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Didactic Praxeological Analysis of Indonesian and Singaporean Mathematics Textbooks: Mathematical Practices in Transformation Geometry

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Abstract: This study aims to compare the contents of mathematics textbooks from Indonesia and Singapore, focusing on the activities of learning mathematics on transformation geometry material at the junior secondary level. We adopt the concept of didactic praxeology, which is a sub-theory of the Anthropological Theory of The Didactic (ATD). Using the ATD framework, our approach considers textbooks as empirical sources that can reveal knowledge that will be taught in the didactic transposition process. This study uses content analysis techniques with a qualitative approach. Textbooks are analyzed using praxeological analysis; types of tasks and techniques (praxis block) and technology and theory (logos block). The results of the didactic praxeological analysis show that there are differences in mathematical practices between the two textbooks. These differences can be used to complement the activity section in understanding the material. We further discuss our findings to outline the theoretical and methodological aspects of this study, which can potentially contribute to future research on textbooks in mathematics education.

Keywords: ATD, praxeology, textbook, transformation geometry.

▪ INTRODUCTION

Mathematics as one of the subjects studied in schools has various objectives, depending on the context. In general, the objectives of learning mathematics include the development of analytical and problem-solving skills, improving critical and analytical thinking skills, improving communication skills, understanding science and technology, understanding the world around us, and applications in various fields of science and everyday life (NCTM, 2000; Kilpatrick et al., 2001; Boaler, 2015). To achieve these goals, various types of activities are needed that involve the use of mathematical concepts, skills, and methods. This can include practice problems, experiments, games, projects, or discussions designed to develop mathematical understanding and abilities. Mathematical practices are often used in educational contexts to help students understand concepts, understand the language of mathematics, develop mathematical reasoning, and apply them in everyday life (Van de Walle, 2013; Swan, 2005; Boaler, 2015; Elisya et al., 2024).

The question arises regarding what activities need to be carried out related to the material being studied. Before a mathematical concept is taught in class, there is a transformation of the definition and description of the concept in teaching practices involving teachers and other parties involved in the field of teaching (Chevallard & Bosch, 2014). This kind of transformation can be called didactic transposition (Chevallard, 2006). Achiam (2014) explains that the process of didactic transposition begins with an object of scientific knowledge (usually resulting from research), then selected and rearranged to become part of the science to be taught (for example part of the curriculum), then translocated and transformed into knowledge that is taught (for example classroom), finally becoming knowledge that is learned (Figure 1). The process of didactic transposition occurs every time someone intends to spread or teach science to

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others. An object of knowledge is translocated between contexts, and undergoes transformation (or adaptation) throughout the process, which finally becomes knowledge acquired by students.

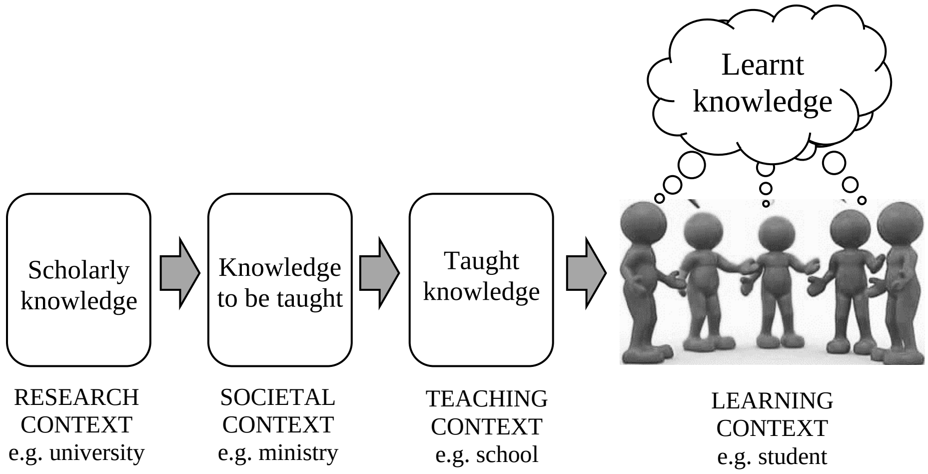


Figure 1. Didactic transposition process

The theoretical framework underlying this research is the Anthropological Theory of the Didactic (ATD) (Chevallard, 2006; Achiam, 2014; Wang et al., 2023; Suryadi et al., 2023). This theory investigates human and didactic activities through praxeology, focusing on the components, dynamics, and conditions that enable their existence and development in a particular institutional environment. Praxeology is a basic part for analyzing human action in general (Chevallard, 2006). The main idea of praxeology as an important means for analyzing all human activities consists of and connects two parts, namely practice (know-how) and theory (knowledge) (Bosch & Gascón, 2014).

Several researchers (Chevallard, 2006; Bosch & Gascón, 2006, 2014; Winsløw, 2012; Chevallard & Bosch, 2020; Suryadi et al., 2023; Hendriyanto et al., 2023) explain that praxeology consists of two main components, namely, the practical component (praxis block) and the theoretical component (logos block) (Figure 2). Each component is divided into two elements. The praxis block consists of the type of task and how to do the task (a set of techniques). While the theory block consists of technology and theory. Etymologically, technology is the technique used to carry out tasks, while theory functions as a basis and support for the technological discourse used to justify these techniques.

