

The Depth of Problem Definition: Evaluating Pre-Service Mathematics Teachers' Initial PCK through Post-Case Study Diagnosis

Silfanus Jelatu^{1,*}, Maximus Tamur¹, Ricardus Jundu², Fulgensius Efrem Men¹, & Ermelinda Bahagia¹

¹Mathematics Education Study Program, Universitas Katolik Indonesia Santu Paulus Ruteng, Indonesia

²Elementary School Teacher Education Study Program, Universitas Katolik Indonesia Santu Paulus Ruteng, Indonesia

*Corresponding email: silfanusjelatu@yahoo.co.id

Received: 24 July 2026

Accepted: 28 September 2026

Published: 21 October 2026

Abstract: This qualitative embedded single-case study examined how pre-service mathematics teachers in Eastern Indonesia define pedagogical problems after school-based case studies and how the depth of those definitions differs across initial pedagogical content knowledge (PCK) categories. Six purposively selected sixth-semester students, two each in low, moderate, and high initial-PCK groups, were selected from 44 questionnaire respondents. Data comprised post-case-study reports, an initial-PCK questionnaire, and semi-structured interviews. The data were analyzed through a reflexive thematic procedure involving data condensation, coding, theme development, and cross-category comparison, with triangulation, peer debriefing, member checking, and an audit trail. All participants identified conceptual and procedural difficulties, prerequisite gaps, and modeling obstacles. However, low-PCK participants predominantly described surface symptoms; moderate-PCK participants mapped error patterns and prerequisite weaknesses using evidence; and high-PCK participants constructed precise, evidence-based, multidimensional diagnoses spanning cognitive, affective, and metacognitive dimensions. Topic complexity was treated as an analytic condition rather than a basis for causal inference by comparing the structure and evidential quality of definitions within each case and then across PCK categories. The findings position problem definition as an observable expression of pedagogical reasoning and support explicit, scaffolded diagnostic-competence learning in mathematics teacher education, especially in resource-constrained and diverse educational settings.

Keywords: pre-service mathematics teachers, problem definition, initial pedagogical content knowledge, eastern indonesia.

Article's DOI: <https://doi.org/10.23960/jpmipa.v27i4.pp1897-1922>

■ INTRODUCTION

Lee Shulman (1987) classified teachers' professional competencies into three main domains: subject-matter Content Knowledge (CK), general Pedagogical Knowledge (PK), and their intersection, known as Pedagogical Content Knowledge (PCK). PCK is understood as a teacher's capacity to transform content knowledge into representations, explanations, examples, and pedagogically robust learning strategies accessible to students (Ball et al., 2008; Shulman, 1987). Subsequent research views PCK as a key component in explaining how

teachers organize, interpret, and respond to students' mathematical thinking in concrete learning contexts (Brodie et al., 2018; Hammer & Ufer, 2023).

As research on PCK has gained momentum, various researchers have identified several derivative competencies that functionally represent the manifestation of PCK in practice, including diagnostic competence (*diagnostic skill*) and the ability to accurately assess and define evidence of student understanding or diagnose evidence of student thinking (Phelps-Gregory & Spitzer, 2018; Reinhold, 2018).

Diagnostic skills refer to a teacher's ability to identify, interpret, and categorize students' errors and ways of thinking, and then relate them to relevant content knowledge and learning process characteristics (Brodie, 2014; Brodie et al., 2018). Teachers with diagnostic competence cannot only recognize correct or incorrect answers but also trace the reasons behind them and use that information to adjust learning strategies to better suit students' needs (Jacobs et al., 2010; Reinhold, 2018).

In this context, two essential pre-instructional activities are identified: (1) the pedagogical diagnosis of students' initial conditions, thinking processes, and difficulties, and (2) the formulation of accurate pedagogical problems accompanied by didactically sound alternative solutions. Research indicates that the ability to analyze tasks, anticipate student responses, and plan for the implementation of high-cognitive-demand tasks is a key indicator of teachers' professional competence and directly influences the quality of both planning and implementation of mathematics instruction (Baumert et al., 2010; Hammer & Ufer, 2023). Teachers with strong PCK can better select and prepare cognitively demanding mathematics tasks, anticipate student responses, and design appropriate instructional interventions (Hammer & Ufer, 2023; Stein et al., 1996). Therefore, diagnostic accuracy and precision in formulating pedagogical problems are prerequisites for planning, managing, and reflecting on teaching practices that prioritize mathematical understanding over mere curriculum completion.

For pre-service teacher students, particularly those in the final phase of their studies and preparing to participate in internship or field practice programs, mastery of diagnostic skills and the ability to define pedagogical problems accurately are concrete manifestations of PCK development. From the very beginning of the teacher education program through the mid-semester (before the internship period), they

already possess what can be termed initial PCK, reflected in how they interpret simulated classroom situations, interpret "student" (audience) responses, design tasks and assessment instruments, and plan follow-up learning activities (Biza & Nardi, 2020; Reinhold, 2018). This is reinforced by mastery of theory gained through pedagogical courses, as well as mathematical expertise. Thus, examining how pre-service mathematics teachers *define* pedagogical problems and articulate their diagnostic findings is a key step in assessing how well teacher education programs have prepared them to enter professional practice in a reflective and evidence-based manner, viewed through the lens of the quality of their initial PCK.

The ability to *define*, as part of diagnostic skills, can be understood as a teacher's capacity to accurately identify, interpret, and formulate learning problems that emerge from data on students' mathematical understanding. In various models of diagnostic competence, this activity encompasses processes such as identifying problems, asking questions, formulating hypotheses, collecting and evaluating evidence, and drawing conclusions based on that evidence (Csanadi et al., 2021; Wildgans-Lang et al., 2020). These abilities are closely related to *professional vision*, the ability to perceive and interpret students' mathematical thinking keenly (Jacobs et al., 2010; Seidel & Stürmer, 2014), and are viewed as part of teachers' dispositions, situational processes, and diagnostic performance within the competency as a continuum model (Leuders et al., 2018; Wildgans-Lang et al., 2020). From this perspective, accurate judgment regarding students' abilities is not merely about the alignment between a teacher's estimates and test scores, but also about the extent to which a teacher can connect available evidence with specific conceptual constructs and misconceptions that explain students' difficulties (Karing et al., 2024; Kolovou et al., 2021; Zurbruggen et al., 2025).

Various studies indicate that the ability to make accurate judgments is an integral part of PCK, particularly the sub-dimension “knowledge of content and students” (KCS), which requires teachers to understand students’ characteristic ways of thinking, common errors, and patterns of misconceptions in specific mathematical topics (Ball et al., 2008; Heinrichs & Kaiser, 2018; Larrain & Kaiser, 2022). Within this framework, diagnostic competence is viewed as a combination of cognitive resources (CK and PCK), situational processes (perception, interpretation, decision-making), and practices manifested in formative assessment, clinical interviews, and the analysis of student tasks and responses (Clarke et al., 2018; Reinhold, 2018; Wildgans-Lang et al., 2020). Intervention studies also indicate that pre-service teachers’ ability to distinguish between relevant and irrelevant evidence, as well as to differentiate between procedural and conceptual aspects, can be enhanced through structured experiences analyzing transcripts or instructional videos and through explicit frameworks for interpreting evidence of student understanding (Kolovou et al., 2021; Phelps-Gregory & Spitzer, 2018; Santagata & Yeh, 2016). Thus, accurate definition can be positioned as central to diagnostic skills: it bridges the gap between pre-service teachers’ professional knowledge and adaptive pedagogical actions that address students’ learning needs within specific mathematical content.

Initial PCK in pre-service mathematics teachers can be understood as a specific dimension of PCK that focuses on knowledge of how students think, typical difficulties, and appropriate teaching strategies for specific mathematical topics, including the ability to diagnose tasks, student responses, and learning contexts before they have full teaching experience (Copur-Gencturk & Li, 2023; Han-Tosunoglu & Lederman, 2021; Leuders et al., 2018). This initial sub-PCK is not separate from content

mastery, but emphasizes how pre-service teachers integrate subject matter knowledge, curriculum, student characteristics, as well as representations and learning strategies into pedagogical decisions, thereby serving as a “bridge” between the content knowledge acquired in college and adaptive teaching practices in the specific context of a mathematics classroom (Ball et al., 2008; Han-Tosunoglu & Lederman, 2021).

In Indonesia, disparities between education in western and eastern Indonesia pose a major challenge for prospective teachers adapting to diverse learning contexts. In Eastern Indonesia, prospective mathematics teachers must contend with varied school conditions, differing levels of student academic readiness, limited learning resources, and sociocultural contexts that demand a more sensitive pedagogical approach. Therefore, teachers cannot generalize how to define pedagogical problems. Prospective teachers must identify mathematics learning problems specific to their students’ context, for example, whether difficulties stem from weak foundational concepts, language barriers in word problems, overreliance on procedural memorization, or emotional experiences that prevent students from activating the knowledge they already possess.

