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Analysis of Mathematics Education Students' Reasoning in Determining the Truth Value of Quantification

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Abstract: Reasoning refers to the structured and logical process of drawing conclusions from given premises. Students of mathematics education, who are being prepared as future educators, are expected to possess strong logical reasoning skills in order to comprehend and convey more advanced mathematical concepts. This study aims to analyze the reasoning abilities of mathematics education students in determining the truth value of quantified statements. The central research question is: How do students' reasoning abilities relate to the accuracy of their problem-solving in the context of quantification? An explorative qualitative approach was employed to explore students' reasoning in evaluating the truth values of quantified statements. The study involved 13 students from the Mathematics Education Department at Universitas Muhammadiyah Ponorogo who were enrolled in the Mathematical Foundations course during the 2024/2025 academic year. Data collection instruments included a set of test questions and open-ended interview protocols. The analysis focused on indicators of mathematical reasoning, including: (1) identifying facts, patterns, properties, and relationships, (2) drawing logical conclusions, (3) making predictions and solving problems, and (4) constructing systematic and valid arguments. Following the written assessment, three students representing different levels of performance were selected for in-depth interviews. The findings revealed that while all three participants demonstrated potential in mathematical reasoning, each exhibited specific areas requiring further development. Subject N was proficient in deconstructing quantification problems and drawing logical conclusions, although further probing during the interview revealed gaps in deeper conceptual understanding. Subject A displayed semantic comprehension of quantification but lacked precision in applying formal logic and integrating numerical analysis. Subject F demonstrated a verbal understanding of truth value criteria and was capable of drawing conclusions, yet faced challenges with formal notation, particularly with singular quantifiers. Overall, the participants exhibited promising reasoning abilities but required reinforcement in interpreting formal notation, constructing valid arguments, and accurately applying logical structures. Misinterpretations of quantifier symbols and formal argument patterns were identified as common errors.

Keywords: mathematical reasoning, truth valu, quantification.

▪ INTRODUCTION

Reasoning refers to the ability to think systematically and logically by identifying patterns and connecting facts or statements – commonly known as premise – through structured and analytical steps to arrive at conclusions. (Budiyono et al., 2020; Muhammad et al., 2024). More concisely, reasoning can be defined as cognitive process aimed at deriving conclusions in the form of knowledge (Ginting & Hasanuddin, 2020). In essence, reasoning is the capacity to utilize existing information to reach a logical conclusion or make informed decisions. This process is a core component of broader

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cognitive functions and is closely linked to problem solving, decision making, and comprehension.

Reasoning is an intellectual ability that plays a major role in human life, as it allows individuals to make assumptions, construct arguments, and draw conclusions (Muhammad et al., 2024). In mathematics learning, reasoning serves as a foundation that allows students to understand and master general mathematical concepts, as well as to solve mathematical problems (Zendrato et al., 2022). The development of students' reasoning abilities is one of the primary objectives that must be achieved in mathematics education (Elsyavira et al., 2024). This underscores the importance of the teacher's role in cultivating accurate and effective reasoning skills in students through instruction.

In order to develop students' reasoning abilities, teachers must themselves possess strong reasoning skills (Bernard & Dudek-Róycki, 2019). Therefore, logical reasoning is a fundamental competency that must be acquired by mathematics education students, who are being prepared to become future teachers. This ability not only forms the basis for understanding more complex mathematical concepts but also plays an important role in problem-solving and mathematical proof (Hansen, 2022; Hasanah et al., 2019).

An assential aspect of logical reasoning is the understanding of quantification, particularly universal (∀) and existential (∃) quantification. Quantification is employed to express the truth value of statements across various mathematical contexts, including set theory, formal logic, and calculus. In mathematics, quantification refers to the process of assigning numerical values or mathematical expressions to objects, phenomena, or relationships in order to describe their properties or characteristics (Piatek-Jimenez, 2010). It enables the construction of formal and structured statements, which are fundamental in mathematical proof and in modelling real-world situations (Dalen, 1983; Huth & Ryan, 2000). In logic, for example, quantification facilitates the representation of propositions that apply to all elements or some elements within a given set (Ebbinghaus et al., 1984).