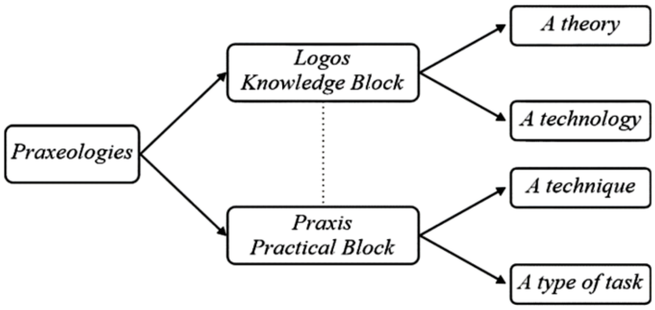


Figure 2. Praxeology model

Based on Figure 2, praxeology is formed from four components, namely the type of task, a set of techniques, technology, and the theoretical basis underlying the technology used. According to dos Santos & Farias (2022), the four components (type of task, a set of techniques, technology, and theoretical basis) are a model for studying human knowledge. Suryadi et al., (2023) explain the praxeology model, namely (ti, τ, θ, Θ), where T is a set of mathematical tasks, ti is one element of T, τ is a set of techniques in completing tasks, θ is a technological discourse, and Θ is a theoretical discourse underlying the technological discourse. Therefore, each ti in T has a set of techniques τ, a technological discourse θ, and a theoretical discourse Θ that underlie the technological discourse in forming a praxeology. The praxis block is usually explicit, but the logos block is often implicit and not easily identified and interpreted (Table 1).

Table 1. Praxeology (description adapted from chevallard & sensevy, 2014)

Praxis Block		**Logos Block**	
Type of Task (T)	Technique (τ)	Technology (θ)	Theory (Θ)
Problems of a certain type	A way to do this type of task	A way to explain and justify the technique	To explain, justify, or produce any part of the technology that may sound unclear or missing

Mathematics textbooks are one example of didactic transformation that plays an important role in mathematics teaching. The didactic transposition series relies heavily on the use of mathematics textbooks (Kang & Kilpatrick, 1992). Mathematics textbooks are widely used as the main source of teaching and curriculum ideas (Pepin et al., 2013). Textbooks are powerful tools to help students understand and master mathematics material (Purnomo et al., 2019; Weinberg & Wiesner, 2011). The use of textbooks has an impact on teachers' teaching in explaining mathematics topics and using their knowledge of learning trajectories in the classroom (Valverde et al., 2002).

Research on mathematics textbooks has been widely recognized as a field of scientific research (Fan, 2013; Fan et al., 2018; Jones et al., 2014; Yunianta et al., 2023; Fathimah et al., 2024). Some researchers believe that studying textbooks from different countries can provide insight and identify similarities and differences in mathematics learning (Haggarty & Pepin, 2002; Kul et al., 2018). Many studies have been conducted on the comparison of mathematics textbooks between countries, such as a comparison of textbooks from England and Shanghai in understanding linear functions (Wang et al., 2015), a comparison of junior high school textbooks in France and Japan on the nature of proofs to be taught in geometry (Miyakawa, 2016), a comparative analysis of the content of first grade elementary school mathematics textbooks in Iran, Japan and America (Reyhani & Izadi, 2018), a comparison of textbooks from the perspective of mathematical creativity in Indonesia and Singapore (Ramelan & Wijaya, 2019), a comparison of junior high school textbooks in Japan and England using a praxeological analysis of symmetry and transformation in geometry (Takeuchi & Shinno, 2019), an analysis of Singaporean and Spanish textbooks on arithmetic material (Vicente et al., 2022), a comparison of fifth grade mathematics textbooks used in Turkey and Singapore (Toprak & Özmantar, 2022), a comparison of Singaporean and Indonesian textbooks using praxeology on set material (Hendriyanto et al., 2023), a comparative study of critical thinking instruction in Japanese and Iranian third-grade mathematics textbooks (Chegini et al., 2023), and many other studies.

Praxiological research through the comparison of mathematics textbooks in Singapore and Indonesia has been conducted by several researchers. Hendriyanto et al. (2023) compared the concepts of Singapore and Indonesian textbooks on the concept of mathematical sets. The results obtained are that there are differences in the presentation of material that considers the culture and character of students, as well as the complexity of task design. Ramelan & Wijaya (2019) compared Indonesian and Singaporean mathematics textbooks on the topic of probability. The aspects studied on mathematical creativity are originality, fluency, flexibility, and elaboration. The results showed that there was a relationship between textbooks and students' mathematics achievement. Indonesian and Singaporean mathematics textbooks both facilitate mathematical creativity. However, Singaporean mathematics textbooks facilitate students' mathematical creativity thinking more.

Comparison of mathematics textbooks in Singapore and Indonesia using praxis on mathematical practices in learning transformation geometry has not been done. Some comparative research of mathematics textbooks on geometry material was conducted by Ogbonnaya (2021), showing the results that the comparison of two textbooks in the applicable curriculum in South Africa contains Euklides geometry content. The material is presented logically and sequentially to enhance learning. Another praxiological study compared Japanese and English mathematics textbooks on symmetry and transformation at junior high school level (Takeuchi & Shinno, 2019). It was found that Japanese textbooks on symmetry and transformation were heavily influenced by the teaching of geometry proofs. While textbooks from the UK on transformation materials are associated with other contexts or content across domains. From this explanation, it is important to analyse textbooks on transformation geometry material. Therefore, praxeological research on transformation geometry material is important to be carried out on textbooks from various other countries, namely through a comparison of mathematics textbooks from Singapore and Indonesia.

