



Analysis of Students' Mathematics Conceptual Understanding Based on Differences in Mathematics Thinking Styles

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Abstract: This study aims to examine students' thinking processes in understanding concepts by considering the variations in mathematical thinking styles. This research is motivated by the finding that students demonstrate different thinking styles and intelligence, which is evident in the distinct ways they represent and design their learning approaches. This study employs a qualitative research method, using interviews based on tasks through the think-aloud method for data collection. The research instrument consists of a written test designed to probe students' thinking processes in understanding mathematical concepts according to the indicators of conceptual understanding used in this study. The researchers used time triangulation techniques to ensure the validity of the research data. The data analysis followed a structured process: data classification, reduction, presentation, interpretation, and conclusion drawing. The research subjects comprised three students with different mathematical thinking styles, namely visual, analytical, and integrated, who shared similarly high mathematical ability levels. The findings show that subjects with a visual mathematical thinking style create representations in the form of images, tables, or graphs for each indicator of understanding. Subjects with an analytical mathematical thinking style create representations in the form of verbal sentences or mathematical models that are rich in mathematical symbols and variables. While subjects with an integrated mathematical thinking style create representations in the form of verbal sentences, images, diagrams, mathematical symbols, tables, mathematical models, or graphs. The study concludes that while the type of mathematical thinking style—visual, analytical, or integrated—does not significantly influence the depth of students' conceptual understanding, it does shape the strategies they employ to construct that understanding. All three students demonstrated the ability to interpret, exemplify, classify, summarize, infer, compare, and explain concepts effectively, a consistency attributed to their shared high level of mathematical proficiency.

Keywords: mathematics, conceptual understanding, mathematical thinking style, cognitive process.

▪ INTRODUCTION

Mathematics is considered a hierarchical knowledge (Šikić, 2022). A strong conceptual understanding of foundational topics facilitates the comprehension of more advanced mathematical concepts ("The Nature Of Mathematics," 2023). This conceptual foundation is crucial, as it supports the development of interrelated mathematical ideas (Andamon & Tan, 2018). Conceptual understanding is the mastery of a number of materials, where it is not just about knowing and understanding but being able to re-express the concept in a form that is easier to understand and being able to apply it in various contexts (Wanabuliandari et al., 2023). Thus, it serves as the core or basis for mastering other higher mathematical concepts (Tsamago & Bayaga, 2023).

In addition to cognitive abilities, learning strategies or models also have an impact on students' ability to understand mathematical concepts (Hutkemri & Zakaria, 2014). Learning that emphasizes procedural understanding more than conceptual understanding

will have an impact on students' low conceptual understanding. The goals in mathematics teaching have shifted towards an emphasis on procedural and conceptual understanding (Ghazali & Zakaria, 2011). Procedural understanding emphasizes the stages in solving operations and symbols used. In mathematics, conceptual knowledge (in the literature interpreted as declarative knowledge) consists of understanding concepts and recognizing their applications in various situations. Procedural knowledge consists of the ability to solve problems through mathematical manipulative skills using pencils and paper, calculators, computers, and others. It is clear that mathematicians find procedures based on mathematical concepts (Gilmore et al., 2017; Hiebert & Lefevre, 2013).

Understanding is a person's ability to connect one object to another (Sierpinska, 2013). Piaget (Mousley, 2005) described understanding as the ability to have several relationships in mind and allow abstraction to occur. In this case, students are said to understand something if they are able to connect ideas in their minds and allow abstraction for the next step. Mousley distinguishes understanding into three general categories: (1) understanding as structured progress, (2) understanding as forms of knowing, and (3) understanding as process. Student understanding can be seen from how students understand the problem, how students carry out the process, and the progress of the structure. Thus, well-organized knowledge can be used as a means to solve various problems in life (Fauziyah & Hani, 2025).