However, in practice, many students struggle to develop a deep understanding of the concept of quantification. This is often due to misinterpretation of symbols or a lack of contextual understanding of the statements involved. Such difficulties may stem from various factors, including limited formal logic training, ineffective instructional approaches, and insufficient integration of theory and application in the learning process. Students frequently encounter challenges in understanding the concept of quantification in mathematics, especially when translating mathematical statements involving universal quantifiers from everyday language into formal expressions. (Rudito, 2009). Error analysis of quantification topics also reveals that prospective mathematics teachers commonly make mistakes in constructing and organizing quantified statements (Tamba, 2020).

The results of this study are expected to contribute to the enhancement of mathematics education quality, particularly in the areas of logic and quantification instruction. The findings may serve as a foundation for developing more effective teaching strategies to support students in mastering the concept of quantification. This study aims to analyze the reasoning abilities of mathematics education students in determining the truth values of quantified statements. The primary focus is to identify which reasoning indicators students are able to fulfill, as well as the factors contributing to their inability to meet certain indicators. The central research question is: How do students' reasoning abilities relate to the accuracy of their problem-solving in the context of quantification?

▪ METHOD

Participants

Quantification is one the topics covered in the Foundations of Mathematics course for first-semester students at Muhammadiyah Ponorogo University. In the 2024/2025 academic year, 13 students were enrolled in this course, all of whom served as participants in this study. However, for the interview stage, three students were selected based on specific criteria. The selection was guided by the level of correctness in problem-solving, categorized as follows: (1) students who answered both problems correctly, (2) students who answered only one problem correctly, and (3) students who answered both problems incorrectly. This criterion was adopted because the test consisted of two items, leading to three possible answer patterns.

Research Design and Procedures

To analyze students' reasoning in determining the truth value of quantified statements, an exploratory qualitative research design was employed. The research-process began with the participant selection and the development of relevant instruments. Data were subsequently collected and analyzed through an in-depth exploration approach. The study was conducted over a three-month period, from November 2024 to January 2025.

Instruments

The test instrument used to assess students' reasoning abilities comprised a single item designed to evaluate the truth value of two quantified statements, accompanied by students' written justifications. The item was constructed to reflect four indicators of mathematical reasoning, namely: (1) identifying facts, patterns, properties, and relationships; (2) drawing logical conclusions; (3) predicting and solving problems; and (4) constructing systematic and valid arguments (Zendrato et al., 2022). In addition to the test, an open-ended interview protocol was employed to further investigate students' cognitive processes beyond their written responses. The validity of the instrument was established through expert judgment. Credibility was ensured through the researchers' continuous engagement throughout the study, regular discussions with the research team, and member checking, wherein participants were asked to verify the accuracy of the data analysis. Dependability was maintained by carefully selecting research methods that aligned with the study's objectives.

Data Analysis

Following the development of the research instruments, data collection was undertaken. All participants were allotted 20 minutes to complete the test. The researchers subsequently reviewed and assessed the students' responses. Based on predefined criteria, three participants were then selected for individual interviews. The data collected were analyzed using content analysis to evaluate students' reasoning abilities according to the established indicators. Coding categories were developed from the collected data through a process of manual coding, which involved carefully reading the responses and identifying meaningful segments. These codes were then organized into categories aligned with the reasoning indicators and further refined into broader themes. The analysis was conducted in a reflective and iterative manner, with detailed documentation to ensure the validity and depth of interpretation. The findings were then synthesized and presented in this publication.

▪ RESULT AND DISSCUSSION

The research participants had previously studied the topic of quantification and the evaluation of its truth value. Therefore, this study commenced directly with the administration of test items to all participants. The questions given are presented in Figure 1.

Tentukan nilai kebenaran kuantifikasi berikut dan berikan alasannya. Domain untuk variabel x dan y adalah bilangan real.

a. $\exists!\, x, (x^2 = 1)$.
b. $\forall x \exists y, (x + y = x - y)$

Figure 1. Reasoning test questions with quantification material

Subsequently, the researchers evaluated the students' responses and categorized them according to the three predetermined criteria. Three students met Criterion 1, having answered both questions correctly. Five students met Criteria 2 and 3, corresponding to those who answered only one question correctly and those who answered both questions incorrectly, respectively. Based on this grouping, one student was selected from each category for the interview. Students N, A, and F were chosen to represent Criteria 1, 2, and 3, respectively. Each student was interviewed individually to ensure the authenticity of their responses. To facilitate data collection during the interviews, a mobile phone recording application was used. The following section presents the results and discussion of the data analysis for each subject criterion.