Textbooks are products of human thought, action, and behavior recorded in mathematics learning documents. The existence of textbooks is considered important. We view textbooks as one description of how mathematics as an activity can be taught. The same textbook does not always mean providing the same classroom practice (Thompson & Senk, 2014). Moreover, different textbooks will offer different learning opportunities (Sievert et al., 2019). Differences in textbooks and learning experiences will result in different learning achievements. Fischer et al (2015) among others found a significant correlation between students' mathematics learning outcomes and the textbooks used.

The curriculum in each country has its characteristics. In Indonesia, the current curriculum provides greater freedom and flexibility to schools, teachers, and students in the teaching and learning process (Permendikbudristek, 2022). So that the use of mathematics learning strategies will provide a quality learning experience by providing real problems or contexts, encouraging interaction and active participation of students, optimizing the use of resources, and using information and communication technology. Meanwhile, the mathematics curriculum in Singapore is in the form of solving mathematical problems that depend on five interrelated components: concepts, skills, processes, attitudes, and metacognition (Kaur, 2019). In this study, researchers compared mathematics textbooks used in Indonesia and Singapore. The selection of Singapore as a comparison for Indonesia was based on the significant differences between the two countries in the results of the PISA study regarding student mathematics achievement. The results of the OECD report show that Singapore's achievement in mathematics in PISA 2022 is ranked first (OECD, 2023), while Indonesia is far below.

This study is expected to provide an overview of the differences in task design in Indonesian and Singaporean textbooks. The results of the analysis can be used to improve textbook design. Therefore, the purpose of this study is to determine the similarities and differences in the content of mathematics textbooks from Singapore and Indonesia regarding the knowledge of learning transformation geometry. In this article, we explain how praxeology allows us to understand how textbooks can provide an overview of teachers' teaching practices. So it is hoped that these findings can contribute to the development of textbooks.

▪ METHOD

The Anthropological Theory of Didactics (ATD) approach is used in this study. The idea of the didactic transposition process makes it clear that it is impossible to interpret school mathematics correctly without taking into account the phenomena related to the way mathematics is introduced and reconstructed in schools. The didactic transposition process underlines the institutional relativity of knowledge and places didactic problems at the institutional level, beyond the individual characteristics of the subjects of the institution under consideration. According to ATD, every human action can be divided into a series of different types of tasks (Chevallard & Bosch, 2020). In this study, the types of tasks are found in textbooks. Thus, textbooks can be studied using the praxeology method because they are products of human action or activity.

Praxeological analysis in ATD allows us to draw a relationship between institutional and individual praxeology (Bosch & Gascón, 2014). The institution is the government, and the individual is the author of the mathematics textbook. The book is the result of someone's writing inspired by the interaction between the community of authors and educational institutions for didactic purposes. The existence of mathematics textbooks is marked by a reciprocal relationship between individuals and institutions (Chevallard & Bosch, 2020).

The didactic analysis in this study uses Brousseau's (2002) didactic situation theory, by examining three key elements in the learning context, namely: student activities, knowledge needed by students, and teacher instructions. The textbook provides assignment instructions that include three key elements that can be revealed through praxeological analysis and continued with didactic praxeological analysis.

The textbooks analyzed from both countries are books created by government institutions. In Indonesia, the mathematics books selected are books written for the current curriculum, Kurikulum Merdeka for Junior High Schools created by the Ministry of Education, Culture, Research, and Technology (Kemendikbudristek), written by Kristanto et al (2022). In Singapore, all textbooks must be approved by the Singapore Ministry of Education before they can be adopted. Since 2008 and 2014, mathematics teaching materials from Singapore have been used in California and France, including a book entitled New Syllabus Mathematics 7th Edition. This indicates that mathematics textbooks from Singapore have a good reputation (Chang et al., 2018). The mathematics textbooks to be analyzed are presented in Table 2.

Table 2. Analyzed textbooks

Country	Textbook series	Abbreviation	Publisher
Indonesia	*Matematika untuk SMP/MTs Kelas IX. Cetakan pertama, 2022*	MSI	*Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi*
Singapore	New Syllabus Mathematics 7th Edition	NSM	Shinglee

The data analysis procedure in this study consists of praxeological analysis and didactic analysis. The first stage is related to praxeological analysis starting from the selection of mathematics textbooks and determining the focus of the research topic, then entering the task design into the praxeological table, coding the identified findings based on theory, and determining the reliability of the data through theory triangulation. Praxeological analysis consists of: (1) praxeological block analysis; (2) theoretical block analysis; and (3) praxeological comparison of Indonesian and Singaporean textbooks. Furthermore, the second stage of didactic analysis according to Bosch & Gascón, (2014) consists of: (1) practical-technical didactic praxeological block; and (2) technological-theoretical didactic praxeological block.

The didactic praxeological analysis framework related to the textbook analyzed using the Yunianta et al (2023) framework is shown in Figure 3. The results of the didactic praxeological analysis can be used to identify barriers to learning and teaching, as well as formulate recommendations or improvement strategies.

