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How Self-Efficacy Shapes Mathematical Analytical Thinking: A Qualitative Study using Action-Process-Object-Schema Theory

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Abstract: Analytical thinking enables students to solve complex problems in a systematic and measurable manner. This study examined students' analytical thinking processes in relation to self-efficacy using the Action-Process-Object-Schema (APOS) theory. The researchers employed a qualitative method with an explanatory approach. They selected mathematics education students from Universitas Muhammadiyah Purwokerto who were enrolled in a sequence convergence course as research subjects. To collect data, the researchers used analytical thinking tests, questionnaires, and interviews. Based on the self-efficacy questionnaire, the researchers classified students into high, medium, and low self-efficacy groups. They selected one informant from each category using purposive sampling. The data analysis involved three stages: data reduction, presentation of findings, and interpretation. The analytical thinking process included collecting, differentiating, organizing, and attributing. The findings showed that students in the high and medium self-efficacy groups demonstrated strong analytical thinking skills. They successfully completed all stages—collecting, differentiating, organizing, and attributing. In contrast, students with low self-efficacy lacked adequate prior knowledge and required assistance in applying mathematical concepts and completing proofs during the organizing stage. These students gathered only limited information at the collection stage due to a poor understanding of the main problem. As a result, they could only partially complete each stage of the analytical thinking process.

Keywords: analytic thinking process, APOS theory, self-efficacy.

▪ INTRODUCTION

Analytical thinking plays a crucial role in helping students logically understand mathematical concepts and apply them to everyday problem-solving (Schumacher & Ifenthaler, 2018). This thinking process promotes conceptual understanding and extends to solving problems in routine contexts (Tian et al., 2014; Yilmaz & Saribay, 2017). In the 21st century, analytical thinking has become essential for effective problem-solving. Robbins (2011) argues that individuals must apply analytical thinking to identify issues, generate ideas, and develop solutions for complex problems. As part of higher-order thinking skills, analytical thinking contributes significantly to the creation of creative ideas (Kao, 2014; Demir, 2022; Yulina et al., 2019). Developing logical thinking skills and correlation analysis between concepts also depend heavily on analytical thinking.

Dilekli (2019) defines analytical thinking ability as the method of solving problems by breaking them into interconnected components, exploring detailed information, identifying correlations among the data, and applying concepts accurately. Anderson & Krathwohl (2001) and Wijaya et al. (2023) explain that analytical thinking involves the stages of differentiating, organizing, and attributing. In the differentiating stage, individuals identify and break down information into structured and interrelated parts during the verification process, which helps generate initial ideas. In the organizing stage, they apply mathematical concepts and manage information to solve problems with

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detailed and accurate reasoning. The attributing stage involves drawing conclusions based on the collected evidence, aligned with the original goals and focus of the problem. This stage marks the conclusion of the reasoning process.

This research adopts the APOS theory, which helps trace students' cognitive processes, including analytical thinking. Arnon et al. (2014) and Dubinsky (2016). explain that the APOS framework has four stages Action, Process, Object, and Schema that focus on how people build their understanding of knowledge (Asiala et al., 1997). In this framework, individuals develop new knowledge by processing stored mathematical objects through actions, which then support problem-solving. As a result, researchers can use APOS theory to observe the cognitive processes students engage in while solving problems, including analytical thinking.

Numerous studies have explored APOS theory. Rahayu et al. (2023) for example, investigated students' problem-solving processes on probability material using APOS theory with a reflective cognitive style. They described problem-solving as involving observing information, presenting the problem, constructing and implementing ideas, and verifying outcomes.

Researchers also recognize the significance of affective factors, such as self-efficacy, in problem-solving (Herminarto et al., 2022; Nurhayati & Nyayu, 2023). Baysal et al. (2010) and Gunbatar (2018) emphasize that self-efficacy influences individuals' analytical thinking abilities. Bandura et al. (1999), Gao (2020), and Šorgo et al. (2017) define self-efficacy as an individual's belief in their ability to achieve specific goals. This belief also affects learning behavior (Öztürk et al., 2020; Widmer et al., 2014). High self-efficacy enables individuals to overcome learning challenges and solve problems, while low self-efficacy often leads to difficulty in finding appropriate solutions. Harahsheh (2017) states that self-efficacy is an inherent and continuous personal trait. Bandura et al. (1999) and Herminarto et al. (2022) explain self-efficacy includes three dimensions: level (confidence in overcoming learning difficulties), strength (perseverance when facing challenges), and generality (belief in the ability to initiate problem-solving actions).

Suyatman et al. (2021) developed indicators of analytical thinking in problem-based learning that include matching, classifying, organizing, and attributing. Qolfathiriyus et al. (2019) used these indicators to understand how students think analytically in two-dimensional geometry and discovered that top students met the basic and intermediate thinking standards. Wijaya et al. (2023) focused on the role of prior knowledge in analytical thinking and highlighted differentiating as a foundational stage. They found that the detail level in explanations during this stage depended on students' prior knowledge. However, past studies rarely examined the stages leading up to differentiating. To break down a problem effectively, students must first comprehend it thoroughly. Students should treat the crucial early stages of gathering information and connecting it with prior knowledge as distinct cognitive activities.

Bintoro et al. (2021) explored spatial thinking through the lens of APOS theory and field-independent cognitive styles. They identified characteristics of spatial thinking in both external and internal holistic learners, beginning with representative understanding and moving on to abstraction and final solutions. APOS theory proved useful in tracing students' cognitive processes and understanding how they develop spatial reasoning. This research focuses on convergence of sequences, a topic that requires mastery of prerequisite knowledge such as the definition of limits, singularity of values, finite sequences, and algebraic convergence properties. These are closely linked to the analytical thinking indicator of collecting. Convergence problems can present multiple scenarios, requiring varied solution strategies, which align with the differentiating and organizing stages of analytical thinking.

Based on this review, the present study introduces novelty by incorporating the collecting stage into the analytical thinking process alongside differentiating, organizing, and attributing. The collecting stage proves essential as it allows students to gather relevant information, relate it to prior knowledge, and form an initial strategy for solving problems. This study analyzes students' analytical thinking characteristics based on their levels of self-efficacy, using APOS theory as the analytical framework. The findings offer foundational insights for designing innovative classroom learning strategies aimed at enhancing analytical thinking. The research looks at this question: "How do the different stages of students' analytical thinking—like collecting, differentiating, organizing, and attributing show up at high, medium, and low levels of confidence in solving sequence convergence problems, when we use APOS theory to analyze it?"

▪ METHOD

Partisipants

The study involved 50 seventh-semester students majoring in mathematics education at Muhammadiyah University, Purwokerto. First, the researchers asked students to complete a self-efficacy questionnaire online. Then, students solved mathematical problems related to sequence convergence. The researchers assessed the questionnaire results and categorized students into low, medium, and high self-efficacy groups. Using purposive sampling, the researchers selected one student from each group to serve as an informant, resulting in three informants representing the self-efficacy categories.

In selecting informants, the researchers also considered students' final scores, answer completeness, verbal communication skills, academic performance, and attitudes. To facilitate the analysis of analytical thinking processes, the researchers labeled the informants as A1 (high), A2 (medium), and A3 (low) self-efficacy. They based the self-efficacy classification on recapitulated questionnaire data, following Hendriana et al. (2017). The classification includes five categories: very high (91–100), high (78–90), medium (65–77), low (0–51), and less (52–64).