Criterion 1-Subject N

Subject N's work serves as an example of correct responses to both problems. The analysis of the written answers was conducted based on the reasoning indicators established in this study. The results of the analysis are shown in Figure 2.

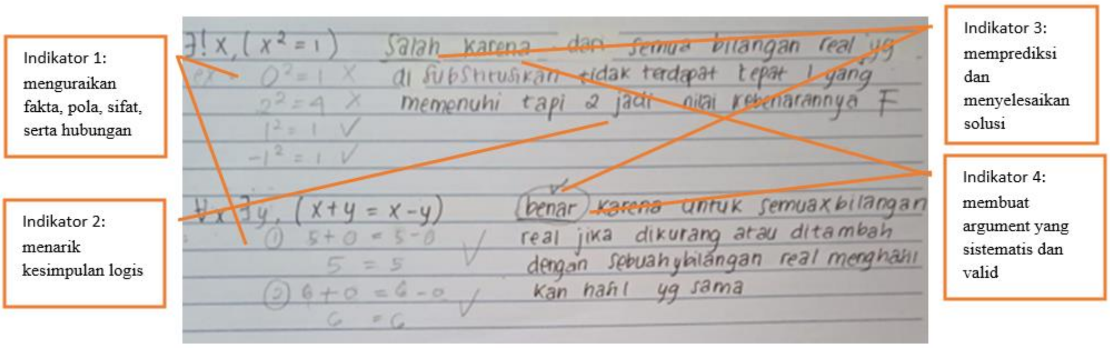

Figure 2. Analysis of the subject's written answer criteria 1

Based on the initial analysis of subject N's written responses, all four reasoning indicators were identified. However, in Question 2, the subject did not explicitly demonstrate the ability to draw logical conclusions. For the first indicator, the subject approached the quantification problem by providing examples through the substitution of several numerical values. This was further supported by the interview findings. The analysis of the interview data revealed that the subject also demonstrated an understanding of the meaning of the quantified statement in the question. Several excerpts from the interview transcript illustrate the presence of the first reasoning indicator.

N02: Well, this is an example to help me find the truth value, if x is 2, it is more than 1 (while pointing to the example x=2)
N03: Then ... for a moment, mum, this example of x=0 squared is wrong, it should be equal to 0 ... yes ...
N18: If 2 plus 2 is the same as 2 minus 2,
N19: 4 equals 0 (then a few moments of silence), must it be the same like this, mum? (while scribbling, choosing the same x and y)

Subject N demonstrated the ability to analyze quantification problems by providing examples derived from the substitution of various numerical values. This skill reflects an understanding of the meaning of embedded in the quantified statements presented in the questions. Interview results further revealed that the subject systematically articulated the problem and showed a clear comprehension of the given quantification. These findings align with previous research suggesting that the ability to generate concrete examples facilitates the understanding of quantification. (Barrera et al., 2019). Such a strategy enhances students' capacity to bridge abstract concepts with practical applications. (Barham, 2020).

The second indicator, namely drawing logical conclusions, was explicitly demonstrated in the subject's written response to Question 1. However, based on the interview analysis, the subject also exhibited the ability to draw logical conclusions for both questions. The subject determined the truth value of each statement using reasoning aligned with the principles they had understood. The following transcript excerpts illustrate evidence of this second reasoning indicator.

N09: So, not exactly one number is correct.
N24: Here, I'm showing y is equal to 0, so whatever x is, it will still be the same, right?
N25: Therefore, the value is correct, because there will be y = 0 that fulfils the condition.

In the written response to Question 1, Subject N explicitly demonstrated the ability to draw a logical conclusion. However, for Question 2, this ability was not fully evident in the written answer. The interview findings clarified that the subject was, in fact, able to draw logical conclusions for both problems, as evidenced by their determination of truth values based on the logical principles they had internalized. This supports the assertion that interviews, as a qualitative data collection method, can uncover deeper levels of understanding that may not be apparent from written responses alone (DeJonckheere & Vaughn, 2019; Nii Laryeafio & Ogbewe, 2023).

The analysis of the written response indicated that the subject followed the third reasoning indicator. Additionally, the interview results demonstrated that the subject predicted and determined the truth value of the quantification by applying the method of parsing through examples. This is evidenced by the statement in the following transcript.

N07: Then the truth value is wrong because there are two numbers when substituted, the result is 1.

N22: If y equals 0, then whatever x is, it will still be true, like my example...maybe it's true.