In the context of pre-service mathematics teachers in Eastern Indonesia, early PCK serves as a diagnostic readiness indicator to identify and define pedagogical problems that arise in mathematics learning, for example, when they must assess the difficulty of tasks, predict student strategies and errors, and weigh information from various task features or classroom situations under time constraints (Copur-Gencturk & Tolar, 2022; Deng et al., 2025). This initial PCK develops through authentic learning experiences such as designing learning programs, analyzing instructional videos, or developing topic-specific PCK instruments that compel pre-service teachers to connect theory with practice and

reflect on their beliefs and knowledge regarding how students learn mathematics, thereby allowing the ability *to define*, as part of diagnostic skills, to be mapped and systematically addressed before they fully enter the teaching profession (Han-Tosunoglu & Lederman, 2021; Phelps-Gregory & Spitzer, 2018).

Substantial disparities in the quality of mathematics education persist in Indonesia, particularly between the Western and Eastern regions. Although access to education has improved, it has not led to equitable quality across learning experiences, instructional practices, or teachers' competence in addressing students' diverse needs. Nationally, low mathematics achievement among Indonesian students remains a significant concern, often attributed to inadequate classroom instruction (Novita, 2022). International assessments such as PISA and TIMSS consistently show that Indonesian students perform below the international average, exhibiting weak problem-solving abilities and limited higher-order reasoning (Darmawijoyo et al., 2025; Nusantara et al., 2025; Wijaya et al., 2024). These disparities are evident both in international comparisons and within Indonesia, with marked differences in educational quality and mathematics achievement between Western and Eastern regions.

This challenge is particularly acute in Eastern Indonesia, where pre-service mathematics teachers face diverse school environments, varying levels of student academic readiness, limited resources, and distinct socio-cultural contexts that require a nuanced pedagogical approach. Consequently, identifying pedagogical problems must be context-specific rather than generic. Pre-service teachers must recognize mathematics learning problems unique to their students' contexts, such as difficulties arising from weak foundational concepts, language barriers in word problems, overreliance on procedural memorization, or affective factors that impede knowledge activation. This

competency corresponds to what Heinrichs & Kaiser (2018) define as the ability to formulate hypotheses about the causes of student errors, specifically "the ability to generate various hypotheses regarding the causes of a specific error, and particularly the ability to identify the specific cause of that error, rather than merely general reasons why the error occurred." In this study, this is termed "defining ability."

In Eastern Indonesia, pre-service mathematics teachers encounter school contexts marked by uneven student readiness, limited access to instructional resources, and linguistic and sociocultural diversity. These conditions are not merely background variables; they shape the classroom evidence teachers must interpret to identify learning difficulties. For instance, students' struggles with contextual word problems may reflect fragile mathematical concepts. Still, they may also reflect the linguistic demands of the instructional language, limited exposure to mathematical discourse, or classroom routines that privilege procedural imitation over explanation. Likewise, limited access to learning materials, technology, and teacher support can restrict opportunities for multiple representations, formative assessment, and feedback. Thus, diagnostic competence in this setting requires pre-service teachers to distinguish between observable errors, plausible cognitive mechanisms, and contextual conditions that influence students' engagement with mathematical tasks (Larrain & Kaiser, 2022; Marmoah et al., 2021; World Bank, 2020).

This orientation is particularly relevant in frontier, outermost, and least-developed regions, where shortages of learning materials, infrastructure, and professional support may constrain instructional practices. However, such conditions should not be treated as deterministic explanations for students' difficulties. Instead, examine them as contextual conditions that may interact with students' prior knowledge, language use, attendance, learning opportunities, and

affective experiences. Accordingly, this study examines whether pre-service teachers use student work, diagnostic assessments, interviews, teacher accounts, and classroom conditions to formulate evidence-based, context-sensitive pedagogical problems. A context-sensitive diagnosis asks not only what students find difficult, but also how the available learning opportunities and school conditions may interact with conceptual, procedural, affective, and metacognitive dimensions of mathematical learning (Heinrichs & Kaiser, 2018; Marmoah et al., 2021; World Bank, 2020).

Accordingly, this study addresses the following research questions: How do pre-service mathematics teachers in Eastern Indonesia define pedagogical problems in mathematics learning,

particularly regarding the “define” aspect of diagnostic skills? How do these abilities vary among groups with low, moderate, and high initial PCK? Specifically, the study aims to (1) describe the quality of pre-service mathematics teachers’ “define” skills in identifying pedagogical problems and the differences in depth based on initial PCK categories, and (2) formulate implications for the development of mathematics teacher education that is more responsive to the specific context of Eastern Indonesia, as a representation of schools with many constraints.

METHOD

Research Design

This research employed a qualitative study to gain an in-depth understanding of how pre-

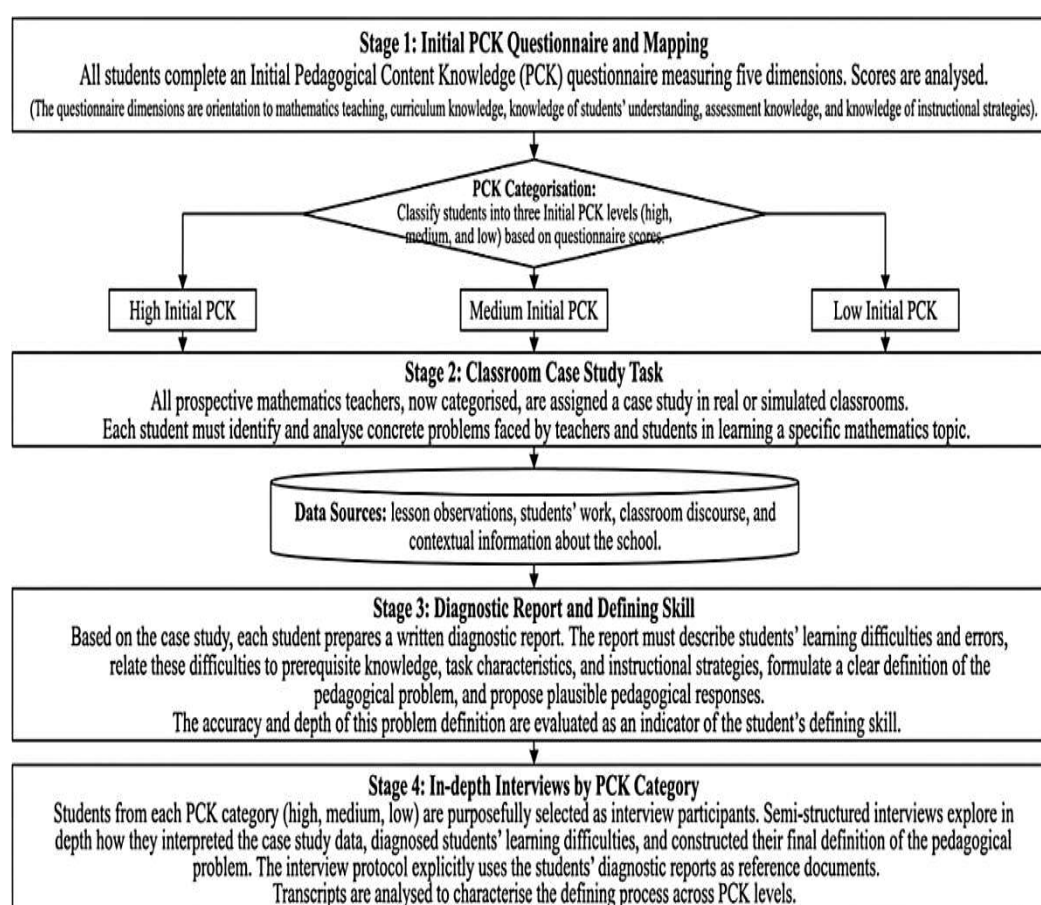


Figure 1. Framework for data collection and analysis methods

service mathematics teachers define pedagogical problems using authentic learning data. The primary bounded system is the Mathematics Education Program at Catholic University of Indonesia Sin Paulus Ruteng, a private institution in East Nusa Tenggara Province, Indonesia. The units of analysis are distinct subgroups of pre-service teachers, categorized by their initial Pedagogical Content Knowledge (PCK) levels: low, moderate, and high. This study enables both a holistic examination of the program's context and a detailed cross-unit analysis of how different initial PCK levels influence the depth of pedagogical diagnosis pre-service teachers formulate when confronting real-world mathematical learning difficulties.

Participants and Sampling

The study began with 44 sixth-semester pre-service mathematics teachers enrolled in the Mathematics Education Program at Catholic University of Indonesia Saint Paul Ruteng, East Nusa Tenggara, Indonesia. All completed the initial PCK questionnaire. For the qualitative phase, six information-rich participants were selected purposively through criterion-based and maximum-variation sampling (Patton, 2015): two participants represented each low, moderate, and high initial-PCK category. Selection also considered the availability of complete post-case-study reports and willingness to participate in a follow-up interview. We used initial PCK as a sampling and interpretive lens, not as an independent variable from which we could claim causal effects on diagnostic performance.

Initial PCK instrument quality. The screening questionnaire initially included 35 items across five PCK dimensions: orientation toward mathematics learning, curriculum knowledge,

knowledge of students' understanding of specific content, assessment, and instructional strategies. We examined empirical internal validity using corrected item-total correlations. Thirty-one items satisfied the retention criterion ($r_{e''} \geq .2564$; $p < .05$). Four items were excluded because they did not meet the criterion: item 1 ($r = .240$, $p = .067$), item 7 ($r = .110$, $p = .406$), item 19 ($r = .191$, $p = .147$), and item 32 ($r = .022$, $p = .867$). Cronbach's alpha was .897 for the 35-item version and increased to .904 after removing the four invalid items, indicating very good internal consistency. The valid 31-item version was used to sample the 44 respondents purposively. This evidence supports the instrument's internal structure and reliability in this screening context; it does not establish the questionnaire as a diagnostic performance measure.

