across each indicator. The generalizing indicator had the greatest effect (r = 0.80), followed by critical thinking (0.77) and problem solving (r = 0.66), which fall into the "large" category. The reasoning indicator also showed a fairly strong effect (0.60), while the communicating indicator had a relatively small effect (r = 0.13).

Qualitative Analysis (Student Responses)

To reinforce the quantitative analysis, this study also included a qualitative analysis of the responses from students in the experimental and control classes. This analysis aimed to identify differences in mathematical thinking structures, problem-solving representations, and the problem-solving strategies employed. Additionally, a qualitative analysis was conducted to better understand how integrating the Gauss-Jordan elimination method facilitated mathematical representation and the development of students' higher-order thinking skills.

Reasoning

The reasoning indicator reflects students' ability to use logical reasoning to analyze relationships between concepts and variables in a physics problem. In physics education, mathematical reasoning skills are an important part of mathematical thinking because they help students systematically connect mathematical representations with interpretations of physics concepts (Devlin, 2021).

Students in the experimental class demonstrated better reasoning skills than those in the control class. This was evident from the students' answers, in which they not only wrote that mass m $> m,$, but were also able to explain the physical consequences of this relationship namely, that object 1 moves downward and object 2 moves upward. These answers indicated that students could logically connect quantitative information with their interpretation of motion in a pulley system.

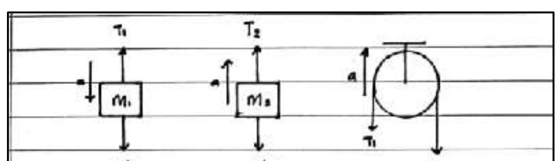
In contrast, students in the control class generally only wrote down the relationship m $> m,$ without providing further explanation regarding the direction of the system's motion or its physical implications. This indicates that the students' thought processes in the control class were still at the information-identification stage and had not yet reached the conceptual reasoning stage that links mass to the dynamics of the system's motion. These findings suggest that learning using the Gauss-Jordan method helps students develop a more systematic and logical way of thinking when analyzing relationships between physical variables.

Generalizing

Generalization is a student's ability to represent physical phenomena in mathematical models. This ability involves systematically linking physical concepts with symbols, diagrams, equations, and other mathematical representations to construct a problem-solving framework (Rasyid et al., 2025). In physics education, the ability to generalize is crucial because it helps students transform concrete phenomena into mathematical models that can be analyzed logically and in a structured manner.

$\Sigma F = m_1 a$	$\Sigma F = m_2 a$	$\Sigma \tau = I\alpha$
$5(10) - T_1 = 5a$	$T_2 - 4(10) = 4a$	$T_1 - T_2 = a$
$T_1 + 5a = 50$	$T_2 - 4a = 40$	

(a)



# object 1	# object 2	# Pulley
$\Sigma F = m_1 a$	$\Sigma F = m_2 a$	$\Sigma \tau = I\alpha$
$m_1 g - T_1 =$	$T_2 - m_2 g = 4a$	$T_1 - T_2 = m_K \cdot a$
$T_1 + 5a = 50$	$T_2 - 4a = 40$	$T_1 - T_2 = a$

(b)

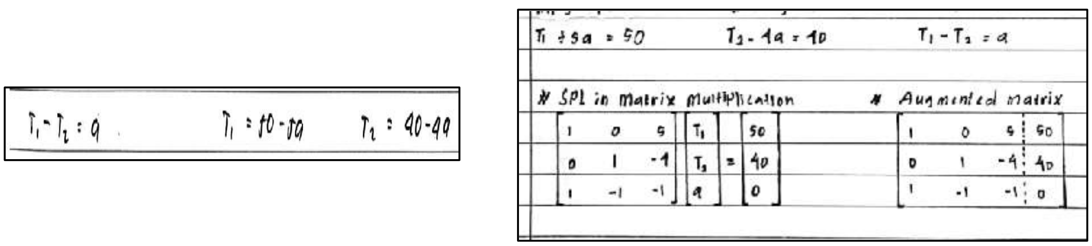
Figure 3. Comparison of students' responses on the "generalizing" indicator in the control class (a) and the experimental class (b)

Figure 3 shows that students in the control Class tended to write down mathematical equations directly without first drawing a free-body diagram. The students' answers indicated that the generalization process was still limited to substituting values into the dynamic equations without explicitly showing the relationships between forces and the system's direction of motion. As a result, the relationships between variables appeared less systematic, and their solutions tended to be procedural.