Furthermore, to find out students' conceptual understanding of mathematical concepts according to NCTM (2000), it can be seen from the students' abilities in: (1) defining concepts verbally and in writing; (2) identifying and creating examples and non-examples; (3) using models, diagrams, and symbols to represent a concept; (4) changing one form of representation to another; (5) recognizing various meanings and interpretations of concepts; (6) identifying the properties of a concept and recognizing the conditions that determine a concept; (7) comparing and distinguishing concepts (NCTM, 2020).

Hiebert & Carpenter in (Begum et al., 2021) suggested that a person's understanding of a concept can be analyzed using several methods, including (1) students' incorrect answers, (2) connections made between symbols and symbolic procedures and their matching with symbolized objects, (3) connections between symbolic procedures and internal problem-solving situations, and (4) connections made between different symbol systems.

(Mayer & Clark, 2003) stated that "Students understand when they build relationships between new knowledge to be acquired and previous knowledge. Both experts wrote about cognitive processes in the category of understanding, including interpreting, exemplifying, classifying, summarizing, concluding, comparing, and explaining. Interpreting is changing from one form of representation to another. Exemplifying is creating a specific illustration of a concept or principle (L. W. Anderson et al., 2001). Classifying is determining that something belongs to a particular category. Summarizing is making a short statement that represents the information presented or abstracts a general theme. Inferring is drawing logical conclusions from the information presented (Mayer & Clark, 2003). Comparing is detecting similarities and differences between two or more objects (J. R. Anderson, 2015). Explaining is building a cause-and-effect model of a system (L. W. Anderson et al., 2001; Tiwari et al., 2011).

In this study, the indicators of conceptual understanding used are interpreting, giving examples, classifying, summarizing, concluding, comparing, and explaining. A person's understanding of mathematical concepts and skills in solving mathematical problems are influenced by many factors, and one of these factors is mathematical thinking style (Güner & Erbay, 2021). Solving these mathematical problems requires a cognitive thinking process (Fauziyah et al., 2019). Mathematical thinking style is the way a person chooses to understand mathematical facts and connections by using internal imagination and/or certain external representations. Mathematical thinking styles are divided into three, namely visual, analytical, and integrated mathematical thinking styles. Visual thinking style tends to think according to using internal pictorial imagination and external pictorial representations. While an analytical thinking style tends to think about a concept using existing symbols or verbal representations. Meanwhile, an integrated thinking style can combine visual thinking and analytical thinking (Ferri, 2012).

Several studies have shown that the main characteristics of students who have a visual thinking style are imagining situations in the form of pictures and using pictographic images. Meanwhile, students with analytical thinking styles have the characteristic of immediately changing into mathematical models and returning to real models only when needed to understand the task better (Risnanosanti, 2017). For students who are taught by teachers with the same mathematical thinking style, the understanding process will be faster because both use the same "mathematical language." Teachers need to be aware of their mathematical thinking style to ensure equal opportunities for a number of students and, on the other hand, to develop their own mathematical potential (Sternberg & Zhang, 2005). Mathematics will be easily understood by students when taught by teachers who are aware of their mathematical thinking style and teach according to their students' mathematical thinking style. However, in reality, not many teachers consider students' mathematical thinking style in the process of learning mathematics in the classroom (Ferri, 2012).

Based on the background above, the researchers are interested in analyzing students' conceptual understanding of mathematics based on differences in mathematical thinking styles. The subjects have received mathematical learning treatment that emphasizes learning by providing meaningful understanding through contextual problems. Thus, the formulation of the problem in this research is: How are the differences in the construction of conceptual understanding built by students in terms of differences in their mathematical thinking styles?

▪ METHOD

Participants of the Research

In determining the research subjects, the researchers used the MaTHSCU instrument developed by Rita Borromeo Ferri. Content validity was carried out by experts before the instrument was used. Then, three students were selected, each of whom had visual, analytical, and integrated thinking styles. Several other variables that might affect the results of this study have been controlled by the researchers, including gender and level of mathematical ability. Because in one class, those who met the three thinking styles with equivalent mathematical ability levels were male students, it was decided that the subjects would be 3 male students. Data on the level of mathematical ability was obtained from the mathematics teacher in the class in the odd semester, and this study

was conducted in the even semester. The selection of this subject has also been consulted with the mathematics teacher in the class.