Research Design

This qualitative study used an exploratory and descriptive approach. The researchers aimed to describe the characteristics of students' mathematical and analytical thinking processes through the lens of self-efficacy and APOS theory. They collected data from students' written solutions to mathematical problems and analyzed their analytical thinking based on the APOS framework.

The research process began with a literature review on analytical thinking, self-efficacy, and APOS theory. After developing and validating the research instruments, the researchers collected data from verified participants. They scored both the analytical thinking tests and the self-efficacy questionnaires. Based on the results, they categorized the students into high, medium, and low self-efficacy groups and selected one representative from each. The researchers then conducted in-depth interviews with the three informants to explore their analytical thinking processes.

In addition to analyzing interview responses, the researchers traced each informant's problem-solving approach using APOS theory. They then described and interpreted the characteristics of each analytical thinking stage. Finally, they drew conclusions based on the initial research question.

Research Instrument

The primary instruments used to assess students' analytical thinking processes included a self-efficacy questionnaire, a test, and an interview guide. The researchers designed the questionnaire based on five self-efficacy indicators: belief in one's abilities, confidence in overcoming challenges, persistence, willingness to seek new experiences, and strong task commitment. Each indicator was represented by three statements, totalling 15 items. The questionnaire followed a Likert scale from 1 to 5: 1 (strongly disagree), 2 (disagree), 3 (strongly agree), 4 (agree), and 5 (strongly disagree). The test consisted of a single question on sequence convergence, developed by the researchers based the content and analytical thinking indicators to assess the four stages of analytical thinking: collecting, differentiating, organizing, and attributing. Figure 1 presents the analytical thinking test used in the study.

Given a pair (X, d) metric spaces and sequence $(x_n) \subseteq X$ defined $(x_n) = \left(\frac{(-1)^n}{n}\right), \forall n \in N$. Answer the following questions related to the convergence of sequences.

a. Write down the known information? Also write down other information that is related/relevant to the information? (Provide an explanation of each information obtained)
b. Write down the meaning of the sequence $(x_n) = \left(\frac{(-1)^n}{n}\right), \forall n \in N$? (Write it according to your understanding and describe it in detail. You can also give examples of members of the sequence)
c. What is the sequence $(x_n) = \left(\frac{(-1)^n}{n}\right), \forall n \in N$ convergence? If so, then specify the point where the sequence (x_n) convergence? Prove. (Elaborate in detail).
d. What is the final conclusion of the above answer?

Figure 1. Analytical thinking test questions

Figure 1 presents four main questions, each designed to measure a specific analytical thinking indicator. The first question (a) targets the collecting indicator, the second question (b) assesses differentiating, the third question (c) evaluates organizing, and the fourth question (d) measures attributing. The researchers conducted validity and reliability testing using SPSS statistical analysis. Table 1 summarizes the SPSS test results.

Table 1. Validity and reliability tests

Aspects	Analytic Thinking Test Questions				Self-Efficacy Questionnaire	Category
	Question 1	Question 2	Question 3	Question 4		
Validation Test	$r_{hit.} = 0.798 > r_{table} = 0.297$	$r_{hit.} = 0.755 > r_{table} = 0.297$	$r_{hit.} = 0.605 > r_{table} = 0.297$	$r_{hit.} = 0.570 > r_{table} = 0.297$	$r_{hit.} = 0.421 > r_{table} = 0.297$	Valid

Reliability Test	Cronbach Alpha Values = 0.626 > 0.60	Cronbach Alpha Values = 0.801 > 0.60	Reliable

Each question in the analytical thinking test showed an r value greater than the critical r from the table. Additionally, the Cronbach's alpha value exceeded 0.60, indicating that the test met both validity and reliability standards. The self-efficacy questionnaire also demonstrated valid and reliable results, with every r value surpassing 0.297 (the table r value) and the Cronbach's Alpha again exceeding 0.60.

To obtain comprehensive data on students' analytical thinking processes, the researchers conducted detailed interviews with the selected respondents. They developed the interview guide based on the indicators of analytical thinking and included open-ended questions. Experts in educational evaluation and psychology validated the instruments. After revising the instruments according to expert feedback, the researchers finalized them as valid and reliable for use in the study.

Data Analysis

Fifty respondents completed an essay-type mathematics problem within 15 to 20 minutes. After submitting their work to the lecturer, the researchers selected three informants one from each self-efficacy category: high, medium, and low—to analyze their responses. The researchers used figures and tables to visually represent the analytical thinking processes of the informants, providing detailed descriptions.

At the end of the results section, the researchers visualized the APOS process derived from the analytical thinking stages. They described the cognitive activities involved in each indicator collecting, differentiating, organizing, and attributing in detail. Then, they coded each activity and grouped the codes into the APOS components: action, process, object, and schema, based on the definitions. This method helped the researchers see how developing analytical thinking and building knowledge in each area was related to the APOS framework.

To deepen their analysis, the researchers conducted individual interviews with the informants to verify the analytical thinking stages. Each interview lasted approximately 25 to 30 minutes. During the interviews, they used the APOS stages as a reference framework to ensure thorough confirmation of the analytical process. After the interviews, the researchers triangulated the data by comparing the students' written responses with the interview results, focusing primarily on how each student approached and solved the mathematical problem. Finally, the researchers drew conclusions based on the triangulated data.

▪ RESULT AND DISSCUSSION

In this section, the researchers present the analysis of students' analytical thinking processes in relation to their self-efficacy levels, as framed by APOS theory. They divide the analysis into three categories based on self-efficacy: high, medium, and low. Each category includes an examination of the four main analytical thinking indicators: collecting, differentiating, organizing, and attributing. Table 2 shows how students' analytical thinking processes differ at three levels of self-confidence, explaining each step in the APOS framework.

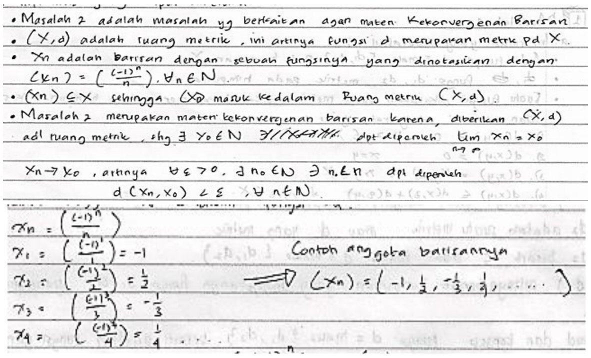
Table 2. Identification of students' analytical thinking processes reviewed from self-efficacy

Category	Stage			
	Collecting	**Differentiating**	**Organizing**	**Attributing**
Low	Clear	Unclear	Unclear. not detailed	Unclear
Medium	Clear. detailed	Clear	Clear	Clear. detailed
High	Clear. detailed	Clear. detailed	Clear. detailed	Clear. detailed

The analytical thinking processes of students with high, medium, and low self-efficacy differ most noticeably in the differentiating and organizing stages. Students with high and medium self-efficacy successfully completed both stages, while those with low self-efficacy failed to do so. As shown in Table 2, the three subjects displayed clear differences in the differentiating stage, particularly in how they divided the problem into distinct parts and articulated the problem definition clearly and in detail. This step made it easier for students to approach complex problems effectively.

Breaking the problem into smaller components also facilitated the organizing stage, where students applied the concept of convergence step-by-step to construct their proof. When students proved each part correctly, they found it easier to reach an accurate final conclusion. Students in the high self-efficacy category successfully divided the problem into parts, applied the definition of convergence to each, and arrived at a correct solution that aligned with the original problem. In contrast, students in the low self-efficacy category did not break the problem into manageable sections, which hindered their ability to construct a valid proof. The following section presents a detailed analysis of students' analytical thinking processes based on their self-efficacy levels.