The analysis of the written responses indicated that Subject N demonstrated the third reasoning indicator. The prediction and determination of the truth value of the quantification were accomplished through the use of relevant examples. The interview findings further supported this, revealing that the subject was able to predict the truth value of quantification after parsing with examples. This is consistent with previous research, which suggests that the use of concrete examples enhances the ability to predict and analyze the truth value in quantification (Nieuwland, 2016; Tamba, 2020).

The fourth indicator was also supported by the interview results. The subject engaged a series of arguments before concluding that the proposed truth value was valid. This is illustrated by the statement in the following transcript.

N07: Then the truth value is wrong because there are two numbers when substituted, the result is 1.

N08: 1 and -1

N09: So there is not exactly one correct number.

T16: Why do you think it's right?

N23: Because all the x's and y's are entered, even though the operation is different, the result is the same.

N24: Here, I'm showing y is equal to 0, so whatever x is, it will be the same, right?

Subject N engaged in a series of logical arguments before gaining confidence in the resulting truth value. The interview findings indicate that the subject's analytical thinking process supported the construction of arguments aligned with the principles of mathematical logic (Corneli et al., 2019; D'Agostino & Modgil, 2018; Faizah et al., 2020; Muhtadi et al., 2020). According to (Cresswell & Speelman, 2020), the ability to construct logical arguments is a crucial step in the development mathematical reasoning.

The development of Subject N's analysis showed that the emergence of mathematical reasoning was not coincidental, but rather a result of a learning approach that fostered exploration, the use of concrete examples, and provided space for reflective thinking through dialogue. The subject was able to demonstrate an understanding of the meaning of quantification and draw logical conclusions because they were encouraged to construct meaning from the problem through value substitution activities and by testing the truth of statements. This approach promotes the internalization of concepts rather than the mere memorization of procedures. These skills were visibly developed through a gradual process: from describing the problem using examples, to making conjectures, and finally, to constructing logical arguments based on their findings. This strategy reflects constructivist principles, which emphasize students' active participation in the reasoning process.

This finding is consistent with previous research that highlights the importance of conceptual scaffolding in developing mathematical reasoning. For example, (Corneli et al., 2019) and (D'Agostino & Modgil, 2018) noted that the construction of logical arguments is influenced by students' involvement in reflecting on relationships between ideas. In addition, (Faizah et al., 2020) emphasized that students' logical thinking skills are more effectively developed when they are allowed to evaluate and verify their ideas. These results are also in line with (Nieuwland, 2016), who highlighted that the use of examples in solving quantification problems strengthens the inference process. Thus, the results of this study not only align with but also extend the understanding of the significance of exploratory approaches in cultivating meaningful mathematical reasoning.

Criterion 2-Subject A

Subject A's work represents a correct response to one of the two problems. The analysis of this written response is conducted based on the same reasoning indicators used in Criterion 1. The results of the analysis are shown in Figure 3.

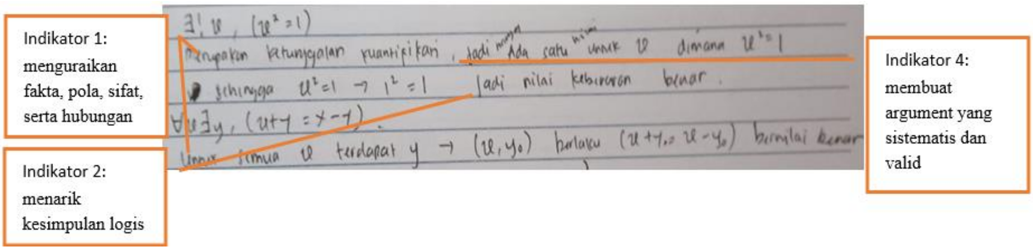

Figure 3. Analysis of the subject's written answer criteria 2

Based on the initial analysis of Subject A's written responses, only three reasoning indicators were identified, and these were not demonstrated in a complete manner. For the first indicator, the subject attempted to parse the quantification problem by interpretating the meaning of quantified statement. This interpretation is further supported by the interview findings, as evidenced by several statements presented in the following transcript.

T20: For this first problem, what can you describe?
A01: It's the uniqueness of quantification, mum, so only exactly one value of x can fulfil x squared equals 1.
A02: It would be correct if there is only one value like that, mum (while pointing to the example of x=1). If two numbers are substituted, the result is 1, well...(showing regret).
A08: This is composed quantification, ma'am (silent for a while).
A09: for every x, there is a y if ... (silent again)
A10: This is true because if for x and y zero, then x plus y zero equals x min y zero (pointing to her written test answer).