In contrast, students in the experimental class demonstrated a more comprehensive generalization process by drawing freehand diagrams of each object before formulating mathematical equations. Based on these visual representations, students then transformed them into appropriate mathematical models. These findings confirm that learning using the Gauss-Jordan method helps students build multi-level representations, ranging from visual representations and mathematical models to organized systems of equations (Supriadi et al., 2026).

Critical thinking

Critical thinking indicators relate to students' ability to select, evaluate, and use problem-solving strategies logically and systematically. This ability encompasses not only applying mathematical procedures but also reorganizing information and determining the most effective representations for solving problems (Rexigel et al., 2024; Wang & Abdullah, 2024). In physics education, critical thinking plays a vital role in helping students understand the connections between concepts and choose the appropriate mathematical approach to problem-solving.



(a) (b)

Figure 4. Comparison of students' responses on the critical thinking indicator in the control class (a) and the experimental class (b)

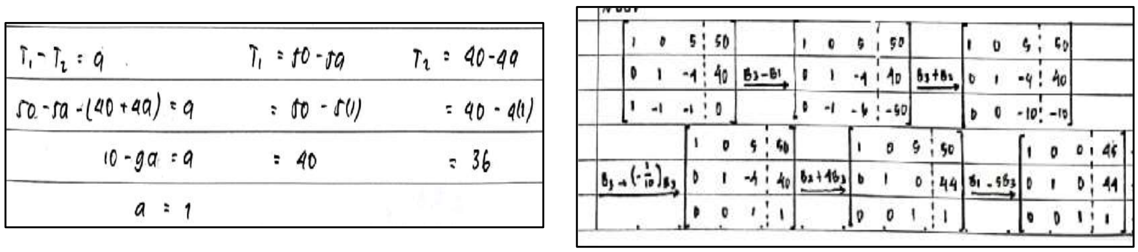
In the control class, students generally wrote down only three linear equations derived from Newton's laws and rotational dynamics, without further transforming them into a more structured mathematical representation. Each equation was treated and solved as a separate step rather than as part of one interconnected system. These answers indicate that the students' thinking process is still at the stage of applying basic procedures and has not yet reached the stage of reorganizing the system of equations into a single unified structure that represents the relationships among all variables.

Meanwhile, students in the experimental class demonstrated better critical thinking skills by transforming linear equations into matrix equations and augmented matrices. Students were not only able to construct mathematical equations but also to represent the relationships between variables in a more organized matrix structure. These findings indicate that integrating the Gauss-Jordan method helps students develop critical thinking skills through the systematic, structured reorganization of mathematical information (Wang & Abdullah, 2024).

Problem solving

Problem-solving is a student's ability to solve problems by applying mathematical strategies, procedures, and steps in a structured manner until the correct solution is obtained. This ability includes integrating concepts, selecting solution methods, and performing mathematical manipulations logically and systematically to solve problems (Polya, 2014). In physics education, problem-solving skills are crucial because students are expected not only to understand concepts but also to apply them to arrive at the correct mathematical solution.

Students in the control class solved the SPL using direct substitution and elimination methods. Although they correctly obtained the final solution for the system's acceleration, they still carried out the solution process step by step and separately for each equation. As a result, the relationships among the variables were not fully apparent. In addition, the students' answers indicated that their mathematical manipulation process was still procedural in nature and relied on repeated substitution.



(a) (b)

Figure 5. Comparison of students' responses on the problem-solving indicator in the control class (a) and the experimental class (b)

In contrast, students in the experimental class demonstrated a more systematic problem-solving process by using the Gauss-Jordan method. The system of linear equations was first represented in augmented matrix form, then solved step by step using the ERO method until a reduced row echelon matrix was obtained. This process shows that students cannot only arrive at the final solution but also organize the solution procedure in a more structured and efficient manner. Using matrix representation helps students see the relationships between equations simultaneously, making the problem-solving process more systematic than the conventional approach. These findings indicate that integrating the Gauss-Jordan method supports the development of problem-solving skills through a more organized and logical mathematical representation.