High Self-Efficacy Category



Translation:

- Problem 2 is a problem related to the material of sequence convergence.
- (X, d) is a metric space, which means that the function d is a metric on X.
- x_n is a sequence with a function denoted by $(x_n) = \left(\frac{(-1)^n}{n}\right), \forall n \in N$
- $(x_n) \subseteq X$ so that (x_n) goes into the metric space X
- Problem 2 is about the convergence of sequences because it is given (X, d) is a metric space so

that $\exists x_0 \in N$ can get $\lim_{n\to\infty} x_n = x_0$

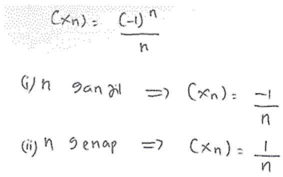
$x_n \to x_0$ it means $\forall \varepsilon > 0, \exists n_0 \in N \ni n \geq n_0$ applies $d(x_n, x_0) < \varepsilon, \forall n \in N$

Figure 2. A1 answer at the collecting stage

At the beginning of the process, students worked to understand the problem. Figure 2 illustrates the core task—proving the convergence of a given sequence. Students identified and explained all relevant information by rewriting the sequence, connecting it to the metric space, and stating the definition of the limit of a sequence. They correctly and clearly wrote the meaning of convergence, which served as a foundation for proving convergence based on the provided statements. Students recorded all problem-related information on their answer sheets.

In addition, students explained that their initial strategy involved applying the limit definition. They listed several terms of the sequence, starting from (x_n) starting from n = 1,2,3,4, and drew conclusions about the pattern of its members. The following interview excerpt supports this process:

R : What idea initially emerged to prove convergence?
A1 : I used the convergent definition.
R : What information did you write?
A1 : I wrote down the essence of the problem, its connection to metric spaces, the written sequence, and the definition of the limit of the sequence.
R : Can the collected information help you find ideas?
A1 : Yes, sir, that's all I could write. This information is interrelated. I imagined the connection between pieces of information until the initial idea emerged.

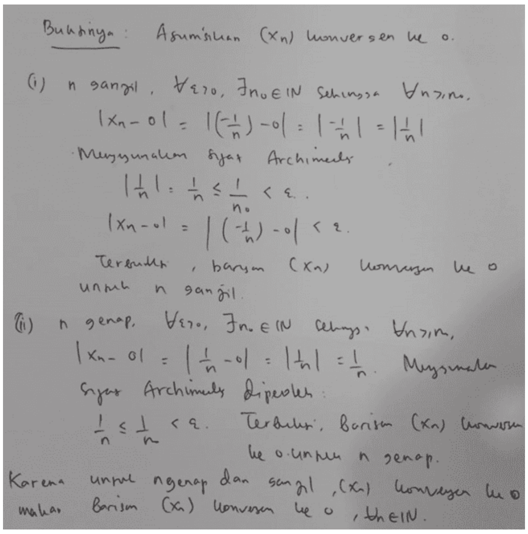


Translation:
$(x_n) = \frac{(-1)^n}{n}$
n odd $\Rightarrow (x_n) = \frac{-1}{n}$
n even $\Rightarrow (x_n) = \frac{1}{n}$

Figure 3. A1 answer at the differentiating stage

Figure 3 shows the results of student work at the differentiating stage. After identifying each member of the sequence based on the value n, students conclude that the value (x_n) is divided into two parts: odd and even n. The writing shows if the value of n is odd, then (x_n)=(-1)/n. Suppose n is even then (x_n)=1/n. As a result, the convergence evidence is also based on these two categories. The interview explains the same information about the odd and even categories and the subsequent verification process.

R : Explain the process of finding ideas into odd and even categories.
A1 : First of all, I wrote down the members and noticed differences if the n values were different. Next, I considered dividing n into two parts, odd and even.
R : What was the initial description of the process of proving convergence?
A1 : The division into categories has an impact on the evidence process. The evidence is carried out in each odd and even n category.

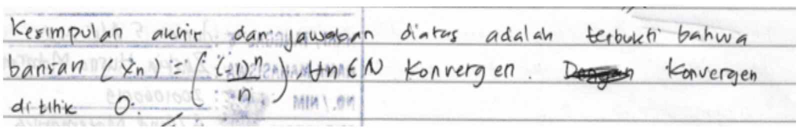

Buktinya: Asumsikan (x_n) konvergen ke 0.
(i) n ganjil, $\forall \varepsilon > 0, \exists n_0 \in \mathbb{N}$ sehingga $\forall n \geq n_0$.
$|x_n - 0| = |(\frac{-1}{n}) - 0| = |\frac{-1}{n}| = |\frac{1}{n}|$
Menggunakan sifat Archimedes,
$|\frac{1}{n}| = \frac{1}{n} \leq \frac{1}{n_0} < \varepsilon$.
$|x_n - 0| = |(\frac{-1}{n}) - 0| < \varepsilon$.
Terbukti, barisan (x_n) konvergen ke 0 untuk n ganjil.
(ii) n genap, $\forall \varepsilon > 0, \exists n_0 \in \mathbb{N}$ sehingga $\forall n \geq n_0$,
$|x_n - 0| = |\frac{1}{n} - 0| = |\frac{1}{n}| = \frac{1}{n}$. Menggunakan sifat Archimedes diperoleh:
$\frac{1}{n} \leq \frac{1}{n_0} < \varepsilon$. Terbukti, barisan (x_n) konvergen ke 0 untuk n genap.
Karena untuk n genap dan ganjil, (x_n) konvergen ke 0 maka barisan (x_n) konvergen ke 0, $\forall n \in \mathbb{N}$.

Translation:
Assume it converges to 0.
Evidence:
i. n odd, $\forall \varepsilon > 0, \exists n_0 \in N \ni \forall n \geq n_0$,
$|x_n - 0| = \left|\frac{-1}{n} - 0\right| = \left|\frac{-1}{n}\right| = \frac{1}{n}$.
using Archimedes' properties
$|x_n - 0| = \left|\frac{1}{n}\right| = \frac{1}{n} \leq \frac{1}{n_0} < \varepsilon$.
It is proven that the sequence converges to 0 for odd n.
ii. n even, $\forall \varepsilon > 0, \exists n_0 \in N \ni \forall n \geq n_0$,
$|x_n - 0| = \left|\frac{1}{n} - 0\right| = \left|\frac{1}{n}\right| = \frac{1}{n}$.
using Archimedes' properties
$|x_n - 0| = \left|\frac{1}{n}\right| = \frac{1}{n} \leq \frac{1}{n_0} < \varepsilon$.
It is proven that the sequence converges to 0 for even n.
Because for even and odd n, the sequence (x_n) converges to 0, so the sequence converges to 0, $\forall n \in N$.

Figure 4. A1 answer at the organizing stage

The next stage assumes that the sequence (x_n) converges to 0. In Figure 4, the evidence process is divided into two categories: odd and even n. In the odd n category, students used Archimedes' definition and properties. Each step is written systematically and in detail. At the end, the initial conclusion is written as part of the confirmation that the sequence (x_n) converges to 0 for odd n. Like the even n category, students proved the convergence of (x_n) with the definition of sequence limits and Archimedes' property. The students wrote the answers clearly and in detail. One piece of evidence is to write the conclusion for n even. The results of interviews with students explored the applied ideas to prove the convergence of the sequence (x_n). Here are the results.