Initial PCK levels were classified using a group-norm approach based on the total scores of 44 prospective mathematics teachers on the 31-item Initial-PCK instrument. Total scores were calculated by summing all item responses, and the resulting distribution had a mean of 105.07 and a sample standard deviation of 11.45. Participants were categorized as having low PCK when their score was below $M - SD$ ($d'' 93$), moderate PCK when their score fell between $M - SD$ and $M + SD$ (94–116), and high PCK when their score was at least $M + SD$ ($e'' 117$). As shown in Table 1, four participants (9.09%) were classified as low, 35 (79.55%) as moderate, and five (11.36%) as high. We then purposively selected six case-study participants, with two from each category; we prioritized representative score positions and varied response patterns, while excluding response profiles that suggested possible straight-lining and required verification.

Table 1. Distribution of initial PCK scores and group-norm categories (N=44)

Initial-PCK category	Group-norm criterion	Operational score interval	Frequency (n)	Percentage (%)
Low	$X < M - SD$	≤ 93	4	9.09
Moderate	$M - SD \leq X < M + SD$	94 – 116	35	79.55
High	$X \geq M + SD$	≥ 117	5	11.36
Total	—	—	44	100.00

Control of topic-related variation. The six participants analyzed different mathematical topics: circle (L1), relations and functions (L2), systems of linear equations in two variables (M1), quadratic equations (M2), integers (H1), and inverse functions, composition, and transformations (H2). Because topics differ in conceptual density, representational demands, and prerequisite structure, the analysis treated topic as a case-level contextual condition. The analysis did not compare raw frequencies of identified difficulties across topics or attribute

differences solely to PCK. Instead, we first analyzed each report within topic using common indicators of problem-definition quality, specificity, evidential grounding, symptom–cause distinction, use of multiple sources, and cognitive, affective, and metacognitive scope. The resulting within-case profiles were then compared across PCK categories through a constant-comparative matrix. Claims therefore focus on patterned associations between initial-PCK category and the depth of defining, while acknowledging that material complexity may contribute to individual variation.

Table 2. Research participants

No	Participant Category	Material Examined by Respondents	Participants Code
1	Low Initial PCK	Cycle	L1
	Low Initial PCK	Relationship and function	L2
2	Initial PCK Moderate	systems of linear equations in two variables (SPLDV)	M1
	Initial PCK Moderate	quadratic equations	M2
3	High Initial PCK	Integers	H1
	High Initial PCK	Inverse functions, composition, transformations	H2

To ensure traceability of quantitative data across the case studies, Table 3 summarizes each partner school's context, the observed class, student enrolment, the number of participants for each instrument, and the diagnostic assessment format. Prospective mathematics teachers conducted the six cases across junior and senior

high schools and addressed diverse mathematical topics. We calculated reported percentages using the number of students who completed the relevant instrument; where multiple instruments were used, we reported the corresponding sample size (N) separately for each data source.

Table 3. Context of partner schools, observed classes, participants, and diagnostic assessment formats

No.	Code	Partner School	Educational Level And Observed Class	Class Enrolment	Diagnostic Assessment Format and Data Sources
1	H1	SMPA	Junior secondary school, Grade VII	15	Written student test on signed-integer operations, contextual word problems, and multiplication and division of integers, with written solution procedures; supplemented by a semi-structured interview with one mathematics teacher.

2	L1	SMPB	Junior secondary school, Grade VIII	28 students	Interviews with three Grade VIII students and one mathematics teacher, complemented by a diagnostic test of conceptual understanding, computation, reasoning, and higher-order thinking skills (HOTS) related to circles.
3	L2	SMPC	Junior secondary school, Grade VIII	25 students	Mathematics teacher interview and student achievement test comprising multiple-choice and constructed-response items. The assessment examined function values, substitution, integer operations, the distinction between relations and functions, and conceptual explanations involving domain and codomain.
4	M1	SMPD	Junior secondary school, Grade IX	23 students	Six-item written diagnostic test and interviews with one mathematics teacher and three students. The test assessed substitution, graphical representation and intersection points, elimination, correction of mathematical models, contextual modeling, and verification of ordered-pair solutions.
5	H2	SMA1	Senior secondary school, Grade XII-C	36 students	Triangulated data collection comprising a written diagnostic test, a Google Forms questionnaire, and in-depth interviews. The test included seven multiple-choice items at Bloom's levels C1–C5 and three constructed-response items at levels C5–C6.
6	M2	SMA2	Senior secondary school, Grade XII	36 students	Mathematics teacher interview, five diagnostic exercises, and student reflection questions. The assessment covered roots of quadratic equations, discriminants, coefficients or parameters, products of roots, factorization, and contextual modeling problems.

Data Collection

The research data sources included: (1) student case study reports; (2) results of the initial PCK questionnaire; and (3) transcripts of in-depth interviews. Data collection involved document analysis and semi-structured interviews. Document analysis focused on the quality of students' problem-defining skills, particularly the clarity of problem formulation, the specificity of context, the use of evidence, the integration of

multiple sources, the differentiation between symptoms and root causes, the accuracy of interpretation, the coherence of argumentation, and the ability to refrain from jumping to solutions. Semi-structured interviews explored students' reasoning in selecting evidence, interpreting data, and formulating research questions, as documented in the accompanying documents and reflective conversations for each PCK category.

Data Analysis

The analysis combined Miles et al.'s (2018) interactive model of data condensation, data display, and conclusion drawing/verification with Braun & Clarke's (2006) reflexive thematic analysis procedures. First, we repeatedly read reports and verbatim interview transcripts and wrote analytic memos to identify segments relevant to the two research questions. Second, we condensed the data by selecting meaning units related to the definition of mathematical-learning problems, and we assigned initial descriptive and interpretive codes (e.g., symptom label, evidence source, prerequisite linkage, misconception explanation, affective factor, metacognitive monitoring, and proposed intervention). Third, the researchers clustered codes into candidate themes representing the depth and dimensionality of problem definition. Fourth, the researchers reviewed themes against coded extracts and the full dataset; they examined discrepant cases and refined code/theme boundaries. Fifth, a cross-case matrix displayed each participant's topic, evidence base, symptom–cause reasoning, and cognitive–affective–metacognitive coverage. Sixth, the researchers compared the resulting within-case profiles across PCK categories and concluded only after returning to raw documents and interview evidence for verification. The process was recursive rather than strictly linear, and the thematic map in Figure 1 presents the final analytic structure.

Trustworthiness and Ethical Considerations.

Credibility was strengthened through triangulation of reports, questionnaire-based PCK classification, and interviews; peer debriefing of codes and theme definitions; limited member checking in which participants reviewed summary interpretations of their interview accounts; and deliberate examination of disconfirming evidence. An audit trail of interview protocols, de-identified transcripts, coding notes, analytic memos, and

successive versions of the cross-case matrix supported dependability and confirmability. Transferability is addressed through thick description of the program context, participant selection, topics, and analytic boundaries. Before participation, all participants received written and oral information about the study's purpose, procedures, voluntary nature, confidentiality safeguards, and their right to withdraw without academic consequences. We obtained written informed consent before access to case-study reports and before interviews; we also requested separate permission for audio recording. We used pseudonyms/codes (L1–H2) throughout, and removed identifying information from transcripts and reports. The study followed the host institution's applicable research-ethics procedures and the ethical principles of voluntary participation, confidentiality, and risk minimization.

■ RESULT AND DISCUSSION

The two research questions organize the results. Research Question 1 examines how pre-service mathematics teachers define pedagogical problems within post-case-study evidence. Research Question 2 then compares the depth, evidential basis, and dimensional scope of these definitions among participants with low, moderate, and high initial PCK. This organization separates the empirical findings from their theoretical interpretation and prevents readers from interpreting the category comparison as a causal test of PCK.

Research Question 1: How Do Pre-Service Mathematics Teachers Define Pedagogical Problems in Mathematics Learning, Particularly Regarding The “Define” Aspect of Diagnostic Skills?

Document analysis indicates that all student groups could identify common patterns in students' mathematics learning difficulties. Four patterns emerged consistently: conceptual

difficulties, procedural difficulties, weak mastery of prerequisite material, and difficulties with modeling or contextual problems. At this level, differences between groups were less about whether they detected problems and more about how deeply they formulated them into operational definitions.

Across the six case-study reports and follow-up interviews, problem definition occurred through three linked moves. First, participants identified an observable learning difficulty, such as an error in a concept, procedure, prerequisite skill, representation, or mathematical modeling task. Second, they selected evidence from student work, test results, classroom observation, teacher accounts, and/or student interviews. Third, they stated the problem's likely pedagogical significance. All participants accomplished the first move; the latter two moves varied substantially in specificity and explanatory depth. Thus, the common empirical pattern was not merely detecting a difficulty, but translating heterogeneous classroom evidence into a bounded, evidence-supported problem formulation.