Communicating

The communicating indicator relates to students' ability to present their problem-solving results clearly, coherently, and in accordance with the mathematical representations and physics concepts used. Mathematical communication skills include the ability to write down symbols, notation, and units, as well as to interpret results step by step so that the problem-solving process can be well understood (Tong et al., 2021). In physics education, mathematical communication skills are important because students must not only arrive at a final answer but also explain the physical meaning of the results they obtain.

In the control class, students generally wrote down only the final results, acceleration and rope tension, without consistently using physics symbols and units. Their written answers were also brief and did not present the results logically. Meanwhile, in the experimental class, students presented their answers more systematically, using appropriate physics symbols and units. However, in both the control and experimental classes, students did not interpret the results or explain their physical meaning; instead, they merely wrote down the numerical values of their calculations.

These findings indicate that students' communication skills remain low compared to other indicators of mathematical thinking. Although the Gauss-Jordan method helps students formulate solutions more systematically, their ability to communicate results conceptually and interpretatively has not yet developed optimally. This indicates that learning focused on mathematical representations needs to be integrated with strategies that specifically train mathematical communication and conceptual explanations in physics.

Gauss-Jordan as a Mathematical Representation

The results of this study suggest that the educational contribution of the Gauss–Jordan method extends beyond its traditional role as a procedure for solving systems of linear equations. Improvements observed in the generalizing and critical thinking indicators indicate that students did more than obtain correct solutions; they also became better at constructing mathematical models and identifying relationships among variables in an integrated way. This interpretation is consistent with previous work emphasizing that mathematical representations play a central role in conceptual understanding and the development of higher-order thinking skills (Castillo et al., 2025).

One possible explanation for this pattern lies in the nature of matrix representations themselves. Representing a system of equations in matrix form makes its mathematical structure more explicit, allowing students to organize information before carrying out calculations. Within physics learning, linking mathematical representations to physical concepts is essential for developing a deeper understanding of phenomena (Neumann et al., 2021; Wasis et al., 2023). In addition, elementary row operations (ERO) provide a structured sequence of reasoning that encourages logical, systematic, step-by-step thinking. Such a process supports mathematical thinking by integrating analytical reasoning, generalization, and problem-solving skills (Devlin, 2021).

The value of this approach becomes more apparent when it is compared with conventional instruction. In substitution- and elimination-based procedures, students usually process relationships among variables sequentially, making it difficult to visualize the structure of the entire system. Consequently, students often follow computational steps without fully recognizing how the variables are connected. In contrast, representing the complete system as an augmented matrix enables students to examine those relationships simultaneously. This characteristic promotes relational understanding rather than merely instrumental understanding (Hiebert & Carpenter, 1992; Skemp, 1976).

A similar pattern was found in the reasoning indicator, which showed a relatively large improvement. The structured sequence required by the Gauss–Jordan method appears to help students organize logical arguments while following a clear problem-solving pathway. This observation agrees with Schoenfeld (1985), who argued that well-structured problem-solving activities strengthen mathematical reasoning and strategic thinking. A different pattern emerged for the communicating indicator. Although students became more capable of solving problems systematically, their ability to explain their reasoning and communicate mathematical ideas improved to a much smaller extent. This finding suggested that students' internal representational development during mathematical modeling was not accompanied by an equivalent improvement in expressing those representations through explanation and interpretation (Duval, 2006). A likely explanation is that classroom activities emphasized representational transformation and matrix manipulation while providing relatively few opportunities for students to verbalize their reasoning or explain the physical meaning of the solutions obtained. This interpretation is consistent with Ainsworth (2006), who argued that being able to use a representation does not necessarily imply being able to communicate that representation without deliberate opportunities for reflection and argumentation.