R : How did you prove the sequence(x_n) is convergent?
A1 : I used the limit definition for sequences and Archimedes' property.
R : What is the first thing you do to get the evidence?
A1 : I made the correct assumption that the sequence (x_n) converges to 0. After that I proved it.
R : Are you sure about the answer?
A1 : I'm sure, sir. The evidence process is carried out one by one based on odd and even categories. I also confirmed that the sequence(x_n) converges to 0 for odd and even n.


Kesimpulan akhir dari jawaban diatas adalah terbukti bahwa barisan $(x_n) = \left(\frac{(-1)^n}{n}\right), \forall n \in \mathbb{N}$ Konvergen. ~~Dengan~~ Konvergen di titik 0.

Translation:
The conclusion from the answer above is that it is proven that the sequence $(x_n) = \left(\frac{(-1)^n}{n}\right), \forall n \in N$ converges to 0.

Figure 5. A1 answer at the attributing stage

At the end of the answer, students provided conclusions regarding the convergence of the sequence. Figure 5 explains that the sequence (x_n) converges to a point. In this section, students could mention the ending of the serial combination. The sequence (x_n) converges to point 0. This conclusion confirms that the evidence is based on the main question. Another activity students carried out was rechecking their work results. Based on interviews, the researchers found that the students still had spare time after working on the questions. Thus, the students checked all the steps for completion. The following is the obtained information from the results of the interview.

R : Are you sure about the answers you have written?
A1 : I'm sure, sir, it's by the question at the beginning. I also proofread it before collecting it.
R : What parts did you check?
A1 : Usually, I check the steps, sir. Checking whether there are errors in using definitions or the like.
R : Do you usually do this activity or not?
A1 : Yes, sir, if there is still time.

The A1 evidence process demonstrates how the student divided the problem into several parts and correctly applied previously acquired mathematical concepts. This approach illustrates the thematization of knowledge and the construction of evidence within analytical thinking. Gunbatar (2018) also found that students with high self-efficacy could solve problems analytically. Analytical thinking ability often reflects a person's level of self-efficacy. Students in the high self-efficacy category confidently and accurately demonstrated every stage of the analytical thinking process. Their retained knowledge helped them generate solution ideas, showing that prior knowledge positively influenced their thinking. Veeck et al. (2023) emphasized that individuals who feel confident in their initial abilities tend to use that confidence to enhance their analytical thinking and solve problems independently.

Figure 6 shows how a student with high self-efficacy thinks analytically, based on APOS theory, while Table 3 explains the symbols used. Students with high self-efficacy revealed a strong mental schematic structure by observing sequence patterns and applying several related mathematical concepts. Their first step to proving convergence involved examining the pattern of the given sequence, which consisted of two distinct parts: odd and even terms. They applied the definition of a sequence limit to each part to facilitate proof. They also used relevant mathematical concepts, including absolute value and the Archimedean property. These activities enabled them to prove the convergence of the sequence analytically, relying on retained and relevant knowledge. This verification process aligns with the application of schemas in the analytical thinking process (Arnon et al., 2014). The schema showed an organized framework made up of various mathematical ideas, created through the interaction of action, process, object, and schema.

Table 3 further explains how the APOS components correspond to the indicators of analytical thinking. The collecting indicator aligns with the action category, while differentiating and organizing fall under the process category. The attributing indicator represents the object category. Altogether, the schema aspect of APOS theory encapsulates all cognitive activities involved in collecting, differentiating, organizing, and attributing.

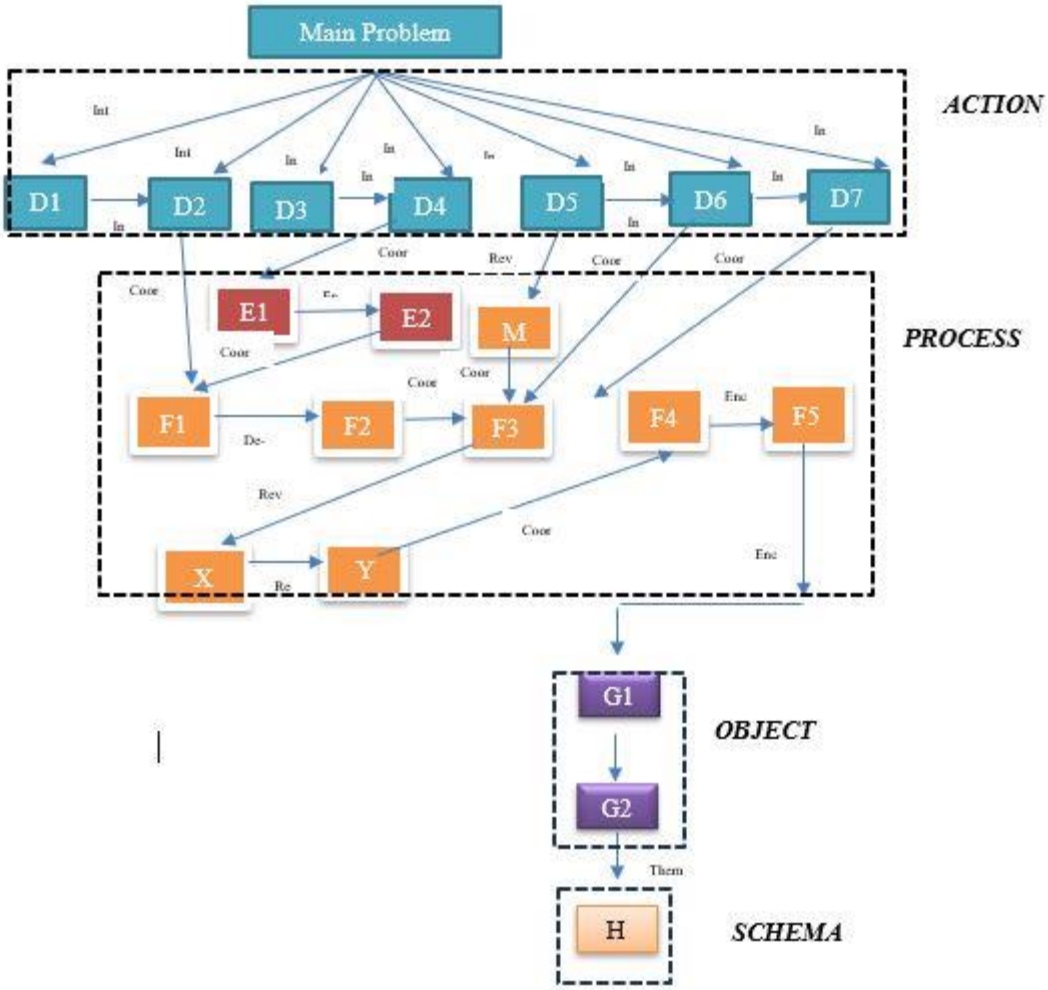


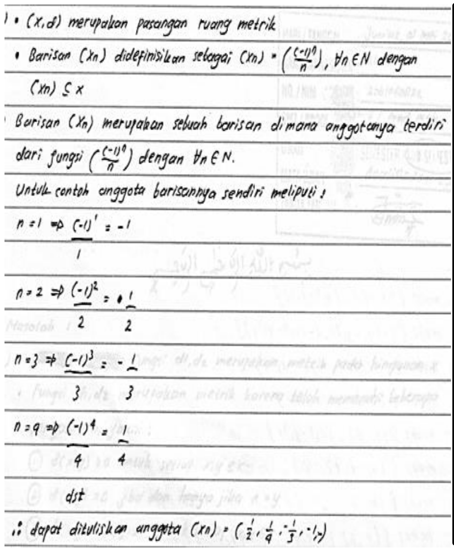
Figure 6. Analytical thinking process of A1