Table 2 visually represents thematic comparisons and coding density in the qualitative analysis across the six included respondents (N = 6). This matrix heat map structurally represents qualitative patterns; it is not a statistical model or a causal ranking. Each subject had varying mathematical demands depending on the topics within the various PCK categories; however, this

analysis focuses on how participants defined a problem based on available evidence, rather than on which topics caused more difficulty. This table is organized along two main axes: first, the quality of problem definitions, both common and category-specific; and second, cross-category variations in diagnostic depth and their implications for diagnostic competence.

Based on four explicit analytical indicators: (a) the specificity of the problem statement, (b) the type and integration of evidence, (c) the distinction between symptoms and plausible underlying mechanisms, and (d) the extent to which cognitive, affective, and metacognitive dimensions are considered, the coding distribution reveals a clear qualitative continuum, consistent with participants' initial levels of PCK. Participants with low PCK (L1, L2) showed high coding density concentrated on vague, superficial symptom descriptions and evidence from a single source. Participants with moderate PCK (M1, M2) shifted toward specific, evidence-based error mapping, although their diagnostic focus remained highly limited to the cognitive dimension. In contrast, participants with high PCK (H1, H2) produced the highest overall coding density, characterized by dense triangulation of data from multiple sources, explicit identification of causal mechanisms, and comprehensive coverage across all cognitive, affective (math anxiety), and metacognitive (illusion of competence) dimensions.

Table 4. Matrix coding heatmap: density of coding references across PCK Levels (N = 6)

Analytic Indicator	Respondents					
	Low Initial PCK		Moderate Initial PCK		High Initial PCK	
	L1	L2	M1	M2	H1	H2
1. Specificity of Problem Statement						
1.1 General / Vague Problem Framing	9	8	3	2	0	0
1.2 Specific & Contextual Problem Statement	1	2	8	9	12	11
2. Kind and Integration of Evidence						
2.1 Single-Source / Surface Evidence	8	7	2	3	0	1

2.2 Multi-Source Triangulation (Test, Interview, Survey)	1	0	5	4	10	11
3. Distinction Between Symptoms and Underlying Mechanisms						
3.1 Symptom-Level / Surface Description	10	9	4	3	1	1
3.2 Identification of Underlying Causal Mechanisms	0	1	6	5	12	13
4. Coverage of Cognitive, Affective, and Metacognitive Dimensions						
4.1 Cognitive Dimension (Conceptual & Procedural)	7	6	11	10	9	8
4.2 Affective Dimension (Math Anxiety & Panic)	0	1	1	2	8	9
4.3 Metacognitive Dimension (Illusion of Competence & Monitoring)	0	0	1	1	10	9
Total Coding Density per Participant	36	34	41	39	62	63

Research Question 2: How Do Variations In Defining Ability Appear Among Groups With Low, Moderate, And High Initial PCK?

The cross-category analysis used the same analytic indicators for every participant: (a) the specificity of the problem statement; (b) the kind and integration of evidence; (c) the distinction between symptoms and plausible underlying mechanisms; and (d) the extent to which cognitive, affective, and metacognitive dimensions were considered. This comparison aimed to identify patterned variation in the quality of defining. Because each participant analyzed a different mathematical topic, the analysis does not compare the absolute frequency of difficulties across categories or claim that initial PCK alone caused the observed differences.

Low initial PCK: Describing observable symptoms

In the subjects with low initial PCK, definitions generally remain descriptive of symptoms. Students show confusion about concepts such as circles, relations, or functions, but their formulations typically stop at general conclusions, such as “students’ conceptual understanding is still low.”

Subjects in this category also demonstrate definitions that remain predominantly descriptive.

In this category, students can generally identify learning difficulties and include evidence from test results and interviews. However, the constructed definitions still describe conditions of difficulty rather than offer a deep pedagogical diagnosis. The definition reflects a limited understanding of the concept of a circle.

“Students are reported to tend to memorize the formulas for circumference and area without understanding the meaning of the concepts, have difficulty distinguishing between radii and diameters, and experience confusion with word problems. This group also presented data indicating that ‘the average student achievement falls within the 45–55% range; only about 25% of students succeeded on higher-order reasoning questions; 68% of students misunderstood the diameter; 72% did not use the constant δ ; and 85% had not yet grasped the relationship between diameter and area’ (Report Document L1).

Referring to this report, the researcher probed the students by quoting from it. The researcher asked, *“Interesting. In your report, you wrote that 72% of students did not use the constant δ and tended to memorize*

formulas,” then asked, “In your opinion, are these students ‘empty-headed’ and know nothing, or do they know but have the wrong concept (misconception)?”

This question was answered:

“It seems more like they have not fully grasped the concept. They know the formulas for area and perimeter by heart, but when given a word problem with a different format, they immediately draw a blank. So, it boils down to just memorization.” (L1)

Next, the researcher posed an analogy: the students were imagined as teachers about to teach in that class (the one being analyzed) and facing a situation where almost all the students were confused by the material or the process. “What would be the first thing they do in the classroom?” Among the respondents, one answered:

Perhaps the first step would be to re-explain the basic concepts from the beginning slowly. Then we would start with easy example problems first, before gradually moving on to word problems.” (L1)

They also demonstrated a pattern similar to that for functional material. They identified that:

“Students have difficulty determining function values and explaining the definition of a function conceptually, and are often confused about distinguishing between relations and functions.” (Report Document L2)

The interviews confirmed this pattern. When asked to elaborate on why students memorize formulas, the students replied,

“What we can conclude from the collected data is that students do indeed struggle to understand the concepts and apply formulas to problems” (L2) and acknowledged that:

“Written reports do tend to describe the nature of the difficulties more than the deeper reasons behind them.” (L2)

This quote indicates that, in the low category, the definition remains at the surface-level identification stage and has not yet reached pedagogical diagnosis. This excerpt of the response also shows that pre-service mathematics teachers with a low initial PCK category exhibit a diagnostic thinking pattern that is still superficial, because although they are quite skilled at identifying students’ procedural and conceptual errors, they tend to describe learning difficulties with general labels such as “confused,” “does not understand,” or “weak” without exploring the underlying cognitive mechanisms, viewing rote memorization as an innate student trait rather than a consequence of teacher-centered learning culture, and consistently responding to widespread comprehension failures with the repetitive solution of “explaining it again more slowly” without utilizing alternative pedagogical strategies, so that the reflective interviews they participated in actually served as an initial intervention that fostered cognitive dissonance and a gradual shift from merely describing symptoms toward formulating pedagogical problems more explicitly.

However, the analysis of the final subject (functional material) primarily concluded that students’ conceptual understanding remains low, without further elaboration on how these difficulties arise or persist in the learning process. Thus, the analysis in the low PCK category still shows an initial ability to identify problems but not yet the capacity to develop them into a sharp, multi-layered pedagogical diagnosis.

Moderate initial PCK: Mapping evidence and prerequisite weaknesses

In the group with moderate initial PCK, the analysis shifted toward more systematic, data-

driven problem mapping. Students linked students' low scores to more specific error types, such as weaknesses in integer operations, coefficient manipulation, substitution, elimination, graphical representation, and contextual modeling. Interviews indicated that they began to reason about the relationship between the target material and the prerequisite material. In a case study report examining the material on systems of two-variable linear equations, it is written:

"Students' learning difficulties with this material are primarily caused by a weak grasp of basic concepts and prerequisite skills, which subsequently result in students' inability to perform solution procedures, represent problems in graphical or mathematical model form, and solve contextual problems." (Report Document M1)

Although the definitions in this category remain predominantly cognitive and procedural and do not yet fully incorporate students' affective or metacognitive dimensions, they are sufficiently strong and systematic. In this group, the definitions combine diagnostic test results with interviews with teachers and students. Their strength lies in mapping types of learning difficulties in greater detail, supported by relatively clear empirical data.

Regarding the SPLDV material, the definitions identify students' primary difficulties as weaknesses in integer operations, coefficient manipulation, substitution, elimination, graphical representation, and contextual modeling. This group reported the following case study results:

"100% of students have not achieved mastery, 91.30% experience difficulties with the substitution method, 95.65% with graphical representation, 86.96% with elimination, and 78.26% with modeling contextual problems." The teacher in the document emphasized that

"the main difficulties lie in calculating coefficients for elimination and integer operations such as ' ' ' and that the impact of basic algebraic skills on SPLDV learning, when rated on a scale of 1–10, was a '10.'" (Report document).

The percentage of 91.30% reported for Systems of Linear Equations in Two Variables (SPLDV) refers to 21 of 23 Grade IX students at SMPD who completed the written diagnostic test. The assessment consisted of six constructed-response items covering solution procedures using substitution and elimination, graphical representation, contextual mathematical modeling, and verification of ordered-pair solutions. Students' responses to the item requiring them to determine x and y through substitution indicated difficulty with the substitution method. Students were classified as experiencing difficulty when they could not formulate an appropriate substitution, made algebraic manipulation errors, or failed to obtain the correct solution.

Rather than treating the percentages as a descriptive list of students' failures, the pre-service teachers used them to identify a coherent diagnostic pattern across tasks. The high prevalence of difficulties in substitution, elimination, and graphical representation, together with interview evidence, indicated a more fundamental weakness in integer operations, coefficient manipulation, and understanding of algebraic symbols. Thus, the percentages served as screening evidence to prioritize the diagnostic focus: students' difficulties with SPLDV were not isolated procedural errors, but interconnected consequences of fragile prerequisite algebraic knowledge that constrained procedural execution, representation, and mathematical modeling.