Research Design and Procedure

This study focuses on three students who each have visual, analytical, and integrated mathematical thinking styles with different levels of intelligence. The approach in this study is categorized as a qualitative approach, while the type of this study is exploratory descriptive research (Merriam, 2009). Qualitative data in general form of words comes from observations, interviews, or documents. Qualitative data has advantages compared to quantitative data; for example, qualitative data is richer in terms of description and explanation (M. Miles et al., 2018).

After obtaining the research subjects, the researchers collected data through task-based interviews to explore the construction of students' conceptual understanding in turn. Furthermore, at a different time, the researchers took the 2nd data using an equivalent instrument; this was done because this study used time triangulation. Furthermore, the data from the first and second excavations were validated to obtain valid data related to how each subject constructed their conceptual understanding as seen from the 7 indicators of conceptual understanding in this study.

Instrument of the Research

The MaTHSCU instrument developed by Rita Borromeo Ferri was used to measure students' mathematical thinking style. Content validity was carried out by experts before the instrument was used. In accordance with the type of research, qualitative research, the main instrument in this study is the researchers' observation, while the auxiliary instrument is a test instrument that can measure indicators of conceptual understanding and interview guidelines. The interview guidelines were developed based on the indicator of conceptual mathematical understanding in table 1. Interview questions were developed according to the expected student abilities in each indicator in table 1. As the main instrument, the researchers act as planners, collectors, analyzers, translators, and reporters of research results. Thus, the researchers must be objective, responsive and neutral.

The material in the problem is developed based on the curriculum applicable at the high school level. The mathematics problem test instrument is used to collect in-depth data to describe the level and strategy in understanding mathematical concepts according to their thinking style. The responses written or expressed by the subject in each step of problem-solving are used as a guide to analyze the level and strategy carried out by the subject in understanding a mathematical concept.

The conceptual understanding test instrument was processed by experts through content validation to validate the problem construction, materials, and language in the problem. The validators were two mathematics teachers and two mathematicians from college. Table 1 shows the mathematical understanding test instrument used in this study, which focuses on understanding the concept of function.

Table 1. Instrument

Indicator	Description	Student's Ability	Instrument
Interpreting	Changing from one form of representation to	Creating another form that is different from the	What is meant by function?

	another	given concept form, for example, changing the form of words into symbols or images.	
Exemplifying	Finding a specific example or illustration of a concept or principle	Provide a specific example or model of a concept.	If there are two sets $A = \{a, b, c\}$ and $B = \{p, q\}$, can you make a function from A to B?
Classifying	Determining that something belongs to a certain category	Create categories of given objects based on similarities or differences in the characteristics of the objects.	Please classify the function below based on the type! Give the reason! a. $f(x) = x^2 + 2x - 3$ b. $g(x) = 5^x - 1$ c. $h(x) = -6x + x^2 + 5$ d. $k(x) = 5 - 2x$ e. $j(x) = \frac{x-3}{2x-8}$ f. $c(x) = 4^{x-1}$ g. $d(x) = (x-1)^4$ h. $m(x) = (x-1)(x-1)$
Summarizing	Producing a short statement that represents presented information or abstracts a general theme	Compose simple sentences that can describe concepts using the characteristics of the concept.	What are the properties of functions that you know?
Inferring	Drawing a logical conclusion from presented information	Making an important emphasis of a concept that characterizes the concept.	Based on the examples above, what are the characteristics of a function that can differentiate it from other concepts?
Comparing	Detecting similarities and differences between two or more objects, events, ideas, problems, or situations	Determine the differences and similarities of the concept with other concepts.	What is the difference between relation and function?
Explaining	Constructing a cause-and-effect model of a system	Explains concepts applied in real problems.	A restaurant runs a one-price program of Rp2,000 each for an additional side dish. Meanwhile, the basic price of a meal package for 1 plate of rice and vegetables is Rp12,000. This price follows the formula $h(x) = 12000 + 2000x$, and x states the number of additional side dishes. Determine

the price of one food package with
 $x = \{x | 0 \leq x \leq 4, x \in \text{integer}\}!$

The researchers used triangulation techniques to ensure the validity of the research data. The type of triangulation used by the researchers is time triangulation. In this triangulation, the researchers compiled a second mathematical comprehension test instrument that is equivalent to the first mathematical comprehension test instrument.