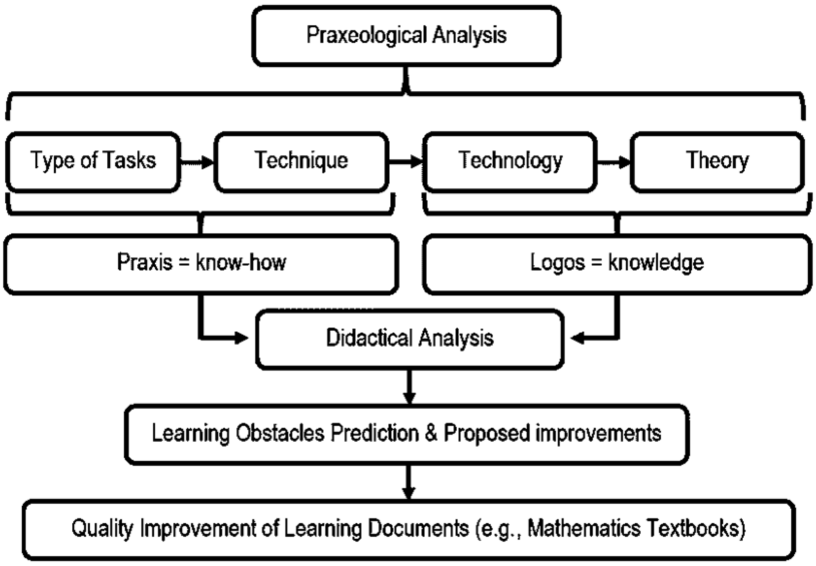


Figure 3. Didactic praxeological analysis framework

The analysis of mathematics textbooks begins with the selection of books according to the chosen topic, namely transformation geometry. The aspects that will be analyzed in Singaporean and Indonesian textbooks are mathematical practices that can facilitate students to understand the material of transformation geometry. Furthermore, the types of tasks and techniques used to complete the tasks in the textbooks of both countries will be identified. This is done to find out the mathematical practices in understanding the material of transformation geometry, which has three themes (Translation, Reflection, and Rotation). Each theme will be analyzed for the types of tasks and techniques used to complete the tasks (Table 3).

Table 3. Geometry transformation topic focus

Indonesia		Singapore	
Sub Chapter	**Objective**	**Sub Chapter**	**Objective**
Translation Reflection Rotation	Describe the geometry of transformations to the coordinate plane and apply it to real problems. The sequence of material delivery begins with translation, then reflection, and rotation Practice mathematics through interactive activities, trying, creative thinking, critical thinking, reflection, and projects	Reflection Rotation Translation	Describe the geometry of transformations to the coordinate plane and apply it to real problems. The sequence of material delivery begins with reflection, then rotation, and translation. Practice mathematics through investigation activities, class discussions, thinking time, journal writing, and performance tasks.

Praxeology here is described as a task that must be completed. The type of task given is done with a technique, namely the methods that students can use to complete the given task. After the technique is selected, technology is needed that is justified by rational reasoning shown in the mathematics textbook. The basis or reference for the technology used is called theory. Furthermore, a review of three main elements is carried out, namely student activities, knowledge needed by students, and teacher instructions in textbooks that involve the results of the disclosure of praxis and logos so as to produce a picture of learning obstacles which then produce recommendations for improvement. More technically it will be explained, the first step is to write down the type of task (T), then the technique (τ) is identified. The technique (τ) used to complete the type of task (T) in the book becomes the technique (τ) that is most likely to be used by students in completing the given practice questions. The selection of the technique (τ) has a reason called technology (θ). Technology (θ) is present to justify the existence of technique (τ). Then from technology (θ) there is a reason called Theory (Θ).

▪ RESULT AND DISSCUSSION

The analysis in this study is divided into two parts, namely praxeological analysis and didactic analysis. The praxeological analysis consists of the analysis of praxis blocks and logos blocks, while didactic analysis analyzes the didactic organization based on the findings of the praxeological analysis. Before conducting the textbook analysis, MSI will represent Indonesian mathematics textbooks, and NSM will represent Singaporean mathematics textbooks. Figures 4 and 5 show the visualization of the introduction to the textbook.