Subject A demonstrated the ability to parse quantification problems by understanding the meaning of quantification. This indicates that the subject possesses a foundational understanding of the concept, although it remains limited to semantic interpretation and is not yet supported by robust numerical analysis. Such an understanding is crucial in helping students grasp the relationship between quantified statements and their corresponding domains. (Huth & Ryan, 2000). In this case, the subject was able to demonstrate the first reasoning indicator only in response to question number 2.

The second indicator – drawing logical conclusions – was also explicitly evident in the subject's written response, though only for question number 1. However, based on the analysis of the interview data, it became clear that the subject attempted to draw logical conclusions for both questions, despite providing an incorrect answer to the first. The subject determined the truth value based on reasoning aligned with the logical principles they understood. Evidence supporting the second indicator is presented in the following transcript.

A03: Yes, ma'am, this point is correct because there is only 1 which, when squared, equals 1.
T203: Are you sure about that?
A04: Yes, ma'am (showing a face full of confidence), the value is correct because it fulfils the requirements of the truth value of the singularity of quantification.
A15: It's true that if I add zero to x, it's still x, and if I subtract 0, it's still x, so for any x, this applies...
A16: So the truth value is correct, right?

Subject A successfully drew a logical conclusion in problem number 1, as evidenced by the written response. However, in problem number 2, although the interview revealed that the subject understood the process of logical inference, the incorrect answer to the first problem highlights a lack of validity in the application of logical reasoning. This suggest that the subject's understanding of formal logic and its practical application remains limited and requires further development, as noted by (Dalen, 1983). The logical principles used by the subject were not entirely accurate in evaluating certain domains, leading to flawed conclusions.

Based on the analysis of the subject's written responses, the third indicator— predicting and solving problems – was not evident. However, the interview results revealed that the subject did attempt to predict and determine the truth value of quantification statements after parsing them with examples. This is illustrated by the following transcript.

A02: It will be true if there is only one value like that, mum (while pointing to the example of x=1).
T202: Anything else?
A03: Yes, ma'am, this point is true because there is only 1 which, when squared, equals 1.
A10: This is correct because if for x and y zero, then x plus y zero equals x min y zero (while pointing to his written test answer).

Based on the analysis of the written responses, Subject A did not demonstrate evidence of the third indicator predicting and solving problems. However, the interview results revealed that the subject was able to predict the truth value of quantified statements after parsing them using examples. This suggests that the subject would benefit from a more exploratory learning approach to better connect abstract concepts with concrete examples. (Tamba, 2020) highlights that the use of diverse and relevant examples can enhance students' ability to understand and verify the truth value of quantified statements.

The fourth indicator is also corroborated by the interview results. The subject engaged in a series of arguments before arriving at what was believed to be the correct truth value. However, in the first problem, the subject incorrectly identified the truth value due to a flawed argument. Specifically, the subject overlooked certain elements of the domain – namely, the inclusion of negative numbers – which led to an incorrect conclusion. This lack of thoroughness indicates a need for greater attention to detail when constructing arguments. The following transcript illustrates this point.

A03: Yes, ma'am, this point is correct because there is only 1 which, when squared, equals 1.
T204: What is the domain for x?
A05: real number
T205: Are negative numbers also real?
A06: Yes, ma'am, I'm wrong.
A07: There is -1 whose square is also 1, ma'am, so the truth value is wrong because there are two numbers if you substitute them, the result is 1, well...(shows regret).

Subject A showed the ability to construct an argument prior to determining the truth value, as evidenced by the interview data. However, in the first problem, the argument presented was invalid due to an oversight in considering the domain of negative numbers. This highlights the need to enhance analytical reasoning skills, particularly in ensuring that all elements of the domain are accounted for in the evaluation process. Such errors are common when students neglect certain aspects of the domain, a challenge noted in previous research (Miseliunaite et al., 2022; Peças et al., 2023; Reinmuth, 2020), which underscores the importance of a comprehensive and systematic approach to problem analysis.