Table 3. The Information Of Codes

Code	Information	Stage	APOS Components
D1	Writing the main questions related to proving the convergence of sequences	*Collecting*	*Action*
D2	Explaining the main topic of the problem		
D3	Writing the sequence (x_n)		
D4	Writing several members of the sequence (x_n)		
D5	Explaining the function *d* is a metric on *X*		
D6	Writing the definition of sequence limit or sequence convergence		

D7	Explaining the initial idea to apply, such as applying the definition of sequence limit		
E1	Observing the members of the sequence (x_n) based on the value of *n*	*Differentiating*	*Process*
E2	Defining the sequence (x_n) based on odd and even *n* values		
M	Recalling the definition of metric space		
F1	Writing the assumption that the sequence (x_n) is convergent	*Organizing*	
F2	Proving the convergence assumption division of two parts based on odd and even *n*-values		
F3	Proving each part of the sequence, such as odd and even with the definition of the limit of the sequence		
X	Remembering and using properties at absolute values		
Y	Remembering and applying Archimedes' properties of nature correctly		
F4	Writing the evidence process confidently, systematically, and in detail		
F5	Providing initial conclusions regarding the evidence of convergence		
G1	Writing the conclusion that the sequence (x_n) converges to 0	*Attributing*	*Object*
G2	Examining in detail each step of the evidence		
H	Creating limitations for definitions, absolute value properties, and Archimedes' properties are the main elements in proving the convergence of sequences.		*Schema*
Int	Interiorizing		
Coor	Coordinating		
Rev	Reversing		
Enc	Encapsulating		
De Enc	De encapsulating		
Them	Thematizing		

Medium Self-Efficacy Category



Translation:

- (X, d) is a metric space pair
- The sequence (x_n) is defined as $(x_n) = \left(\frac{(-1)^n}{n}\right), \forall n \in N$ by $(x_n) \subseteq X$
- Sequence (x_n) is a sequence with members consisting of functions $\left(\frac{(-1)^n}{n}\right)$ with $\forall n \in N$.
- Examples of line members include:

$$n = 1 \Rightarrow \frac{(-1)^1}{1} = -1$$
$$n = 2 \Rightarrow \frac{(-1)^2}{2} = \frac{1}{2}$$
$$n = 3 \Rightarrow \frac{(-1)^3}{3} = -\frac{1}{3}$$
$$n = 4 \Rightarrow \frac{(-1)^4}{4} = \frac{1}{4}$$

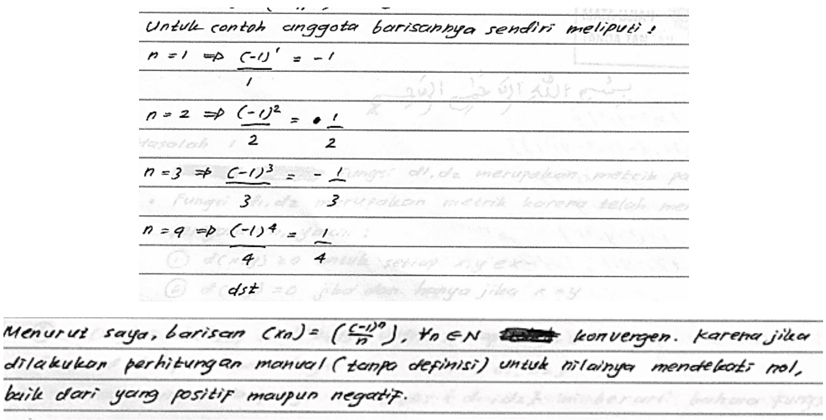
Can be described as a member of $(x_n) = \left(\frac{1}{2}, \frac{1}{4}, -\frac{1}{3}, -1\right)$

Figure 7. A2 answer at the collecting stage

Figure 7 explains students' cognitive activities at the collecting stage. Students successfully wrote the related information to the main problem. The written information includes metric spaces, sequences (x_n), and sequence members. As an initial illustration, students wrote the four members of the first line for n = 1, 2, 3, 4. The results of the interview directly obtained other information. The attempt to explain the main question began with proving the convergence of the sequence (x_n) and expressing the initial idea to use. The respondent explained that the idea to use was – the definition of convergent sequence. Here are the interview excerpts.

R : What is being asked in this problem?
A2 : Proving the convergence of the sequence (x_n).
R : What can you write about the problem?
A2 : I wrote down its relation to metric space, the sequence (x_n) was clarified again, and I wrote down several members of (x_n) as many as the first four.
R : Why did you write down the members?
A2 : I want to see what number the sequence pattern leads to. This helps to confirm my answers.
R : What initial idea will you use to prove the sequence's convergence (x_n)?
A2 : I will use the definition of a convergent sequence studied previously.

The students wrote the members of the sequence to see the sequence pattern. Based on Figure 8, students looked at the sequence members; if n is an odd number, the sequence member is opposing. If n is an even number, its members have a positive value. Based on this pattern, students could explain the division of the problem into two parts: odd and even n. Students also explained the exact direction of the solution by proving the convergence of each odd and even part using the definition of a sequence limit. The student reinforced that if the process continued, the line members would approach 0. Here are the explanations of the student while being interviewed.



Untuk contoh anggota barisannya sendiri meliputi :

$n=1 \Rightarrow \frac{(-1)^1}{1} = -1$

$n=2 \Rightarrow \frac{(-1)^2}{2} = \frac{1}{2}$

$n=3 \Rightarrow \frac{(-1)^3}{3} = -\frac{1}{3}$

$n=4 \Rightarrow \frac{(-1)^4}{4} = \frac{1}{4}$

dst

Menurut saya, barisan $(x_n) = \left(\frac{(-1)^n}{n}\right)$, $\forall n \in N$ konvergen. karena jika dilakukan perhitungan manual (tanpa definisi) untuk nilainya mendekati nol, baik dari yang positif maupun negatif.

Translation:

- Examples of line members include:

$n = 1 \Rightarrow \frac{(-1)^1}{1} = -1$

$n = 2 \Rightarrow \frac{(-1)^2}{2} = \frac{1}{2}$

$n = 3 \Rightarrow \frac{(-1)^3}{3} = -\frac{1}{3}$

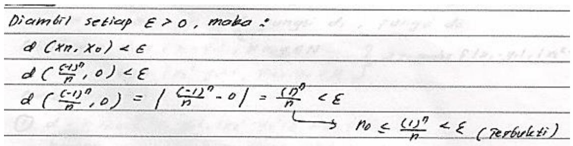
$n = 4 \Rightarrow \frac{(-1)^4}{4} = \frac{1}{4}$

The sequence $(x_n) = \left(\frac{(-1)^n}{n}\right), \forall n \in N$ is convergent. If you do a manual calculation (without definition), the value is close to zero, both positive and negative.

Figure 8. A2 answer at the differentiating stage

R : What can be obtained from the pattern of the sequence members?
A2 : If n is odd, then the value x_n<0. However, if n is even then x_n>0.
R : What can you conclude?
A2 : Proving the convergence of sequences is divided into two categories, namely for odd and even n.
R : What will be used to prove the convergence of a sequence?
A2 : I still use the definition of sequence limit and Archimedes' property.
R : Where will the line members go if you continue the process?
A2 : The line goes to number 0.