Figure 4. Introduction to Indonesian textbooks

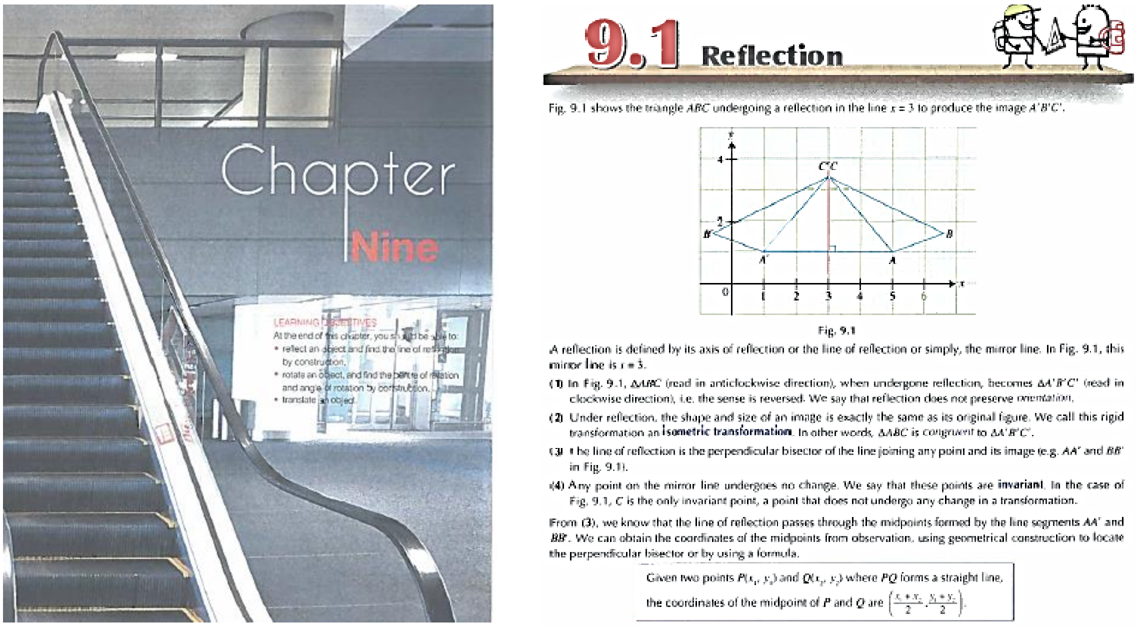


Figure 5. Introduction to Singapore textbooks

The introduction section of the textbooks in Indonesia (MSI) and Singapore textbooks (NSM), is expected to provide students with an overview of the material on transformation geometry. In the MSI textbook, students are introduced to symmetrical forms that can be found in everyday life. In a work of art, symmetrical forms can be found in batik designs. In the form of house architecture, the concept of symmetry can be found in traditional house architecture. In the NSM textbook, at the beginning, the form of line reflection on the coordinate plane is shown. The book provides information that reflection lines can be obtained using observation, geometric construction, or midpoint formulas. From the beginning of the material, differences in ideas contained in each textbook are already visible. The differences are apparent in the order in which the material is presented. The textbook in Indonesia (MSI) begins with the sub-material on translation, reflection, and rotation. While the textbook in Singapore begins with the sub-material on reflection, rotation, and translation.

The difference in the order of the material is shown in Figure 6. This indicates a difference in students' initial starting point in understanding the material on transformation geometry.

a. Table of Contents of Indonesian Textbooks

CHAPTER 9

b. Table of Contents of Singapore Textbooks

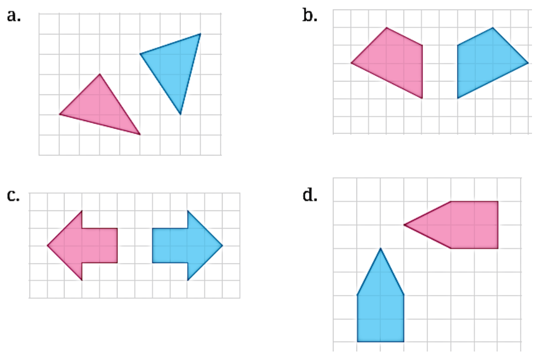
Figure 6. Table of contents of Indonesian and Singapore textbooks

Analysis of Practical Blocks

The type of task (T) in the practical block is the type of task in the mathematics textbook. There is a practical block of learning material for the concept of transformation geometry in MSI presented in eight types of tasks ($T1,T2,\ldots,T8$). The eight types of tasks in MSI are broadly divided into three categories, namely understanding the concept of translation ($T1,T2,T3$), being able to identify the shadow image from the transformation results ($T4,T5$), and being able to make the shadow image from the transformation results ($T6,T7,T8$) (Table 4). Meanwhile, NSM presents six types of tasks ($T1,T2,\ldots,T6$) with different forms. The six types of tasks in the NSM are categorized into three, namely understanding the concept of reflection ($T1,T2,T3$), being able to estimate the shadow of the transformation result ($T4$), and being able to create a shadow of the transformed image ($T5,T6$) (Table 5).

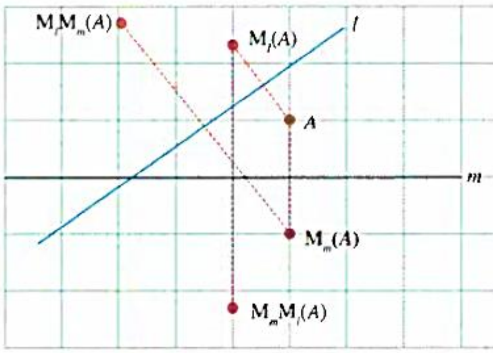
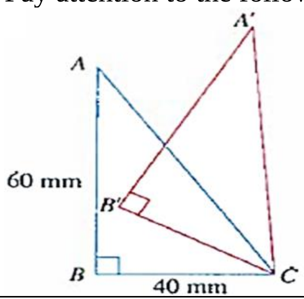
Technique (τ) in the practice block explains the method for solving a particular type of T. The type of technique (τ) used in this study adopts the findings of Takeuchi & Shinno (2020) which consists of four types of techniques (τ), namely perceptual ($\tau1$), physical ($\tau2$), operational ($\tau3$), and algebra ($\tau4$). $\tau1$ is the solution of T using visual assessment based on the appearance of the form presented; $\tau2$ is the solution of T carried out using physical aids such as rulers, compasses, or others, including conducting validation or discussion; $\tau3$ is the solution of T that directs students to conduct investigations or discoveries by developing their understanding, and $\tau4$ is the solution of T that uses mathematical expressions. The description of how to do a type of task in the textbook uses the stages of Van Hiele (1986); Fuys et al. (1988), namely visualization, analysis, sequencing or informal deduction, deduction, and rigor.