The development of Subject A's analytical responses indicates that, while the subject possessed an initial understanding of the concept of quantification, the application of reasoning remained limited and lacked full accuracy. This limitation appears to stem from a reliance on semantic comprehension of quantification rather than numerical validation or systematic verification against the relevant domain. Although the subject recognized that the problem involved the uniqueness of quantification, the method used to confirm the truth of the statement did not involve a thorough evaluation of all domain elements – evident in the failure to consider negative real numbers. This suggests that the subject’s formal reasoning strategy was insufficiently robust to support valid inferential conclusions.

In addition, the interviews data revealed that while the subject was capable of constructing arguments and drawing logical conclusions, these arguments were frequently unsupported by concrete evidence or comprehensive logical structure. In the first problem, for instance, the subject believed that only one value satisfied the quantifier, yet this assumption was not rigorously tested, leading to a misjudgment of the statement’s truth value. This indicates that, although the logical thinking process had begun to develop, the quality and precision of the arguments required further refinement. This finding aligns with the work of (Miseliunaite et al., 2022), who reported that reasoning errors in quantification tasks often result from students' failure to fully consider the scope of the domain.

When compared with existing studies, these results exhibit both alignment and divergence. The findings are consistent with those of (Tamba, 2020) and (Nieuwland, 2016) who argue that students’ understanding of quantification improves with the use of concrete examples. However, they contrast with (Faizah et al., 2020), who found that valid logical arguments tend to emerge when learners are given the opportunity to thoroughly evaluate and reflect on their ideas. Subject A appeared not to have reached that evaluative stage independently. Thus, the present study reinforces the notion that while semantic comprehension of quantification provides a foundation for reasoning, more advanced strategies – particularly those fostering critical evaluation and rigorous domain analysis – are needed to enhance the accuracy and validity of logical reasoning.

Criterion 3-Subject F

Subject F's work represents an example of incorrect responses for both problems. The analysis of the subject's written answers was conducted using the reasoning indicators established in this study. The results of this analysis are presented in Figure 4.

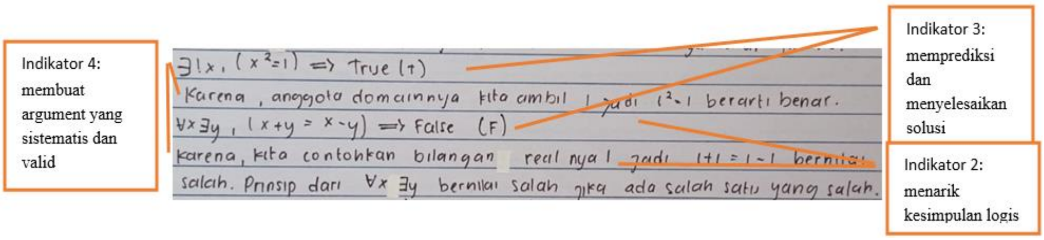

Figure 4. Analysis of the subject's written answer criteria 3

Based on the initial analysis of Subject F's written responses, three reasoning indicators were identified. The subject did not explicitly demonstrate the first indicator – describing facts, patterns, properties, and relationships – in the written answers. However, the interview analysis revealed that the subject articulated an understanding of the problem by explaining the criteria for determining the truth value of quantification statements. Several excerpts from the interview transcript illustrate evidence of this first indicator.

F01: This is an existential quantification, mum.
F02: The truth value is true if there is something that fulfils f(x).
F08: (reading the written answer) This is the universal quantor that limits x and the existential quantor that limits y, right?
F09: The truth value will be wrong if one of them is wrong.

Although subject F did not explicitly decompose facts, patterns, properties, and relationships in the written response, the interview showed that the subject understood the criteria for determining the truth value of quantification statements. Through verbal explanations, the subject identified logical relationships, indicating that conceptual understanding was present, although it had not yet been translated into formal representations. According to research, conceptual understanding in quantification should encompass the ability to integrate factual knowledge with formal representations to prevent ambiguity.

The second indicator – drawing logical conclusions – was explicitly evident in the subject's written answers. Furthermore, the interview analysis confirmed that the subject demonstrated the ability to draw logical conclusions for both problems, although the conclusions themselves were not entirely accurate. The subject determined the truth value based on reasoning aligned with the principles he understood. The transcript illustrating the second indicator is presented as follows.

F04: At first, I thought that this fulfils something, so the value is correct. I took the example of x=1 so that the square is equal to 1 (while pointing to the written reason), so the value is correct because it fulfils the requirements of the existential truth value.