Data Collection and Analysis Procedures

Data was collected using task-based interviews with the think-aloud method. Subjects worked on the mathematics comprehension test by writing answers on the answer sheet provided and expressing what they were thinking. The mathematics comprehension test was created to measure students' conceptual understanding according to the indicators used in this study (table 1). If the subject did not express their thinking process, the researchers asked open-ended questions to obtain responses. All subject activities when completing the mathematics comprehension test were recorded with an audio recorder. The think-aloud method was applied effectively to obtain qualitative research data. The think-aloud research method has a strong theoretical basis with valid data sources about what is on the subject's mind (Charters, 2003).

Time triangulation was used to ensure the validity of the data in this study, assisted by the 2nd concept comprehension test instrument, which was equivalent to the first concept comprehension test instrument. Data collection was carried out at school outside of school hours and was carried out repeatedly for each subject to complete the data needed. This research was conducted in the even semester of the 2024/2025 school year.

The qualitative data analysis technique used in this study is the data analysis technique proposed by Miles & Huberman (M. B. Miles et al., 2014) which begins with data collection through various methods, namely interviews, observation, and document analysis of the subject's answer sheets. Each of these data is analyzed alternately based on the research subject. The subject's answer sheets are analyzed step by step according to the indicators of conceptual understanding used in this study. The indicators are interpreting, giving examples, classifying, summarizing, concluding, comparing, and explaining.

To analyze the data from task-based interviews, the stages are carried out, namely starting from data categorization, data reduction, data presentation, and drawing conclusions. Data categorization in this study is based on 7 indicators of conceptual understanding. The data reduction stage is carried out through a selection process, focusing on simplification and data transformation in the field. Data reduction in this study is an activity that refers to the process of selecting data related to 7 indicators of mathematical concept understanding. The reduced data is presented visually or narratively using tables to facilitate data interpretation. Furthermore, the data that has been selected and identified is coded to find out the data source. The use of codes for the data in this study is as follows:

P/VMTS/AMTS/IMTS, In, Tn, Ni

P: Researchers

VMTS: Subject with visual mathematical thinking style

AMTS: Subject with analytical mathematical thinking style

IMTS: Subject with integrated mathematical thinking style

In: nth concept understanding indicator

Tn: nth concept understanding test

Nn: Sequence of nth interview questions or answers

Coding PI1T2N3 is the researchers' question on the first indicator (interpreting) in the second mathematical concept understanding test, sequence of questions 3. Coding AMTSI4T1N8 is the answer of the subject with an analytical mathematical thinking style on the 4th indicator (summarizing) in the first mathematical concept understanding test, sequence of questions 8.

The next stage is data presentation, which is writing down a collection of data that has been organized and categorized so that it is easy to interpret the data in order to draw conclusions from the data. The last step is drawing conclusions; namely, the researchers combined the results of the analysis findings to formulate comprehensive conclusions and provide insight into the phenomena being studied. This technique provides a systematic and holistic approach in processing qualitative data, resulting in accuracy and reliability in analysis.

▪ RESULT AND DISCUSSION

Conceptual understanding based on the difference of mathematical thinking style

The following is data from three subjects related to conceptual understanding in understanding the concept of function based on the differences in their mathematical thinking styles. Data were taken from the results of written tests and task-based interviews with the think-aloud method. The data presented here is data that has been consistent after time triangulation. In addition to being consistent, this data has been reduced in such a way that it is adjusted to the needs to answer the problem formulation in this study. This data will be presented for each indicator of conceptual understanding for the three subjects.