Table 4. MSI Praxis block

Type of Task (T)	**Technique (τ)**	**Description of Each Textbook's Techniques (Van Hiele's Theory)**
A square *ABCD* with *A*(1.1). *B*(3.1). *C*(3.3), and *D*(1.3) is translated 4 units to the left and 2 units up.		
T_1: State the above case in translation form $\binom{a}{b}$	$\tau_1. \tau_3$	Writing the translation form based on previous perception (memorial)
T_2: Determine the image of the translation result of square *ABCD*	$\tau_1. \tau_3$	Understanding the shadow of translation form (analysis)
T_3: Draw square *ABCD* and its shadow on the coordinate plane	$\tau_1. \tau_2. \tau_3$	Acting drawing shadows using the processes of perception and memory (analysis)
Pay attention to the following picture 		
T_4: Is the blue image a reflection of the red image?	$\tau_1. \tau_3$	Understanding the relationship between properties and geometric shapes to determine the results of image reflection (visualization)
T_5: If yes, draw the reflection line	$\tau_1. \tau_2. \tau_3$	Understanding the relationship between properties and geometric shapes to determine the results of image reflection (visualization)
Given ΔDEF with the coordinates of the corner points at *D*(3,2), *E*(8,1) and *F*(8,5). Draw the image of the result of the transformation ΔDEF if:		
T_6: Reflected about the *x*-axis then rotated 90° counterclockwise with the center of rotation *O*(0,0)	$\tau_1. \tau_2. \tau_3$	Acting drawing the transformed shadow using the process of perception and memory (analysis)
T_7: Rotated 180° with the center of rotation *O*(0,0) then translated a distance of $\binom{-2}{3}$	$\tau_1. \tau_2. \tau_3$	Acting drawing the transformed shadow using the process of perception and memory (analysis)

T_8: Rotated 90° clockwise with the center of rotation $O(0,0)$ then reflected about the y-axis after which it is translated a distance of $\binom{-2}{3}$	$\tau_1.\ \tau_2.\ \tau_3$	Acting drawing the transformed shadow using the process of perception and memory (analysis)

Table 5. NSM practice blocks

Type of Task (T)	Technique (τ)	Description of Each Textbook's Techniques (Van Hiele's Theory)
Pay attention to the following picture  Point A is reflected on two lines l and m. We present its reflection on line l by M_l and on line m by M_m. So $M_mM_l(A)$ is the reflection of point A on line l followed by line m while $M_lM_m(A)$ is the reflection of point A on line m followed by line l.		
T_1: What do you notice about the two points represented by $M_mM_l(A)$ and $M_lM_m(A)$?	$\tau_1.\ \tau_3$	Understanding the position of two points formed from the results of a transformation (informal deduction)
The coordinates of A and B are (-3,1) and (-1,5) respectively. The line connecting A and B is reflected on the x-axis to $A'B'$		
T_2: Determine the coordinates of A' and B'. $A'B'$ is then reflected on the line $x = 2$ resulting in $A''B''$	$\tau_1.\ \tau_2.\ \tau_3$	Acting drawing the transformed shadow using the process of perception and memory (analysis)
T_3: Determine the coordinates of A'' and B''. Show your work on a piece of graph paper.	$\tau_1.\ \tau_2.\ \tau_3$	Acting drawing the transformed shadow using the process of perception and memory (analysis)
Pay attention to the following picture. 		

The triangle $A'B'C'$ is the image of the triangle ABC under a clockwise rotation of 25º about C.			
T_4:	Calculate, giving your answer correct to the nearest 0.5º. $C\hat{A}A'$	$\tau_1.\ \tau_2.\ \tau_3.\ \tau_4$	Understanding the position of two points formed from the results of a transformation (informal deduction)
A_1 is the translation $\binom{3}{-7}$ and A_2 is the translation that will move the point (2.5) to (5.9).			
T_5:	Find the shadow of point A(2.4) under T_1	$\tau_1.\ \tau_3$	Performing the action of drawing the transformed shadow using the process of perception and memory (analysis)
T_6:	What is the shadow of point B(4.-8) below T_2?	$\tau_1.\ \tau_2.\ \tau_3.\ \tau_4$	

Overall, the MSI practical block from $T1$ to $T8$ is dominated by $\tau1$, $\tau2$, and $\tau3$. This shows that the task design in MSI directs students to conduct investigations or discoveries by developing their understanding. The description of textbook techniques based on Van Hiele's Theory shows the development stage of geometric understanding at the visual (T4,T5) and analysis (T2,T3,T6,T7,T8) stages. The development stage of geometric understanding has not yet reached informal deduction. This results in a lack of learning opportunities for students to achieve high-level abilities. Meanwhile, in NSM, knowledge formation is mainly carried out through observation, identification, and estimation steps. Knowledge formation in the NSM practical block involves all τ, namely $\tau1,\tau2,\tau3,\tau4$.

In the NSM textbook, students are given various experiences in learning transformation geometry material, including students are given activities to conduct experiments using various tools and media to validate and discuss, and students are asked to estimate a given problem. This gives students a learning experience with open answers. Learning opportunities like this can develop and improve high-level thinking skills. Overall, the characteristics of τ that are developed build a structured and continuous learning trajectory according to the intended formulation (theory). The description of the textbook technique based on Van Hiele's Theory shows the stages of development of understanding geometry at the analysis stage (T2,T3,T5,T6) and informal deduction (T1,T4). The stage of development of understanding geometry in the NSM textbook has reached the informal deduction stage. This shows that students' thinking processes have been directed at high-level thinking skills.

Logos Block Analysis

In the context of this study, technology (θ) refers to the justification behind the technique (τ) chosen for each type of task, while theory (Θ) in this context refers to the theoretical basis of the technique. The results obtained in the MSI textbook for type T1 tasks were completed using the $\tau1,\tau3$ technique. The first three types of T ($T1$, $T2$, and $T3$) in MSI serve to understand the translation shadow of a shape by $\theta1$ (change in the position of an object with a certain distance and direction). $\theta1$ justifies $\tau1,\tau2,\tau3$, because the solution of $T1$,$T2$, and $T3$ is based on students conducting investigations or discoveries by developing the understanding gained. Two types of tasks ($T4$,$T5$) in MSI are advanced understanding of the shift of a shape using the techniques $\tau1,\tau2,\tau3$ resulting in $\theta 2$ (change in the position of an object through reflection of the shadow). Likewise, the last three types of T ($T6$,$T7$,$T8$) in MSI are interrelated using the techniques $\tau 1$,$\tau 2$,$\tau 3$ which function to understand the rotation of a shape by $\theta 3$ (change in the position of an object by rotating an object).