F10: For example, if I choose x and y as 1, then the value is wrong. That means the truth value is wrong (while pointing to his written test answer).

Subject F demonstrated evidence of this indicator through the written answers. The interview also confirmed that the subject was able to draw logical conclusions for both problems. However, the confidence in an incorrect answer highlights a limitation in validating conclusions using formal principles of logic. This aligns with the findings of (Lazić et al., 2022; Matušek, 2021), who emphasize the necessity of reinforcing logical foundations in mathematics education to enhance the accuracy of reasoning.

The analysis of the written answers also showed that the subject had identified the third indicator. Interview results supported this finding, indicating that the subject predicted and determined the truth value of quantification statements after parsing them using examples. This is evidenced by the following excerpt from the interview transcript.

F04: At first, I thought this fulfilled something, so the value is correct. I took the example of x=1 so that the square is equal to 1 (while pointing to the written reason), so the value is correct because it fulfils the requirements of the existential truth value.

F10: For example, if I choose x and y as 1, then the value is wrong. That means the truth value is wrong (while pointing to her written test answer)

F12 : ok, Mom, this means...ummm...in my opinion, for all x, there is a y that x plus y is equal to x minus y. I think this is wrong because there is no way that it can be fulfilled

F13: Yes, it's impossible, mum, for all x that can be added or subtracted from y, the result is the same, like my example earlier.

The subject demonstrated the third indicator in the written answer, which was further supported by the interview. The subject predicted the truth value of quantification statements through decomposition using concrete examples. This reflects an explorative approach, which plays a crucial role in developing an understanding of quantification concepts. Research by (Tamba, 2020) confirms that exploration through systematic examples enhances students' ability to accurately determine truth values.

The fourth indicator is also corroborated by the interview findings. The subject constructed a series of arguments before accepting the assumed truth value. However, the reasoning was flawed due to the formulation of invalid arguments. In the first problem, the subject failed to correctly interpret the notation related to the uniqueness of quantification. In the second problem, the subject also showed an incomplete understanding of nested quantifiers. This is illustrated in the following transcript excerpt.

F04: At first, I thought that this fulfils something, so the value is correct. I took the example of x=1, so the square is equal to 1 (while pointing to the written reason), so the value is true because it fulfils the requirements of existential truth value.

F12: ok, Mom, this means...ummm...in my opinion, for all x, there is a y that x plus y is equal to x minus y. I think this is wrong because there is no way that it can be fulfilled

F16: (frowning and thinking for a while) If y=0 means adding or subtracting, then it's the same, mum, so x equals x, so it's right, mum, I was wrong (smiles regretfully).

Subject F formulated a series of arguments to support his belief regarding the truth value of the statements. However, errors in arguments construction were evident in both problems. In the first problem, the subject demonstrated a limited understanding of the notation associated with the singularity of quantification. In the second problem, the subject had not yet fully grasped the interpretation of nested quantifiers. These errors highlight the necessity for a more in-depth understanding of formal notation and the logical complexity inherent in quantification, as emphasized by (Bellucci et al., 2018).

The analysis of Subject F's performance showed that, although both written responses were incorrect, the subject demonstrated engagement with the reasoning process, which included three key indicators. This suggests that the difficulties may stem from a limited understanding of quantification in its formal representation, despite the subject's ability to verbally articulate the criteria for determining the truth value of quantification statements. Subject F also attempted to draw logical conclusions based on self-generated examples. However, the formulation of these conclusions did not consistently adhere to the principles of valid formal logic. These findings support the assertions of (Lazić et al., 2022) and (Matušek, 2021), who argue that students often struggle to validate conclusions due to insufficient mastery of fundamental logical principles in mathematics.

Furthermore, the subject's ability to predict truth values using concrete examples indicates a potential for developing an exploratory approach, aligning with (Tamba, 2020) Findings on the effectiveness of systematic exploration in enhancing the understanding quantification. However, errors persist due to the construction of invalid arguments, such as the subject's limited understanding of the notions of singularity and nested quantification. These shortcomings underscore the need to strengthen students' comprehension of formal logical structures and the meanings embedded in quantification notation.