Interpreting

Subjects with a visual mathematical thinking style

Based on the data in Figure 1, the subject interpreted the function in 2 different forms of representation, namely in verbal form and in the form of an arrow diagram image. In understanding the definition of a function, the subject emphasized the components in it, namely the domain and codomain. Meanwhile, he mentioned two mandatory requirements for a function concept after being asked in more depth by the researchers. The two requirements are (1) mapping each member of the domain and (2) exactly one to the codomain member. However, based on the image created, all domain members have a partner, and their partner is exactly one. This means that the image shows that it is an example of a function concept from 2 sets.

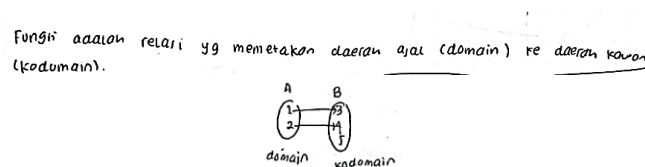


Figure 1. Subject 1 interpretation in understanding the concept of function

The finding is reinforced by the results of interviews with the subject.

- Researcher : What is the answer to question number 1?
 Subject : A function is a relation that maps from the origin to the friend's area, or domain to codomain.
 Researcher : Are there any other requirements for a function?
 Subject : Hmm... Basically mapping from domain to codomain?
 Researcher : Is it permissible to map more than one member to a codomain? and explain the reason!
 Subject : There must be 1 mapping; the pairing must be 1.
 Researcher : Is it permissible for a domain member not to be mapped to a codomain?
 Subject : No, all must have a pair.
 Researcher : Is it permissible for a codomain member not to have a pair?
 Subject : Yes.
 Researcher : Is it permissible for a codomain member to have more than one pair from the domain?
 Subject : Yes.

Subjects with an analytic mathematical thinking style

Based on the data in Figure 2, the subject interprets the function in 1 form of representation, namely in verbal form. In understanding the definition of the function, the subject emphasizes the components in it, namely the first set as the domain and the second set as the codomain. Meanwhile, two mandatory requirements of a function concept are not mentioned. The two requirements are (1) mapping each member of the domain and (2) exactly one to the member of the codomain.

Fungsi adalah pemetaan antara domain (daerah asal) dengan kodomain (daerah kawan).

Figure 2. Subject interpretation in understanding the concept of function

This data is also strengthened by the results of interviews with the subject.

- Researcher : What do you think is meant by function?
 Subject : A function is a mapping between a domain or area of origin and a codomain or area of the opponent using a certain relation. (while writing)
 Researcher : A function is a relation that maps from an area of origin to an area of a friend, or a domain to a codomain.
 Researcher : Are there any other requirements for a function? Is it permissible to map more than one member to a codomain?
 Subject : No.
 Researcher : Can there be a domain member that is not mapped to a codomain?
 Subject : No, all must have a partner.
 Researcher : Can a codomain member have more than one partner from the domain?
 Subject : Yes.
 Researcher : Can there be a codomain member that does not have a partner?
 Subject : Yes.

Subjects with an integrated mathematical thinking style

Interpreting

Based on the data in Figure 3, the subject interprets the function in two different forms of representation, namely in verbal form and in the form of an arrow diagram image. In understanding the definition of a function, the subject emphasizes the components in it, namely the existence of a domain and codomain. While the two mandatory requirements of a function concept are not mentioned. The two requirements are (1) mapping each member of the domain and (2) exactly one to the codomain member. From the representation in the form of an arrow diagram, the subject understands that the function tends to be a one-to-one correspondence. Which one-to-one correspondence is a special form of a function.

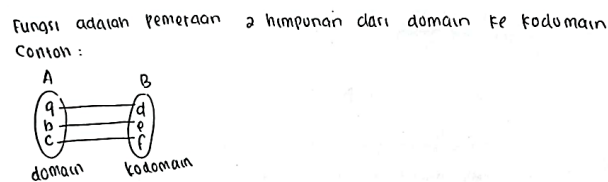


Figure 3. Subject 2's method of explaining concepts in solving real problems.