In follow-up questions probing the reasons for their answers, respondents stated:

"We see that the low scores are merely the result. What is more important is the

nature of the difficulties, such as students being confused about moving terms, incorrectly multiplying negative signs, or not knowing when to use substitution and when to use elimination.” (M1)

Next, the researcher asked them to rephrase or re-explain the students’ mistakes or difficulties. What would you write? One respondent answered:

“We would write that students’ learning difficulties with this material are primarily caused by a weak grasp of basic concepts and prerequisite skills, which then results in students’ inability to follow problem-solving procedures, represent problems in the form of graphs or mathematical models, and solve contextual problems.” (M1)

Quadratic equations also demonstrated a fairly systematic analytical pattern. They found that students’ main difficulties with quadratic equations lie in factoring, the discriminant, basic arithmetic, and application problems. Their data showed this.

“72.22% of students struggled with factoring, while 80% of students were unable to analyze and solve application problems. Teachers in the report stated that students “are more comfortable with mechanical procedures than with analytical processes such as factoring” and that “the root of the problem is a weak grasp of basic mathematical operations.” (Report Document M2)

The 72.22% reported for quadratic equations refers to 26 of 36 Grade XII students at SMA2 who participated in the diagnostic assessment. The assessment comprised five practice-based diagnostic tasks and reflection questions covering determining roots, using the discriminant, determining parameter values, the product of roots, and mathematically modeling a contextual problem involving consecutive odd

integers. The researchers identified difficulties with factorization when students could not determine appropriate factor pairs to find the roots of a quadratic equation, particularly when the coefficients or signs required more advanced analysis. The reasons behind this report were explored by asking:

“In your opinion, why have these foundational weaknesses persisted into the current material? Is it because students have forgotten, lack practice, or is there a learning approach that makes their understanding fragile?”

Respondents answered: *“... It is likely due to a combination of several factors. It could be because students have not fully mastered the foundational material, and previous practice was limited to routine problems. When new material is introduced, students immediately face more complex procedures. So the old weaknesses carry over.” (M2)*

The researcher asked the participants to imagine they were the teachers in that class the next day and to consider what they would do. One respondent replied:

“We would start by reassessing the students’ foundational skills, then reinforce the concepts and operations that serve as prerequisites. After that, we would provide gradual practice, starting with simple problems and progressing to contextual problems, so that students are not immediately overwhelmed.” Another respondent added, “...Yes, we see that instructional interventions must be tailored to the nature of the difficulty; it is not enough to simply repeat the same material.” (M2)

Although quite robust, the “define” phase in the moderate PCK category still shows limitations compared to the high PCK category. Their “define” phase has mapped students’

difficulties specifically and in a data-driven way. However, it has not yet fully developed into an analysis that encompasses affective, metacognitive, or broader interconceptual dimensions. Students with moderate-level PCK demonstrate a more systematic process of defining problems than the low-PCK group, as they can formulate pedagogical problems by referring to error patterns, test results, and confirmation from classroom teachers. However, the limits of their analytical abilities remain evident: they still interpret learning difficulties primarily as issues of prerequisites, conceptual understanding, and procedures, while aspects such as students' self-monitoring, anxiety, self-confidence, and the learning experiences that perpetuate these difficulties remain underexplored. Thus, their position can be likened to that of a laboratory analyst who carefully reads numbers but has not yet fully assumed the role of a "Clinical Reasoning in Education" capable of fully linking symptoms, causes, and students' learning experiences within a comprehensive pedagogical diagnostic framework.

High initial PCK: Constructing multidimensional diagnoses

In the high-initial-PCK group, the analysis evolved into a more comprehensive pedagogical diagnosis. Analysis of the report documents shows that this group not only identifies misconceptions regarding composition, inverses, and translations but also links these symptoms to illusions of understanding, math anxiety, and weak self-monitoring among students. Interviews clarify this depth.

Students explained that they often *"feel they understand when the teacher explains, but that is an illusion. As soon as the teacher's scaffolding is removed during an exam, their understanding collapses immediately because their conceptual foundation is fragile."*

In fact, when asked to formulate a more conceptual definition, the informant stated that *"the difficulty in learning mathematics is not merely an inability to answer questions, but a systemic failure involving fragile concepts, a loss of metacognitive control when facing a roadblock, and negative emotional responses to the classroom learning environment"* (H1).

The group representing high PCK demonstrated the most comprehensive definition. This group's main strength lies in its integrated use of multiple data sources: *Google Forms*, face-to-face interviews, and written tests. In their document, they stated that these three data sources were used to strengthen and improve the reliability of the findings, so issues identified across multiple sources were considered genuine problems. Furthermore, the problem definition in this subject is explicitly structured with clear criteria. The subject asserts that a good problem formulation must be "specific, data-driven, verifiable, measurable, and conceptual." This statement indicates that students not only identify students' difficulties but also understand the principles of problem definition formulation as part of pedagogical reasoning.

The strength of the students' problem definition is also evident in the breadth of problem dimensions they successfully identified. They identified misconceptions about the order of function composition, the meaning of inverse functions, and the direction of translation, and also noted students' affective and metacognitive dimensions. One of their key findings is that.

"91.7% of students felt they understood when the teacher explained, but became confused when they had to work on problems on their own. This finding is reinforced by a student's quote: 'When the teacher explains, I feel I understand because the steps are clear. However, when I try to solve a problem on my own,

I suddenly get confused about the first step.” (Report Document H2)

The percentage data reported for the functions case study were obtained from Grade XII-C at SMA1, which comprised 36 students. Data were collected through three complementary instruments: in-depth interviews with the mathematics teacher and three purposively selected students, a Google Forms questionnaire completed by 12 students, and a written diagnostic test administered to 36 students. Specifically, the 91.7% figure refers to 11 of the 12 questionnaire respondents who reported understanding inverse and composite functions. This self-reported result should, however, be interpreted alongside the diagnostic-test and interview findings, which revealed persistent misconceptions concerning the notation $f^{-1}(x)$, the order of function composition $g \circ f(x)$, and the direction of graph translation. Therefore, this percentage does not represent the entire school population; rather, it reflects the responses of the questionnaire participants in Grade XII-C.

Supporting this report, the authors argue that students’ learning difficulties cannot be separated from their emotional experiences. A total of 66.7% of students reported feeling nervous or anxious before a math exam. In their report, they included a quote from one of their student interviews, which contained the student’s response:

“I felt extremely panicked when I was about to start the test. I had studied beforehand, but when I first started working on it, I did not know where to begin.” (H2).

To delve deeper into this observation, the researchers asked, “As pre-service teachers, how do you connect this panic (affective) with your misconceptions regarding the order of function composition and the direction of translation (cognitive)?” The respondents answered

“In our view, this is a recurring cycle, Sir. Because they know their understanding is fragile (relying solely on procedural memorization), anxiety arises. Then, during the exam, this panic overloads their working memory. As a result, distinguishing between positive and negative translation directions or the order of functions, for example, becomes very chaotic. Anxiety blocks access to those already weak concepts.” (H2)

When asked what they would do if they were asked to teach that class and whether they would explain it again, the respondents replied

‘Absolutely not, sir! Explaining it again would only reinforce that illusion of competence. First, we will work on reducing their math anxiety. Perhaps by giving them problems (partially solved examples) and then asking students to complete them and explain their reasoning to their classmates. We want to build a sense of security while also forcing them to monitor their own thought processes, not just copy formulas from the blackboard.’” (H2)

The researcher’s final question: “One last question. Based on your journey in unraveling this issue, how do you now define ‘Mathematics Learning Difficulty’? The respondent answered:

“Mathematics learning difficulties are not merely an inability to answer questions, but rather a systemic failure involving fragile concepts, a loss of metacognitive control when facing a roadblock, and negative emotional responses to the classroom learning environment.” (H2)

Thus, the definition in the high PCK category not only highlights students’ conceptual weaknesses but also explains how these difficulties are experienced and perpetuated in the learning process. The research results show a

clear gradation in the quality of the “define.” The low-initial PCK group tended to write “direct reports” on students’ difficulties; the intermediate group began constructing evidence-based problem maps; and the high group formulated “defines” that were more diagnostic, multidimensional, and closer to the foundation of pedagogical intervention design.

Students in the high initial PCK category demonstrated a critical attitude toward surface-level symptoms; for example, they did not simply accept test scores at face value but spontaneously triangulated them with other data sources, thereby revealing the depth of their pedagogical reasoning. They can also debunk the illusion of competence by recognizing that students often equate following the teacher’s blackboard explanations with genuine conceptual understanding, meaning the diagnostic process they construct extends into students’ metacognitive domain. Furthermore, this group can explain causally how mathematics anxiety disrupts working memory capacity during exams, resulting in a holistic diagnosis that does not separate cognitive aspects from students’ affective dimensions. When asked to formulate interventions, they did not revert to the “re-explaining” solution. However, they proposed more precise strategies, such as worked examples and peer tutoring, specifically aimed at strengthening metacognition and reducing math anxiety.