Synthesis of Findings and Implications for Learning Mathematical Logic

The analysis of the three subjects revealed several recurring difficulties in understanding and applying the concept of quantification in mathematical logic. One of the most prominent challenges lies in comprehending formal quantification notations, particularly with respect to singular and nested quantifiers. Subjects A and F, for instance, demonstrated misconceptions in interpreting number domains and showed limited precision in grasping the universal and existential meanings of quantifiers (Huang et al., 2024; Wehmeier, 2024). Another major issue emerged in the validation of logical inferences: although the subjects were able to construct arguments, they often failed to verify the logical validity of these inferences using formal reasoning principles. This was evident in their reliance on isolated examples without a comprehensive examination of all possible cases.

The findings also point to varied reasoning strategies among the subjects, ranging from concrete example-based reasoning and semantic parsing to more formal argumentation. Subject N employed a more systematic and complete approach, whereas Subjects A and F displayed limitations in both validating and explicitly articulating their reasoning processes. These observations highlight the spectrum of students' logical reasoning abilities and suggest that such abilities may not always be visible through written responses alone. Consequently, incorporating interview-based evaluations or reflective verbal tasks becomes essential to uncover deeper levels of understanding that written assessments may fail to capture (Wingate et al., 2024).

The implications for teaching in Foundations of Mathematics are significant. There is a clear need to enhance exploration-based learning environments that prioritize students' reasoning processes alongside the accuracy of their answers. Instructors should develop instructional strategies that foster students' abilities to translate formal notation into semantic understanding, utilize varied examples to test logical statements, and engage in deliberate validation of their conclusions. Pedagogical approaches that incorporate reflective dialogue, structured argumentative practice, and explicit mapping of domain elements can provide more effective support for cultivating rigorous mathematical and logical reasoning skills.

▪ CONCLUSION

In general, all three subjects demonstrated potential in their reasoning abilities; however, they still require further reinforcement in understanding formal notation, drawing logical conclusions, and constructing valid arguments. While they were able to identify relevant information and made attempts to apply basic logical principles, several errors were observed, particularly in the interpretation of quantifier symbols and formal argument structures. These errors were evident in their inconsistent distinctions between universal and existential statements, as well as in their testing of the truth value of a statement based on contextual parameters. Furthermore, their reasoning processes and justifications tended to be intuitive and lacked systematic structure, underscoring the need for continued practice in constructing coherent arguments grounded in formal logic.

Regarding the first indicator – understanding the concept of quantification – Subjects N, A, and F exhibited varying levels of success. Subject N demonstrated a systematic approach to breaking down quantification problems, supported by the use of concrete examples, which strengthened conceptual understanding through practical application. Subject A showed a semantic understanding of quantification but had yet to integrate a robust numerical or domain-based analysis, suggesting a limited depth of understanding. Similarly, Subject F articulated an understanding of the criteria for the truth value of quantification statements during the interview; however, this understanding was not fully evident in the formal written responses.

For the second indicator – the ability to draw logical conclusions – the performances of the three subjects also differed. Subject N effectively drew logical conclusions for both problems, though not always explicitly in the written responses, illustrating the importance of interviews in uncovering deeper reasoning. Subject A managed to draw a logical conclusion in the first problem but lacked precision in applying formal logic to assess domain-specific truth values, indicating an area for further development. Subject F demonstrated the ability to draw conclusions in both written and oral responses; however, the persistence of incorrect answers revealed limitations in validating these conclusions according to formal logical standards.

The analysis revealed that Subjects N and F explicitly demonstrated the third indicator – predicting and determining the truth value of quantification – through the use of relevant concrete examples, as evidenced in both their written responses and interview data. This highlights the significance of employing exploratory approaches in developing an understanding of truth values within quantification. In contrast, Subject A did not demonstrate this indicator in the written response; however, the interview revealed the ability to predict truth values after decomposing the problems using examples. This suggests that Subject A would benefit from more structured exploratory activities to strengthen conceptual understanding.

Further analysis indicated that Subject N exhibited a strong ability to construct logical arguments prior to confirming the truth value. Subject A also attempted to construct logical arguments but demonstrated imprecision, particularly in accounting for the domain of negative numbers. Subject F formulated arguments to justify conclusions; however, persistent errors were noted in interpreting formal notations, such as those related to singular and nested quantification.

Based on these findings, it is recommended that structured instructional strategies be implemented, incorporating exploratory approaches with concrete examples and emphasizing mastery of formal logic principles. These strategies aim to enhance mathematics education students' reasoning abilities, particularly in understanding the concept of quantification. Future research should consider examining reasoning abilities with a larger sample size to broaden the generalizability of the findings, as this study was limited to a small number of subjects.

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