In the differentiating stage, students with the medium self-efficacy category did not divide complex problems into several parts in writing. However, based on interviews, students could explain the solution's direction by looking at the line members, consisting of two parts. The underlying necessity was the need for more experience and knowledge regarding the studied material. Paleeri (2015) found that individual knowledge was useful for breaking down complex problems into parts to solve. The retained knowledge became the basis for the problem-sharing process to solve problems more systematically. Chonkaew et al. (2016) and Anwar and Mumthas (2014) found the differentiating stage positively influenced the complex problem solution. The students coordinated two structures in action and then conducted the process of interiorizing two or more obtained information in the previous stage to produce a new structure (Mudrikah, 2015).


Diambil setiap $\varepsilon > 0$, maka :
$d(x_n, x_0) < \varepsilon$
$d\left(\frac{(-1)^n}{n}, 0\right) < \varepsilon$
$d\left(\frac{(-1)^n}{n}, 0\right) = \left|\frac{(-1)^n}{n} - 0\right| = \frac{(1)^n}{n} < \varepsilon$
$\rightarrow n_0 \leq \frac{(1)^n}{n} < \varepsilon$ (Terbukti)

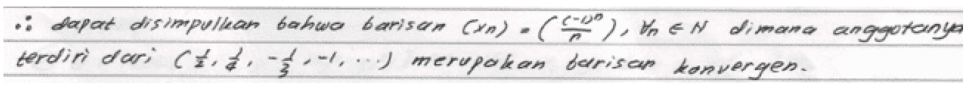
Translation:
Taken every $\varepsilon > 0$, then:
$d(x_n, x_0) < \varepsilon$
$d\left(\frac{(-1)^n}{n}, 0\right) < \varepsilon$

$d\left(\frac{(-1)^n}{n}, 0\right) = \left|\frac{(-1)^n}{n} - 0\right| = \frac{(1)^n}{n} < \varepsilon \left(n_0 \leq \frac{(1)^n}{n} < \varepsilon\right)$

Figure 9. A2 answer at the organizing stage

Figure 9 explains students' answers at the organizing stage. Students used the definition of sequence limits and Archimedes' property to prove the convergence of the sequence (x_n). Students could also claim that the sequence (x_n) converges to 0 and then use the absolute value rule. Several things required corrections in carrying out algebraic operations and assistance to write the obtained steps in detail. The results of interviews with students explored information about the evidence process. The interview revealed that students needed more confidence in their evidence, so there were several wrong steps.

R *: Explain your ideas for proving the sequence (x_n) is convergent.*
A2 *: I claim that the sequence (x_n) converges to 0, and then I prove it.*
R *: What do you use to prove it?*
A2 *: I use the definition of limit and Archimedes'' property.*
R *: Are you sure the answer is correct?*
A2 *: Not sure, sir, there are evidence steps that still need to be determined. I should have proven every odd and even part.*
R *: In your opinion, have the answers been written in detail?*
A2 *: I don't think so, sir.*


∴ dapat disimpulkan bahwa barisan $(x_n) = \left(\frac{(-1)^n}{n}\right)$, $\forall n \in N$ dimana anggotanya terdiri dari $\left(\frac{1}{2}, \frac{1}{4}, -\frac{1}{3}, -1, \cdots\right)$ merupakan barisan konvergen.

Translation:
It can be concluded that the sequence $(x_n) = \left(\frac{(-1)^n}{n}\right), \forall n \in N$, whose members consist of $\left(\frac{1}{2}, \frac{1}{4}, -\frac{1}{3}, -1\right)$, is a convergent sequence.

Figure 10. A2 answer at the attributing stage

Figure 10 shows the last activity by students, providing conclusions. Students wrote the conclusion that the sequence (x_n) was convergent. This conclusion is in line with the given question. Students completed the solution as requested. In addition, before collecting the worksheets, students rechecked each evidence step, starting from the given information, the given questions, and the steps to prove. The students explained that the process of rechecking answer results was carried out every time they worked on a mathematics problem to ensure the correctness of the explanation and the connection to the given question.

R : *What can you conclude from the results obtained?*
A2 : *The sequence (x_n) convergent.*
R : *After you finished working on it, did you correct it again?*
A2 : *Yes, sir.*
R : *What did you correct?*
A2 : *Start from what is known and asked, then move on to the steps.*
R : *Do you usually carry out re-examinations?*
A2 : *Yes, sir. I usually check again just to be sure.*

In the analytical thinking process, A2 performed the division of problems into systematic parts. The student applied the mathematical concepts correctly at the organizing stage. Figure 11 shows the analytical thinking analysis based on the APOS with successful evidence of the sequence's convergence analytically and correctly. Table 4 continues the explanation of symbols in the APOS theory.

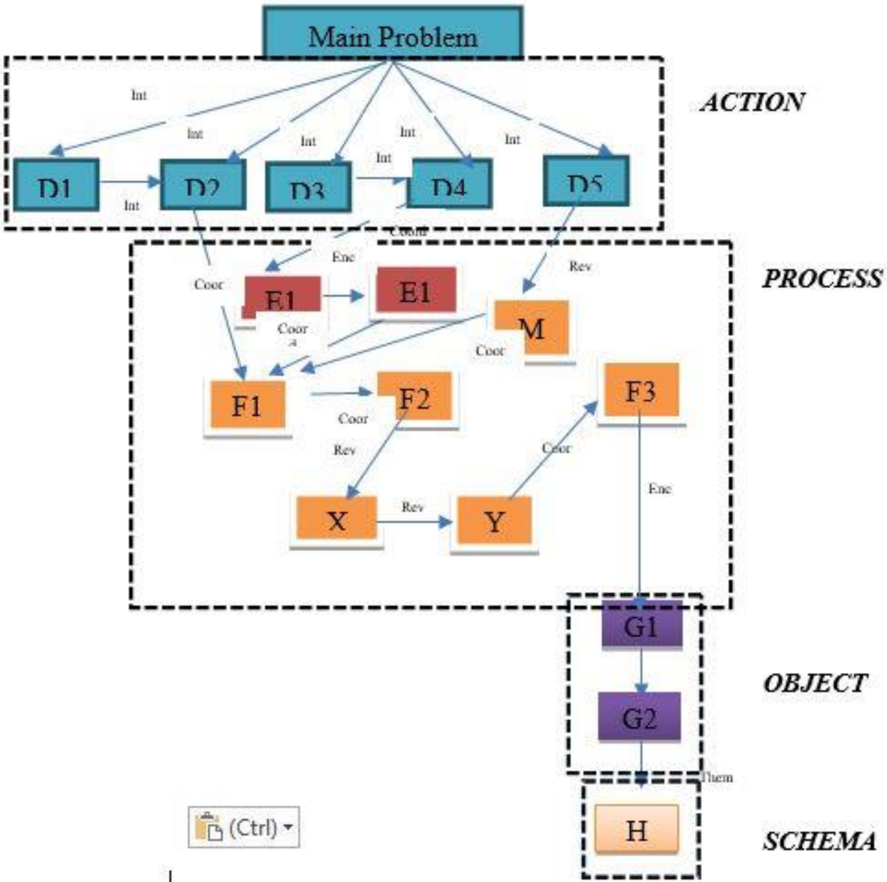


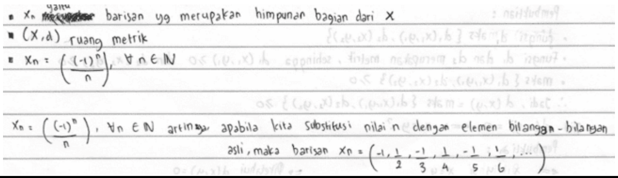
Figure 11. Analytical thinking process of A2