The high-PCK case further shows that diagnostic reasoning can incorporate contextual resources without treating them as direct causes of learning difficulties. The participants used a Google Forms questionnaire, written diagnostic tasks, and interviews to identify a discrepancy between students’ self-reported understanding and their demonstrated conceptual performance. In this case, the questionnaire provided additional evidence of affective and metacognitive conditions; it did not replace analysis of students’ written work or imply that digital access itself

improved conceptual understanding. By triangulating these sources, the participants identified that students’ apparent confidence in inverse and composite functions coexisted with misconceptions concerning $f^{-1}(x)$, $g \circ f(x)$, and graph translation. This use of multiple sources reflects the principle that diagnostic judgments should be based on converging evidence about students’ reasoning, misconceptions, and learning conditions rather than on a single indicator of performance or self-report (Clarke et al., 2018; Larrain & Kaiser, 2022; Wildgans-Lang et al., 2020).

Together, the two research questions show that “defining” is not equivalent to naming a student difficulty. In this study, it ranged from broad descriptions of visible symptoms to evidence-based, multilayered formulations of pedagogical problems. The low-PCK cases were marked by general labels such as “students do not understand” and by a tendency to move rapidly to re-explanation. The moderate-PCK cases more consistently linked specific errors, test patterns, and prerequisite weaknesses, but their explanations focused primarily on cognitive and procedural factors. The high-PCK cases drew on multiple evidence sources and considered the interplay among conceptual fragility, affective responses, and learners’ self-monitoring. This is an interpretive cross-case pattern, not a hierarchy of personal worth or a causal estimate of PCK effects.

The case reports indicate that diagnostic reasoning became more robust when participants connected evidence of students’ mathematical thinking with the instructional conditions in which that thinking developed. In the SPLDV case, the moderate-PCK participants interpreted students’ errors in substitution, elimination, graphical representation, and contextual modeling alongside the teacher’s account of weak integer operations, limited access to instructional media, and students’ difficulty understanding mathematical

explanations. Rather than treating these errors as isolated procedural failures, they identified an interconnected pattern in which fragile prerequisite knowledge was compounded by limited opportunities for visualization and by the linguistic demands of symbolic and contextual tasks. This interpretation is consistent with research showing that diagnostic competence involves not only noticing student errors but also generating and evaluating plausible hypotheses about their causes before selecting an appropriate pedagogical response (Heinrichs & Kaiser, 2018; Larrain & Kaiser, 2022; Seifried & Wuttke, 2010).

By contrast, the low-PCK circle case identified students' dependence on memorized formulas and their difficulty with word problems, but the explanation remained primarily cognitive and descriptive. The participants did not sufficiently examine how lecture-dominated instruction, limited visual representations, and translating contextual language into mathematical representations might sustain the observed difficulties. This difference suggests that structural constraints become diagnostically meaningful only when pre-service teachers treat them as evidence-based explanatory conditions, rather than as broad descriptions of educational disadvantage. This distinction matters because teachers' diagnostic work requires moving from recognizing an error to interpreting possible causes and developing a response appropriate to both the mathematics content and the learning context (Benecke & Kaiser, 2023; Heinrichs & Kaiser, 2018).

This pattern is consistent with the diagnostic-competence literature, which characterizes diagnosis as a knowledge-based process of noticing relevant evidence, interpreting its meaning, and making a defensible pedagogical judgment. Research on teacher noticing likewise suggests that teachers often attend more readily to visible features of student work than to the

underlying mathematical reasoning or to formulating responsive action. Recent reviews report that teachers notice surface features more reliably than they interpret them, and that a strongly procedural conception of mathematics can narrow what teachers notice (Jacobs et al., 2010; Leuders et al., 2018; Melissa et al., 2026; Wildgans-Lang et al., 2020).

The results also align with evidence that diagnostic competence can be developed through structured, content-specific opportunities to analyze student errors and explain their possible causes. Such learning opportunities are particularly relevant for the low and moderate groups: candidates need support to separate an observed error from an explanatory hypothesis, seek corroborating evidence, and articulate uncertainty instead of treating a single source as definitive. However, the present results do not imply that every diagnosis must include affective and metacognitive dimensions. Rather, consider these dimensions when data support them and when they plausibly help explain students' mathematical activity. A rigorous diagnosis remains anchored in topic-specific evidence while considering the learner and classroom context.

For teacher education, the findings support a sequenced curriculum in which candidates first learn to identify mathematical evidence, then link evidence to conceptual and prerequisite structures, and subsequently practice constructing and challenging multidimensional diagnostic hypotheses. Case reports, annotated student work, short clinical interviews, and video-based discussions can make these stages visible. Repeated practice is essential because diagnostic competence is situated and topic-sensitive; it strengthens when candidates analyze how students reason within specific mathematical domains rather than applying generic labels across topics.

The findings indicate that assessment quality concretely reflects pre-service teachers' fulfillment

of pedagogical competencies. Assessment activities, which begin with diagnostic activities, are an important component of teachers' professional competencies. Diagnostic competencies fall into two areas: skills and knowledge. Teachers need to assess student performance accurately so evaluations are fair and objective and so they can teach adaptively, that is, by basing instruction on students' prior knowledge. For pre-service teachers, the accuracy of justification is defined in many studies as the degree of alignment between a pre-service teacher's assessment and external criteria, such as performance on standardized tests or evaluations by highly trained and experienced assessors known as assessment experts, based on the initial foundation of content and pedagogical understanding framed under a single term: initial pedagogical content knowledge (PCK).

This study shows that the quality of definitions students construct varies by their category of initial Pedagogical Content Knowledge (PCK). These differences are evident in students' accuracy in identifying core learning issues, the breadth of evidence they use to support problem formulations, and the depth of their interpretation of students' learning difficulties. These findings reinforce the view that PCK is not merely knowledge of subject matter and teaching methods, but rather professional knowledge that enables teachers or pre-service teachers to interpret how content is understood, misunderstood, or fails to be understood by students within a specific learning context (Ball et al., 2008; Depaepe et al., 2013; Shulman, 1987). From an initial PCK perspective, the results indicate that students with high initial PCK not only possess more information about students but can also process that information into specific, layered, and pedagogically meaningful problem focuses. Findings in the high PCK group also suggest that a strong definition requires more than

describing symptoms. Students in this category can distinguish general complaints from pedagogical problems that warrant intervention. Thus, their definitions move toward pedagogical diagnosis. More mature PCK is evident when pre-service teachers can interpret how students understand, misunderstand, or fail to understand content, rather than merely noting that students are wrong (Gess-Newsome et al., 2019).

In this study, students in the high PCK category demonstrated sharper, more argumentative defining abilities than those in the moderate and low PCK categories. They did not merely state that students were experiencing learning difficulties but also formulated problems in a more specific, data-driven, and conceptual manner. This pattern aligns with previous research findings indicating that teachers or pre-service teachers with higher PCK tend to be better able to anticipate student misconceptions, identify the sources of learning difficulties, and select representations or strategies that better suit students' learning needs (Hill & Chin, 2018; König et al., 2020; Krauss et al., 2008).

In the low PCK category, the definitions that emerge remain at the level of symptoms. When students say that pupils "memorize more than they understand" but cannot explain why that pattern arises, they are only at the initial noticing stage. In fact, the solutions that emerge tend to be normative, such as "rephrasing," "providing step-by-step examples," or "using media." This pattern indicates that pre-service teachers cannot yet fully connect evidence of student errors to the pedagogical mechanisms that produce them. In teacher education, this condition should be interpreted as a sign that students still need scaffolding to move from symptom detection to problem framing. Students in the low PCK category tend to produce more descriptive definitions. They can identify students' learning difficulties, but their problem formulations generally stop at general conclusions, such as low

conceptual understanding or weak problem-solving skills in word problems. This finding suggests that prior knowledge of learning difficulties does not automatically lead to a strong problem definition. The difference lies in students' ability to elaborate on, structure, and interpret data as operational learning problems. Thus, the results support the view that the quality of PCK is evident not only in what pre-service teachers know, but in how they use that knowledge to interpret learning situations (Demirci, 2010; Gess-Newsome., 2019; Wildgans-Lang et al., 2020).

The intermediate PCK category demonstrates a significant leap. Students have become more systematic and stronger at linking test data to teacher observations. They are beginning to recognize that difficulties with the target material often stem from deficiencies in prerequisite knowledge. However, their definitions remain focused on the structure of observed errors. They have not consistently distinguished whether student failure stems from misconceptions, an inability to self-monitor errors, or anxiety that hinders performance when solving problems. In other words, they can already analyze patterns but have not yet fully become pedagogical diagnosticians who interpret interactions among cognition, affect, and the learning context.

In this study, pre-service teachers with PCK have demonstrated systematic patterns of student difficulties. However, the frameworks they construct still tend to focus on cognitive and procedural aspects. They can identify that students struggle with basic operations, symbolic manipulation, or mathematical representations, but have not consistently linked these findings to affective dimensions, metacognitive aspects, or broader learning dynamics. This pattern aligns with findings from numerous studies showing that pre-service teachers' PCK often develops unevenly: knowledge of student difficulties may emerge first, while the ability to integrate that

knowledge into comprehensive pedagogical analysis develops more slowly (Kind, 2009; Li, 2025; Marshman & Porter, 2006).