Furthermore, the first three types of T ($T1$,$T2$, and $T3$) in NSM function to understand the shadow of a shape through reflection using the technique $\tau 1$,$\tau 2$,$\tau 3$ resulting in $\theta 1$ (change in position of an object through reflection of the shadow). Task $T4$ in NSM is an understanding of the translation shadow of a shape using the technique $\tau 1$,$\tau 2$,$\tau 3$,$\tau 4$ by $\theta 2$ (change in position of an object with a certain distance and direction). Likewise, the two types of tasks ($T5$,$T6$) in NSM are advanced understanding of the translation of a shape using the technique $\tau 1$,$\tau 2$,$\tau 3$,$\tau 4$ which results in $\theta 3$ (change in position of an object through reflection of the shadow).

Comparison of MSI and NSM Textbooks

In this section, the researcher discusses the findings of MSI (Table 6) and NSM (Table 7) textbooks.

Table 6. MSI logos block

Type of Task (T)		Technique (τ)	Technology (θ)	Theory (Θ)
T_1:	Expressing the translation results in the form $\begin{pmatrix} a \\ b \end{pmatrix}$	τ_1: perceptual τ_3: operational	θ_1 : Application of changing the position of an object with a certain distance and direction	Θ_1 : Change of position with the concept of translation
T_2:	Determining the shadow of the translation result	τ_1: perceptual τ_3: operational		
T_3:	Creating an object's shadow on the coordinate plane	τ_1: perceptual τ_2: physical τ_3: operational		
T_4:	Identifying objects that are the result of reflection	τ_1: perceptual τ_3: operational	θ_2 : Change in position of an object through reflection of shadow	Θ_2 : Change of position with the concept of reflection
T_5:	Able to create a shadow of the transformation result	τ_1: perceptual τ_2: physical τ_3: operational		
T_6:	Able to create shadow images resulting from rotation in the opposite direction	τ_1: perceptual τ_2: physical τ_3: operational	θ_3 : Change in position of an object through rotation	Θ_3 : Change of position with the concept of rotation
T_7:	Able to create a shadow image resulting from rotation which is continued with translation	τ_1: perceptual τ_2: physical τ_3: operational		
T_8:	Able to create a shadow image of the result of rotation reflected against the coordinate axis	τ_1: perceptual τ_2: physical τ_3: operational		

Table 7. NSM logos block

Type of Task (T)	Technique (τ)	Technology (θ)	Theory (Θ)
T_1: Identifying two points on the coordinate plane	τ_1: perceptual τ_3: operational	θ_1 : Application of changing the position of an object through shadow reflection	Θ_1 : Change of position with the concept of reflection
T_2: Determine the coordinates of the reflection result on a line	τ_1: perceptual τ_2: physical τ_3: operational		
T_3: Determine the coordinates of the reflection result on a line and create a graph	τ_1: perceptual τ_2: physical τ_3: operational		
T_4: Perform an estimate of the change in angle that has shifted	τ_1: perceptual τ_2: physical τ_3: operational τ_4: algebra	θ_2 : Change in the position of an object with a certain distance and direction	Θ_2 : Change of position with the concept of rotation
T_5: Determining the shadow of a point from the shift result	τ_1: perceptual τ_3: operational	θ_3 : Change in position of an object through reflection of shadow	Θ_3 : Change of position with the concept of translation
T_6: What is the shadow of point $B(4,-8)$ under T_2?	τ_1: perceptual τ_2: physical τ_3: operational τ_4: algebra		

Based on the results of the discussion that has been explained, the researcher has described T, τ, θ, and Θ in each textbook in the praxis block and logos block. The results of the analysis of the praxis block and logos block can ultimately compile a new type of task (T) to accommodate all T in the textbooks of both countries. Table 5 is the MSI logos block, it can be seen that the elements of praxeology [T,τ,θ,Θ] are formed. In the MSI textbook, [$T1/\tau/\theta1/\Theta1$], [$T2/\tau/\theta1/\Theta1$], [$T3/\tau/\theta1/\Theta1$], [$T4/\tau/\theta2/\Theta2$], [$T5/\tau/\theta2/\Theta2$], [$T6/\tau/\theta3/\Theta3$], [$T7/\tau/\theta3/\Theta3$], [$T8/\tau/\theta3/\Theta3$] are formed. From these results, it can be seen that the praxeology of 3 types of tasks T1, T2, T4 uses the same technique, namely $\tau1$ and $\tau3$ (perceptual and operational). Meanwhile, the 5 types of tasks T3, T5, T6, T7, T8 use the same technique, namely $\tau1$, $\tau2$, and $\tau3$ (perceptual, physical, and operational). The types of tasks used are stating the translation results in the form (■(a@b)), determining the shadow of the translation results, and creating a shadow of an object on the coordinate plane. Furthermore, the technology used to complete the task uses the application of changing the position of an object with a certain distance and direction, using the concept of translation. Another type of technology used to support tasks T4 and T5 is changing the position of an object through shadow reflection.