Table 4. The information of codes

Code	Information	Stage	APOS Components
D1	Writing the main questions related to proving the convergence of sequences	*Collecting*	*Action*
D2	Explaining the main topic of the problem		
D3	Writing the sequence (x_n)		
D4	Writing several members of the sequence (x_n)		
D5	Explaining the function d is a metric on X		
M	Remembering the definition of a metric function	*Differentiating*	*Process*
E1	Observing the members of the sequence (x_n) based on the value of n		
F1	Writing the assumption that the sequence (x_n) is convergent	*Organizing*	
F2	Proving the convergence of the sequence		
X	Remembering and using properties at absolute values		
Y	Remembering and applying Archimedes' properties of nature correctly		
F3	Writing the evidence process uncertainly and not in detail. There are still some incorrect parts.		
G1	Writing the conclusion that the sequence (x_n) converges to 0	*Attributing*	*Object*
G2	Examining in detail each step of the evidence		
H	The definitions of limit, absolute value properties, and Archimedes' property are applied to prove sequences' convergence. However, some parts still need to be corrected using the three concepts.		*Schema*
Int	Interiorizing		
Coor	Coordinating		
Rev	Reversing		
Enc	Encapsulating		
De Enc	De encapsulating		
Them	Thematizing		

Based on table 4, the collecting indicators are included in the action category, starting from D1 to D5. The process aspect consists of cognitive activities M to F3 which are included in the category of differentiating and organizing indicators. The object aspect includes the overall cognitive activity on the attributing indicator. The schema aspect part includes all cognitive activities from collecting indicators to attributing.

Low Self-Efficacy Category

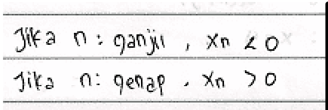

• x_n yaitu barisan yg merupakan himpunan bagian dari X
• (X,d) ruang metrik
• $x_n = \left(\frac{(-1)^n}{n}\right), \forall n \in N$
$x_n = \left(\frac{(-1)^n}{n}\right), \forall n \in N$ artinya apabila kita substitusi nilai n dengan elemen bilangan-bilangan asli, maka barisan $x_n = \left(-1, \frac{1}{2}, \frac{-1}{3}, \frac{1}{4}, \frac{-1}{5}, \frac{1}{6}, \ldots\right)$

Translation:
- x_n is a sequence that is a subset of X
- (X, d) metric space
- $(x_n) = \left(\frac{(-1)^n}{n}\right), \forall n \in N$
- $(x_n) = \left(\frac{(-1)^n}{n}\right), \forall n \in N$ means that if we substitute the value of n with elements of natural numbers, the sequence $x_n = \left(-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \frac{1}{6}, \ldots\right)$

Figure 12. A3 answer at the collecting stage

Students with low self-efficacy category could write the given information and explain the given question. Figure 12 shows the students write the meaning of the sequence (x_n) , pairs of metric spaces, rewrite the sequence (x_n) , and the sequence members. Students wrote the sequence members to determine the leading number of the sequence pattern. This can strengthen the initial assumption of sequence convergence. The interviews with students showed that the obtained information assisted in finding initial ideas although the students did not use all of them. The students also did not use all the information to clarify. Apart from that, the students also explained the objective of the problem based on the attempt to prove the sequence convergence. Here are the interview results.

R : In your opinion, what is being asked in the problem?
A3 : I was asked to prove that the sequence (x_n) is convergent.
R : What information can you write down?
A3 : I tried to write about metric spaces and sequence members.
R : Does this information help you in finding initial ideas?
A3 : Yes, sir, because I understand the direction of the row pattern. Which way does the series converge? This is all I can explain, sir

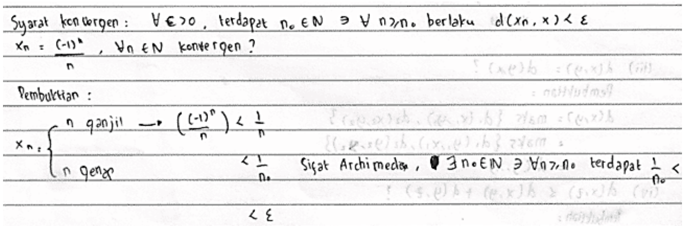

Jika n : ganjil , $x_n < 0$
Jika n : genap , $x_n > 0$

Translation:
If n: odd, $x_n < 0$
If n: even, $x_n > 0$

Figure 13. A3 answer at the differentiating stage

After writing the members of a set, the next stage was to divide the sequence pattern into two parts. Figure 13 shows the students can define the other parts of a sequence. If n is odd, then x_n<0, while x_n>0 if n is even. This division facilitated students to carry out mathematical evidence. The results of interviews with students explained similar information that facilitated students to verify.

R : What do you do next in the process of proving the convergence of the sequence?
A3 : After identifying the members, I provide a simple conclusion based on the n value.
R : Can you try to explain in detail?
A3 : If I substitute n odd, the sequence obtained has a negative value. Conversely, if n is even, then the sequence is positive.
R : Can this definition help you?
A3 : Yes, sir, this helps me in the further evidence process. To prove the convergence of the sequence, it is divided into two case studies, odd and even.



Translation:
Convergent condition: $\forall\varepsilon > 0$, exist $n_0 \in N, d(x_n, x) < \varepsilon$.
$(x_n) = \left(\frac{(-1)^n}{n}\right), \forall n \in N$ A convergent?
Evidence:
$$x_n = \begin{cases} n\ odd \rightarrow \left(\frac{(-1)^n}{n}\right) < \frac{1}{n} < \frac{1}{n_0} < \varepsilon\ (Archimedes\ Property) \\ n\ even \end{cases}$$

Figure 14. A3 answer at the the organizing stage

At the organizing stage, students wrote the correct definition of sequence convergence. Figure 14 shows students also use Archimedes' properties. The definition of Archimedes properties is written correctly. However, the students could not apply the evidence correctly. The students had to write the solution steps systematically but they were unclear. Thus, they needed to correct the evidence. The students could also prove based on the promoted parts in the previous stage, namely for odd and even n. The solution steps need to be more detailed with correct written definitions. The results of the interviews found that students had no proven habits by using definitions. On the other hand, the convergence material tended to be new and made them have difficulties.

R : What do you use to prove the convergence of a sequence?
A3 : I use the convergent definition and Archimedes' property.
R : Can you apply it correctly to the problem?
A3 : I doubt it, sir.
R : Why are you not sure?

A3 : I could be more fluent in proving properties or theorems. Apart from that, I have only just studied the material on sequence convergence, so it isn't easy, sir.

The final stage is attribution. In the answer sheet, the students had to provide clear and visible conclusions. Apart from that, students also did not carry out a thorough re-examination. The interviews with students suggested they check because they did not need to figure out writing. Apart from that, the limited time made the students have difficulties checking the completion steps. However, students usually provided conclusions and correct answers on other occasions. Here are the interview results.

R : Before the answers are collected, do you recheck the results of the work?
A3 : No, sir.
R : Why don't you do that?
A3 : I'm unsure about the explanation, so I'm not motivated to double-check.
R : Do you usually give conclusions and check answers after finishing?
A3 : Yes, sir. apart from the above, the processing time has also run out.