These findings also point to a practical implication for classroom instruction. Rather than leaving this issue solely for future investigation, teachers can explicitly integrate communication activities into Gauss–Jordan learning. For example, students may be asked to present their augmented matrices and reduced row echelon forms while explaining each elementary row operation in their own words. Another possibility is to require written justifications describing the physical relationship represented by every row operation. Such activities complement the matrix-solving procedure by encouraging students to externalize reasoning already developed internally through representational transformation.

Taken together, the findings indicate that the effectiveness of the Gauss–Jordan method is not determined simply by its role as a computational technique for solving systems of linear equations. Its educational value is more closely related to its function as a mathematical representation framework that guides students in transforming physical situations into formal mathematical models. Through this process, students do more than obtain numerical answers; they establish relationships among equations, develop mathematical models, and generalize concepts more systematically. From this perspective, the present study addresses a limitation in previous research, which has largely viewed the Gauss–Jordan method as a computational procedure, by demonstrating the importance of its representational function in supporting students' mathematical thinking.

The present findings further suggest that mathematical representation acts as the principal link between conceptual understanding in physics and the development of higher-order thinking skills. Unlike approaches that primarily emphasize obtaining the final solution, a representation-oriented approach helps students understand the structural relationships among physical quantities through successive representational transformations. Accordingly, this study's contribution extends beyond improving learning outcomes. It also strengthens the conceptual foundation of physics instruction by positioning mathematical modeling as an integral part of scientific thinking.

From a practical perspective, the Gauss–Jordan method may serve as an alternative instructional strategy for physics topics involving multivariable relationships, including rotational dynamics. Matrix representations help students organize relationships among variables while supporting the development of reasoning, generalizing, and critical thinking, all of which are essential components of mathematical thinking (Ainsworth, 2006; Devlin, 2021; Duval, 2006). At the same time, the relatively modest improvement observed in the communicating indicator suggests that future implementations should incorporate discussion, presentations, and scientific argument writing so that mathematical communication develops alongside representational competence (Neumann et al., 2021; Redish, 2003).

■ LIMITATIONS

This study has several limitations that should be considered when interpreting its findings. First, the sample was selected using purposive sampling based on the recommendations of the school's physics teacher rather than random assignment, a decision made necessary by the constraints of randomizing intact classes in a real-world school setting. While this approach helped ensure comparability in students' initial ability levels between the experimental and control classes, it also introduces a potential selection bias. It limits the extent to which the findings can be generalized beyond the specific school context and student population studied. Additionally, this study did not include prior semester report card scores in physics and mathematics as a supplementary indicator of baseline academic equivalence between the two classes because of limited access to students' academic records from the school. The equivalence of students' initial ability was instead established through the pretest of mathematical thinking skills (Table 4), which showed no significant difference between the experimental and control classes ($p = 0.861$), as well as the physics teacher's professional judgment in recommending comparable classes. Future research employing random sampling across multiple schools would help establish the broader generalizability of the Gauss–Jordan representation framework's effectiveness.

CONCLUSION

The present study demonstrates that integrating the Gauss–Jordan method as a mathematical representation framework effectively promotes students' mathematical thinking in rotational dynamics. Its contribution extends beyond solving systems of linear equations by supporting the transformation of physical phenomena into mathematical models, allowing students to establish relationships among variables and develop higher-order thinking skills through mathematical representation. From this perspective, the educational value of the Gauss–Jordan method lies not only in its computational function but also in its role as a representation framework that strengthens mathematical thinking in physics learning.

One aspect that remains less developed is mathematical communication. Although students became more capable of solving problems systematically, they still need further support to explain their reasoning, interpret mathematical representations conceptually, and communicate scientific ideas more effectively. Future studies may therefore combine the proposed representation framework with instructional models emphasizing discussion, scientific argumentation, or multiple representations to foster more balanced development across all dimensions of mathematical thinking.

DECLARATION OF GENERATIVE AI USAGE IN THE WRITING PROCESS

During the writing of this manuscript, the authors used DeepL solely for English-language translation. DeepL was not used to generate the scientific content, analyze data, interpret results, or draw conclusions. The authors carefully reviewed and edited all translated content and assume full responsibility for the published article.

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