Next, Table 6 explains about the NSM logos block, it can be seen that the praxeology elements [T,τ,θ,Θ] are formed, namely [$T1/\tau/\theta1/\Theta1$], [$T2/\tau/\theta1/\Theta1$], [$T3/\tau/\theta1/\Theta1$], [$T4/\tau/\theta2/\Theta2$], [$T5/\tau/\theta3/\Theta3$], dan [$T6/\tau/\theta3/\Theta3$]. From these results, it can be seen that the praxeology of 2 types of tasks T1, T5 uses the same technique, namely $\tau1$ and $\tau3$ (perceptual and operational). As for the 2 types of tasks T2, T3 use the same technique, namely $\tau1$, $\tau2$, and $\tau3$ (perceptual, physical, and operational). While the 2 types of tasks T4, T6 use the same technique, namely $\tau1$, $\tau2$, $\tau2$, and $\tau4$ (perceptual, physical, operational, and algebraic). The types of tasks used are identifying two points on the coordinate plane, determining the coordinates of the reflection results on a line, determining the coordinates of the reflection results on a line, and making a graph. Furthermore, the technology used to complete the task uses the application of changes in the position of an object through shadow reflection, using the concept of reflection. The technology used to complete the shadow task from the shift result is shadow reflection.

Suryadi et al (2023) stated that in mathematical praxeology, it is necessary to consider and study the relationship related to the design of mathematical tasks. Hendriyanto et al (2023) explained that tasks (T) can be completed using various techniques (τ), and technology (θ) can involve various techniques. Generally, tasks (T) can be completed using various techniques (τ), and technology (θ) can involve various techniques. A series of types of tasks (T) and task completion techniques are called timely organizations. Technology usually justifies various techniques for various types of tasks, becoming local organizations. If students have difficulty with tasks, the teacher's role is to guide students to achieve higher understanding and performance than they could achieve on their own (Zakiah, 2020), and equip students with the competencies to face life in the future (Zakiah & Fajriadi, 2020). However, textbooks that only provide simple context may be good and basic but are considered burdensome and do not fall into medium or high levels of cognitive mastery (Sianturi et al., 2021). This can trigger epistemological obstacles when students face mathematical problems that require higher cognitive abilities (Fuadiah et al., 2019).

The difference in textbooks between the two countries is in the structure of the material. In the MSI textbook, it starts with translation, reflection, and rotation. While in the NSM textbook, it starts with reflection, rotation, and translation. The Indonesian textbook on the material of transformation geometry begins the initial activity using the context of everyday life, and there is an introduction to the culture that exists in Indonesia. The use of this is possible so that the material being studied can be meaningful to students.

Several things that facilitate students in understanding the transformation geometry material in Singapore textbooks are seen in the learning scheme, namely the attention section, problem-solving tips, information, recall, just for fun, and internet resources. The textbook contains important information or certain mathematical rules that students have previously learned, guidance on how to approach problems, interesting stories about mathematics as enrichment for students, and guides students to search for information via the internet for independent learning. Based on the above explanation, the mathematical practices in Singapore textbooks facilitate investigation activities, class discussions, thinking time, journal writing, and performance tasks.

Different from the activities that facilitate students to understand the transformation geometry material in Indonesian textbooks, which is seen in the learning scheme. At the end of the learning achievement of phase D, students can perform single transformations (reflection, translation, rotation, and dilation) of points, lines, and plane shapes on the Cartesian coordinate plane and use them to solve problems. In textbooks using learning principles, namely students are the center of learning, the development of conceptual understanding is continued with procedural fluency, development of mathematical and language competencies, and integration of technology in learning is aimed at creating an interactive learning environment. Based on what has been explained, the mathematical practice in Indonesian textbooks facilitates interactive activities, trying, creative thinking, critical thinking, reflection, and projects.

The results of the praxeological analysis show that mathematical practices in Singapore textbooks facilitate students to understand the material of transformation geometry starting with investigation activities, class discussions, thinking time, journal writing, and performance tasks. The visual elements of Singapore textbooks are highly emphasized in the presentation of discussion topics, making them richer and easier to understand (Erbas et al., 2012). This pattern has become part of the Singapore mathematics education curriculum that guarantees mathematical concepts (Ibrahim & Othman, 2010). Meanwhile, mathematical practices described in Indonesian textbooks facilitate students to understand the material of transformation geometry through digital interactive activities, activities to try out concepts that have been learned, giving projects, thinking creatively, and thinking critically to explore the material.

▪ CONCLUSION

Textbooks are one of the important factors that influence how students learn mathematics. However, teachers have a more crucial role in determining what kind of learning experience their students will experience. The praxeology research that has been carried out aims to determine the differences in the content of mathematics textbooks from Indonesia and Singapore, with a focus on learning mathematics on the material of transformation geometry at the junior high school level. Using praxeology, because it is in line with ATD that textbooks are empirical sources that can show the knowledge that must be taught during the didactic transposition process.

This study shows that praxeological analysis helps identify mathematics learning activities in textbooks. There are differences in the contents of the two textbooks, in terms of material arrangement, learning activities, and assignments. Mathematical practices in Singaporean textbooks facilitate students to understand transformation geometry material through investigation, class discussion, thinking time, journal writing, and performance tasks. Meanwhile, mathematical practices in Indonesian textbooks facilitate students to understand transformation geometry material through interactive activities, trying, creative thinking, critical thinking, reflection, and projects. Furthermore, the results of this study can be used by teachers, decision-makers, and other related parties in considering mathematics textbooks from various perspectives. The advantages and disadvantages of each textbook can be used as a reference for evaluation actions in studying the next textbook.

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