A3's analytical thinking process does not run smoothly because the application of mathematical concepts is not implemented excellently. Figure 15 shows no appropriate organizational stage that makes no evidence for the analytical process of solving the problem. The thought process analysis is based on the APOS theory below and table 5 describes the description of the symbols used.

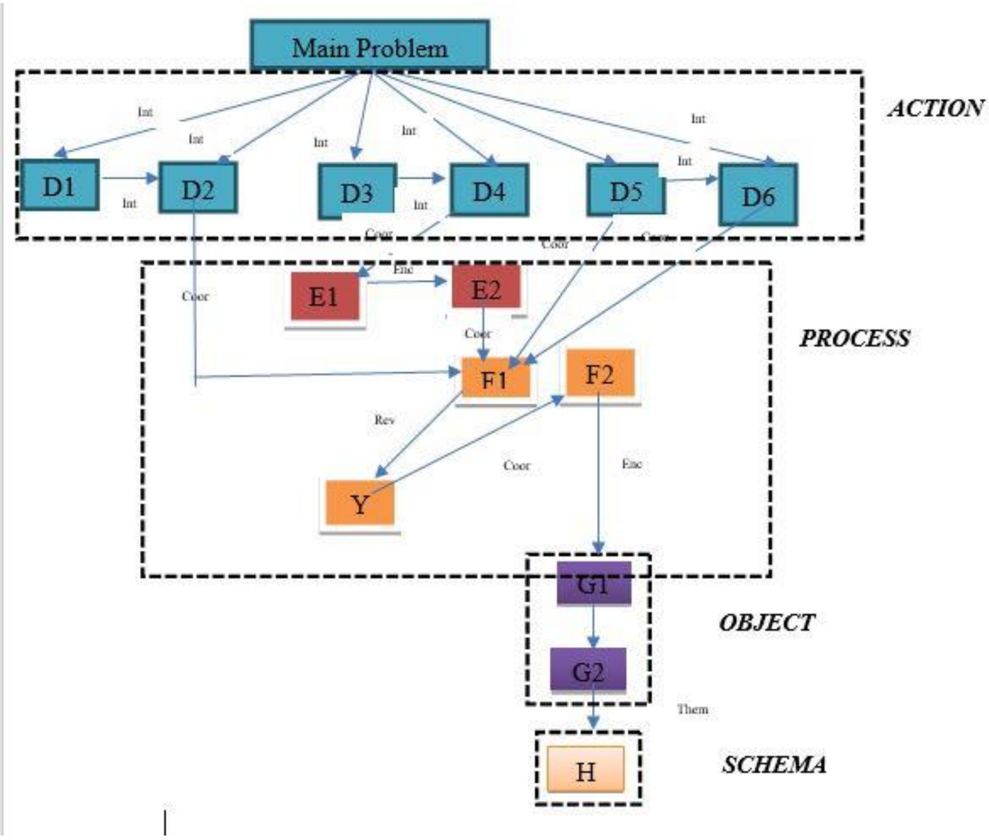


Figure 15. Analytical thinking process of A3

Table 5. The information of codes

Code	Information	Stage	APOS Components
D1	Writing the main questions related to proving the convergence of sequences	*Collecting*	*Action*
D2	Explaining the main topic of the problem		
D3	Writing the sequence (x_n)		
D4	Writing several members of the sequence (x_n)		
D5	Writing the definition of sequence limit or sequence convergence		
D6	Explaining the initial idea to use by applying the definition of sequence limit		
E1	Observing the members of the sequence (x_n) based on the value of *n*	*Differentiating*	*Process*
E2	Defining the sequence (x_n) based on odd and even *n* values		
F1	Proving each part of the sequence is odd and even with the definition of the limit of the sequence	*Organizing*	
Y	Remembering and applying Archimedes' property of nature correctly		
F2	Writing the evidence process confidently, systematically, and in detail		
G1	Writing the conclusion that the sequence (x_n) converges to 0	*Attributing*	*Object*
G2	Examining in detail each step of the evidence		
H	Creating limitations of definitions, absolute value properties, and Archimedes' properties are the main elements in proving the convergence of sequences.		*Schema*
Int	Interiorizing		
Coor	Coordinating		
Rev	Reversing		
Enc	Encapsulating		
De Enc	De encapsulating		

Students in the low self-efficacy category struggled to apply mathematical concepts independently and required assistance to solve problems. They needed to write answers systematically and in detail. However, their uncertainty hindered the effectiveness of their analytical thinking process, as they lacked habitual engagement with related mathematical concepts. Gloudemans et al. (2013) emphasized that individuals’ prior experience with mathematical concepts significantly influences their confidence in problem-solving. Similarly, Wijaya et al. (2023) found that students with limited knowledge and experience often needed guidance to break down complex problems into simpler components. Unlike students in the high self-efficacy category, who used their confidence and knowledge effectively during the differentiating stage, students with low self-efficacy faced difficulties in organizing the steps for constructing a valid proof. Consequently, their mental structures comprising the action, process, and object stages remained underdeveloped (Maharaj, 2014).

Table 5 illustrates the correlation between analytical thinking indicators and APOS theory. The action component includes all cognitive activities related to the collecting indicator, from D1 to D6. The process component encompasses differentiating and organizing activities, while the object component aligns with attributing indicators, specifically G1 and G2. The schema component integrates all four indicators—collecting, differentiating, organizing, and attributing—providing a holistic representation of the student's cognitive activity.

In addition to self-efficacy, other factors such as learning motivation and the use of innovative learning strategies also influence students' analytical thinking. Miele and Wigfield (2014) concluded that students' intrinsic motivation driven by internal enjoyment and desire to solve problems enhances their analytical capabilities. Demir (2022). supported the findings by stating that students with high analytical thinking also tend to demonstrate strong critical thinking skills. Furthermore, Brandt and Lorié (2024) asserted that instructional approaches can significantly enhance analytical thinking. Methods such as problem-based learning (Suyatman et al., 2021; Theabthueng et al., 2022) and inquiry learning (Ramadani et al., 2021) have proven effective in fostering deeper analytical engagement.

▪ CONCLUSION

Analytical thinking involves four stages: collecting, differentiating, organizing, and attributing. Students in the high and medium self-efficacy categories successfully completed each of these stages. They collected relevant information related to the problem in a clear and detailed manner (collecting). They divided the problem into two parts, each requiring separate proof of convergence (differentiating). They carried out the proof process systematically, applied several mathematical concepts accurately (organizing), and demonstrated strong confidence throughout. At the end of the process, they provided appropriate conclusions and reviewed their work for accuracy (attributing).

In contrast, students with low self-efficacy needed assistance in applying relevant mathematical concepts during the proof process. Their limited confidence in their conceptual understanding hindered their problem-solving abilities. Although they attempted to demonstrate sequence convergence, their evidence remained incomplete. These students managed to perform certain tasks in the analytical thinking stages but struggled to fully engage with each step. In the differentiating stage, they could divide the problem into smaller parts, which made problem-solving more manageable. However, they failed to connect all steps into a cohesive and comprehensive proof.

The researchers explored the analytical thinking process using APOS theory, which illustrates the cognitive structure involved in applying mathematical concepts during problem-solving. This framework helped identify how students at different self-efficacy levels approached the components of action, process, object, and schema.

This research contributes to the scientific understanding of analytical thinking processes in relation to self-efficacy. The findings suggest the potential to design a learning model grounded in analytical thinking stages: collecting, differentiating, organizing, and attributing. Educators can adapt this model to foster students' analytical thinking skills at the primary, secondary, or higher education levels.

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