Overall, the analysis indicates that the quality of defining skills is directly related to the subjects' initial PCK level. Subjects with high initial PCK can formulate pedagogical problems more specifically, drawing on students' learning symptoms and contextual relevance to mathematics learning. In contrast, those with moderate initial PCK remain at the stage of partial identification, while those with low initial PCK tend to produce general, normative problem definitions that do not address the core pedagogical issues.

These findings have important implications for mathematics teacher education in Eastern Indonesia. In educationally diverse and resource-constrained settings, pre-service teachers need explicit opportunities to learn how contextual conditions shape diagnostic reasoning without reducing them to generic deficit explanations. They should be prepared to examine whether an observed mathematical error is more plausibly associated with conceptual fragility, weak prerequisite knowledge, the language demands of a task, limited access to representations and formative assessment, affective responses, or an interaction among these conditions. Broad claims that students are weak or confused are unlikely to support adaptive instruction. In contrast, evidence-based and context-sensitive problem definitions can guide feasible pedagogical responses, such as adapting the linguistic framing of contextual tasks while maintaining mathematical demand, using concrete and visual representations where digital resources are limited, conducting brief diagnostic interviews, and designing formative tasks that make misconceptions visible (Marmoah et al., 2021; Walter et al., 2025; World Bank, 2020)

Diagnostic activities reflected in the "defining" outcome pose a significant challenge,

and their complexity and diversity are difficult to address fully in pre-service teacher education; thus, teacher education programs must address them continuously. Diagnostic activities that encompass all information regarding students' learning opportunities, status, processes, and outcomes form a crucial foundation for teachers to understand each student's thinking patterns and learning levels (Chambre & Anderson, 2025; Margevièa-Grinberga & Dûmane, 2024; Napanoy et al., 2021). This needs to be recognized and continuously developed, given that the results of this study and several previous studies consistently show that although teachers are often able to identify difficult tasks, they frequently struggle to analyze *why* those tasks have the potential to create learning barriers, identify common misconceptions, describe errors in students' solutions, or trace the root causes of those errors (Blömeke et al., 2020; Brunner et al., 2021; Sidenvall et al., 2026; Walter et al., 2025). Sometimes, they even select remedial tasks that do not align with the diagnosed difficulties. This situation underscores that diagnostic competence (*define*) is not merely a technical ability but a lifelong learning process that requires trained skills to make adequate diagnoses. Appropriate conclusions can be drawn from them.

■ CONCLUSION

This embedded qualitative study found a progressive pattern in how six pre-service mathematics teachers defined pedagogical problems following school-based case studies. Across all cases, participants identified conceptual and procedural difficulties, prerequisite gaps, and challenges in mathematical modeling. The main difference lay in the quality of formulation: low initial-PCK participants mostly labeled observable symptoms; moderate initial-PCK participants used test and interview evidence to map error patterns and prerequisite

weaknesses; and high initial-PCK participants formulated more precise, evidence-based, multidimensional diagnoses that linked cognitive evidence with affective and metacognitive conditions. Initial PCK was therefore useful as a sampling and interpretive lens for describing these patterns, but the study does not establish it as the sole cause of diagnostic depth.

The findings should be interpreted with important limitations in mind. First, the study involved only six focal informants from one mathematics education program; it aimed for analytic depth rather than statistical generalization. Second, participants analyzed different mathematical topics with different conceptual structures and prerequisite demands. Although the analysis controlled for this variation through common quality criteria, within-case analysis, and a cross-case matrix, topic complexity may still have influenced the depth of individual diagnoses. Third, the evidence represents post-case-study reports and reflective interviews, not direct observation of participants' subsequent teaching decisions. Future research should include larger, more diverse samples; comparable cases within the same topic; longitudinal observation during practicum; and explicit measures of how diagnostic formulations inform instructional action.

For mathematics teacher education and curriculum development, specify diagnostic problem definition as an explicit learning outcome rather than assuming it will emerge from content or methods courses. Programs should sequence practice in analyzing student work, conducting short diagnostic interviews, interpreting video and classroom evidence, distinguishing symptoms from explanatory hypotheses, and considering cognitive, affective, and metacognitive dimensions. Curriculum developers should embed topic-specific case banks and common diagnostic rubrics across methods, assessment, and practicum courses, while providing differentiated scaffolding for candidates who remain at a

descriptive level. Such design is especially relevant in educationally diverse and resource-constrained settings, where context-sensitive, evidence-based problem definitions are essential to equitable mathematics instruction.

■ STATEMENT ON THE USE OF GENERATIVE AI IN THE WRITING

During the writing of this manuscript, the authors used Gemini, DeepL, and Grammarly to refine the language and proofread the manuscript. The authors have reviewed and edited the content generated by these tools and take full responsibility for the content of the published article.

■ REFERENCES

- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching. *Journal of Teacher Education*, 59(5), 389–407. <https://doi.org/10.1177/0022487108324554>
- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., & Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 133–180. <https://doi.org/10.3102/0002831209345157>
- Benecke, K., & Kaiser, G. (2023). Teachers' approaches to handling student errors in mathematics classes. *Asian Journal for Mathematics Education*, 2(2), 161–182. <https://doi.org/10.1177/27527263231184642>
- Biza, I., & Nardi, E. (2020). Scripting the experience of mathematics teaching: The value of student teacher participation in identifying and reflecting on critical classroom incidents. *International Journal for Lesson and Learning Studies*, 9(1), 43–56. <https://doi.org/10.1108/IJLLS-02-2019-0017>
- Blömeke, S., Kaiser, G., König, J., & Jentsch, A. (2020). Profiles of mathematics teachers' competence and their relation to instructional quality. *ZDM*, 52(2), 329–342. <https://doi.org/10.1007/s11858-020-01128-y>
- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101. <https://doi.org/10.1191/1478088706qp063oa>
- Brodie, K. (2014). Learning about learner errors in professional learning communities. *Educational Studies in Mathematics*, 85(2), 221–239. <https://doi.org/10.1007/s10649-013-9507-1>
- Brodie, K., Marchant, J., Molefe, N., & Chimhande, T. (2018). Developing Diagnostic Competence Through Professional Learning Communities. In *Diagnostic Competence of Mathematics Teachers* (pp. 151–171). Springer International Publishing. https://doi.org/10.1007/978-3-319-66327-2_8
- Brunner, K., Obersteiner, A., & Leuders, T. (2021). How prospective teachers detect potential difficulties in mathematical tasks – an eye tracking study. *RISTAL (Research in Subject-Matter Teaching and Learning)*, 109–126. <https://doi.org/10.23770/RT1845>
- Chambre, S. J., & Anderson, A. (2024). Identifying sources of preservice teachers' dyslexia knowledge to guide teacher education. *Frontiers in Education*, 9. <https://doi.org/10.3389/feduc.2024.1468923>
- Clarke, D. M., Roche, A., & Clarke, B. (2018). Supporting Mathematics Teachers' Diagnostic Competence Through the Use of One-to-One, Task-Based Assessment Interviews. In *Diagnostic Competence of Mathematics Teachers* (pp. 173–192). Springer International Publishing. https://doi.org/10.1007/978-3-319-66327-2_8

- doi.org/10.1007/978-3-319-66327-2_9
- Copur-Gencturk, Y., & Li, J. (2023). Teaching matters: A longitudinal study of mathematics teachers' knowledge growth. *Teaching and Teacher Education*, 121, 103949. <https://doi.org/10.1016/j.tate.2022.103949>
- Copur-Gencturk, Y., & Tolar, T. (2022). Mathematics teaching expertise: A study of the dimensionality of content knowledge, pedagogical content knowledge, and content-specific noticing skills. *Teaching and Teacher Education*, 114, 103696. <https://doi.org/10.1016/j.tate.2022.103696>
- Csanadi, A., Kollar, I., & Fischer, F. (2021). Pre-service teachers' evidence-based reasoning during pedagogical problem-solving: better together? *European Journal of Psychology of Education*, 36(1), 147–168. <https://doi.org/10.1007/s10212-020-00467-4>
- Darmawijoyo, D., Zulkardi, Z., Putri, R. I. I., Hapizah, H., & Syutaridho, S. (2025). How do students use mathematical reasoning to solve PISA-type mathematics problems based on making kite contexts? *Infinity Journal*, 14(4), 1065–1080. <https://doi.org/10.22460/infinity.v14i4.p1065-1080>
- Demirci, C. (2010). Constructing a philosophy: Prospective teachers' opinions about constructivism. *Procedia - Social and Behavioral Sciences*, 9, 278–285. <https://doi.org/10.1016/j.sbspro.2010.12.150>
- Deng, F., Zhang, L., Zhu, J., Dou, B., & Lan, W. (2025). Chemistry teachers' pedagogical content knowledge: A review of empirical research published in SSCI Journals from 1986 to 2024. *Teaching and Teacher Education*, 165, 105109. <https://doi.org/10.1016/j.tate.2025.105109>
- Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education*, 34, 12–25. <https://doi.org/10.1016/j.tate.2013.03.001>
- Gess-Newsome, J., Taylor, J. A., Carlson, J., Gardner, A. L., Wilson, C. D., & Stuhlsatz, M. A. M. (2019). Teacher pedagogical content knowledge, practice, and student achievement. *International Journal of Science Education*, 41(7), 944–963. <https://doi.org/10.1080/09500693.2016.1265158>
- Hammer, S., & Ufer, S. (2023). Professional competence of mathematics teachers in dealing with tasks in lesson planning. *Teaching and Teacher Education*, 132. <https://doi.org/10.1016/j.tate.2023.104246>
- Han-Tosunoglu, C., & Lederman, N. G. (2021). Developing an instrument to assess pedagogical content knowledge for biological socioscientific issues. *Teaching and Teacher Education*, 97. <https://doi.org/10.1016/j.tate.2020.103217>
- Heinrichs, H., & Kaiser, G. (2018). Diagnostic Competence for Dealing with Students' Errors: Fostering Diagnostic Competence in Error Situations. In *Diagnostic Competence of Mathematics Teachers* (pp. 79–94). Springer International Publishing. https://doi.org/10.1007/978-3-319-66327-2_4
- Hill, H. C., & Chin, M. (2018). Connections Between Teachers' Knowledge of Students, Instruction, and Achievement Outcomes. *American Educational Research Journal*, 55(5), 1076–1112. <https://doi.org/10.3102/0002831218769614>
- Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional Noticing of Children's Mathematical Thinking. *Journal*

- for *Research in Mathematics Education*, 41(2), 169–202. <https://doi.org/10.5951/jresmetheduc.41.2.0169>
- Karing, C., Rausch, T., & Artelt, C. (2024). Teacher Judgement Accuracy—Measurements, Causes and Effects. In S. Weinert et al. (Ed.), *Educational Processes, Decisions, and the Development of Competencies from Early Preschool Age to Adolescence* (pp. 263–280). https://doi.org/10.1007/978-3-658-43414-4_10
- Kind, V. (2009). Pedagogical content knowledge in science education: Perspectives and potential for progress. *Studies in Science Education*, 45(2), 169–204. <https://doi.org/10.1080/03057260903142285>
- Kolovou, D., Naumann, A., Hochweber, J., & Praetorius, A.-K. (2021). Content-specificity of teachers' judgment accuracy regarding students' academic achievement. *Teaching and Teacher Education*, 100, 103298. <https://doi.org/10.1016/j.tate.2021.103298>
- König, J., Bremerich-Vos, A., Buchholtz, C., & Glutsch, N. (2020). General pedagogical knowledge, pedagogical adaptivity in written lesson plans, and instructional practice among preservice teachers. *Journal of Curriculum Studies*, 52(6), 800–822. <https://doi.org/10.1080/00220272.2020.1752804>
- Krauss, S., Brunner, M., Kunter, M., Baumert, J., Blum, W., Neubrand, M., & Jordan, A. (2008). Pedagogical Content Knowledge and Content Knowledge of Secondary Mathematics Teachers. *Journal of Educational Psychology*, 100(3), 716–725. <https://doi.org/10.1037/0022-0663.100.3.716>
- Larrain, M., & Kaiser, G. (2022). Interpretation of Students' Errors as Part of the Diagnostic Competence of Pre-Service Primary School Teachers. *Journal Fur Mathematik-Didaktik*, 43(1), 39–66. <https://doi.org/10.1007/s13138-022-00198-7>
- Leuders, T., Dörfler, T., Leuders, J., & Philipp, K. (2018). Diagnostic Competence of Mathematics Teachers: Unpacking a Complex Construct. In *Diagnostic Competence of Mathematics Teachers* (pp. 3–31). Springer International Publishing. https://doi.org/10.1007/978-3-319-66327-2_1
- Li, K. (2025). Factors influencing learners learning interest when using AI chatbots-assisted math leaning in higher education. *Frontiers in Psychology*, 16. <https://doi.org/10.3389/fpsyg.2025.1716913>
- Margevièa-Grinberga, I., & Dûmane, I. (2024). Prospective Teachers' Perspectives On Pedagogical Challenges Experienced During Work-Based Learning. *Society. Integration. EDUCATION. Proceedings of the International Scientific Conference, 1*, 194–207. <https://doi.org/10.17770/sie2024vol1.7796>
- Marmoah, S., Colomeischi, A. A., & Duca, D. S. (2021). Defining problems and solutions of education for frontier, outermost, and least developed regions program in Indonesia. *Elementary Education Online*, 20(1), 36–52. <https://doi.org/10.17051/ilkonline.2021.01.013>
- Marshman, M., & Porter, G. (2006). Pre-service Teachers; Pedagogical Content Knowledge Implication for Teaching. *Proceedings of the 36th Annual Conference of the Mathematics Education Research Group of Australasia*, 474–481.
- Melissa, M. M., Anuniwat, D., & Pratini, H. S. (2026). A review of mathematics teachers'

- noticing of students' algebraic thinking. *Frontiers in Education*, 11. <https://doi.org/10.3389/feduc.2026.1897747>
- Miles, M. B., Huberman, A. M., & Saldaña, J. (2018). *Qualitative Data Analysis: A Methods Sourcebook*. SAGE.
- Napanoy, J. B., Gayagay, G. C., & Tuazon, J. R. C. (2021). Difficulties Encountered by Pre-service Teachers: Basis of a Pre-service Training Program. *Universal Journal of Educational Research*, 9(2), 342–349. <https://doi.org/10.13189/ujer.2021.090210>
- Novita, P. (2022). The Quest for Teacher Education Quality in Indonesia: The Long and Winding Road. In *Handbook of Research on Teacher Education* (pp. 651–673). Springer Nature Singapore. https://doi.org/10.1007/978-981-16-9785-2_32
- Nusantara, D. S., Zulkardi, & Putri, R. I. I. (2025). Investigating Students' Mathematical Argumentation Ability in Solving PISA-like Mathematics Tasks on Change and Relationships. *Mathematics Education Journal*, 19(4), 807–826. <https://doi.org/10.22342/mej.v19i4.pp807-826>
- Phelps-Gregory, C. M., & Spitzer, S. M. (2018). Developing Prospective Teachers' Ability to Diagnose Evidence of Student Thinking: Replicating a Classroom Intervention. In *Diagnostic Competence of Mathematics Teachers* (pp. 223–240). Springer International Publishing. https://doi.org/10.1007/978-3-319-66327-2_12
- Reinhold, S. (2018). Revealing and Promoting Pre-service Teachers' Diagnostic Strategies in Mathematical Interviews with First-Graders. In *Diagnostic Competence of Mathematics Teachers* (pp. 129–148). Springer International Publishing. https://doi.org/10.1007/978-3-319-66327-2_7
- Santagata, R., & Yeh, C. (2016). The role of perception, interpretation, and decision making in the development of beginning teachers' competence. *ZDM*, 48, 153–165. <https://doi.org/10.1007/s11858-015-0737-9>
- Seidel, T., & Stürmer, K. (2014). Modeling and Measuring the Structure of Professional Vision in Preservice Teachers. *American Educational Research Journal*, 51(4), 739–771. <https://doi.org/10.3102/0002831214531321>
- Seifried, J., & Wuttke, E. (2010). Student errors: how teachers diagnose and respond to them. *Empirical Research in Vocational Education and Training*, 2(2), 147–162. <https://doi.org/10.1007/BF03546493>
- Shulman, L. S. (1987). Knowledge and Teaching: Foundations of the New Reform. *Harvard Educational Review*, 57(1), 1–22.
- Sidenvall, J., Boström, E., Granberg, C., Lithner, J., Säfström, A. I., Andersson, C., Palmberg, B., & Palm, T. (2026). Teachers' in-the-moment diagnosis of students' problem-solving difficulties in mathematics. *Frontiers in Education*, 11. <https://doi.org/10.3389/feduc.2026.1831773>
- Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Tasks Used in Reform Classrooms. *American Educational Research Journal*, 33(2), 455–488.
- Walter, D., Bergmann, A., Maibach, M., Huethorst, L., Reinartz, L., Grünwald, N., Selter, C., & Harter, A. (2025). How pre-service teachers can be supported to increase their diagnostic skills in

mathematics—design and evaluation of a learning platform for university teacher training. *Frontiers in Education*, 10, 1510828. <https://doi.org/10.3389/feduc.2025.1510828>

Wijaya, T. T., Hidayat, W., Hermita, N., Alim, J. A., & Talib, C. A. (2024). Exploring contributing factors to PISA 2022 mathematics achievement: Insights from Indonesian teachers. *Infinity Journal*, 13(1), 139–156. <https://doi.org/10.22460/infinity.v13i1.p139-156>

Wildgans-Lang, A., Scheuerer, S., Obersteiner, A., Fischer, F., & Reiss, K. (2020). Analyzing prospective mathematics teachers' diagnostic processes in a simulated environment. *ZDM - Mathematics Education*, 52(2), 241–254. <https://doi.org/10.1007/s11858-020-01139-9>

World Bank. (2020). *The Promise of Education in Indonesia*. World Bank.

Zurbriggen, C. L. A., Schwab, S., Kalkusch, I., Neuhauser, A., Lanfranchi, A., & Klaver, P. (2025). Explaining teachers' judgement accuracy of students' subjective well-being by students' individual characteristics and the teacher-student relationship. *Learning and Individual Differences*, 122, 102754. <https://doi.org/10.1016/j.lindif.2025.102754>