This data is also supported by the results of interviews with the subject.

Researcher : Try to read and understand the question.

Subject : (Writing the answer on paper) So, the definition of a function is a mapping of two sets from domain to codomain. This is the domain (writing set A as the domain), this is the codomain (writing set B as the codomain). Now later there is something called range. The range is the result of this (pointing to set B). So, this (pointing to set A) is the original area, and this (pointing to set B) is the resulting area. Now from the original area to the resulting area is called range.

Researcher : Are there any other requirements as a function?

Subject : Well, after that, if the function is from domain to codomain, it is one-to-one, but if the codomain cannot be obtained, it doesn't matter because this (pointing to set A) must be obtained.

The findings above show that the subject with a visual mathematical thinking style prefers to interpret concepts using both verbal dan visual representations, such as words and images. The subject with an analytic mathematical thinking style relies solely on verbal representations to interpret concepts. Meanwhile, the subject with an integrated mathematical thinking style utilizes both verbal and visual forms, combining language with diagrams or images. These findings indicate that in a class, students interpret concepts using their own way of thinking, either in the form of words or pictures or both. This finding aligns with Richard Mayer's Cognitive Theory of Multimedia Learning, which posits that individuals learn more effectively when information is presented through both words and images, rather than through words alone (Moreno & Mayer, 1999). Mayer further emphasized that presenting information in various formats helps students process and integrate information meaningfully.

This finding resulted in a recommendation that learning by presenting information in two forms will reduce cognitive load according to John Sweller's theory (Sweller, 2018). Clear, relevant, and unambiguous presentation will reduce extraneous cognitive load. The presentation in the form of words is represented in the form of images, and both are in line with the semiotic theory (Mazzola et al., 2022) that images (or visual signs) are a form of representation that is different from words (verbal signs).

However, if seen from the level of conceptual understanding, the subjects understand it very well. This is because the three subjects have equal mathematical abilities, and they are classified as having high mathematical abilities. This is in accordance with Piaget & Vygotsky's theory that individuals with high mathematical abilities tend to be at the formal operational stage, namely being able to think abstractly, logically, and systematically—which is needed to interpret mathematical concepts in depth (Moore & Piaget, 1971). The same thing from the mathematical competence model theory, that people with high mathematical abilities tend to have strong conceptual understanding and adaptive reasoning, two very important aspects in the ability to interpret concepts (Findell et al., 2001). This is in line with research conducted by Rodríguez-Naveiras et al., (2024) who found that students with high mathematical abilities excel in interpreting and applying mathematical concepts effectively (Rodríguez-Naveiras et al., 2024).

Exemplifying

Subjects with a visual mathematical thinking style

Based on the data in Figure 4, the subject provides an example of a function in the form of an image, namely an arrow diagram. With the right representation, even from the image, it appears that there are codomain members that have 2 pairs in the domain, and that is not a problem from the concept of function.

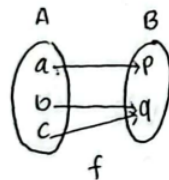


Figure 4. Example of a function concept described by subject 1

This data is also reinforced by the results of interviews with the subject.

Researcher : What shape did you draw in number 2? (pointing to Figure 2)?

Subject : Arrow diagram.

Researcher : Are there any other shapes that can describe a function?

Subject : There are namely Ordered pairs

Based on the data in Figure 5, the subject gave an example of a function in the form of a set of ordered pairs showing that each member of the domain has exactly one pair to the codomain member. With the right representation, even from the set of ordered pairs created, it appears that there are codomain members that have 2 (two) pairs in the domain, and that is not a problem from the concept of function.

$$x = \{(a, p), (b, q), (c, q)\}$$

Figure 5. Examples of function concepts described by the subject

This data is also reinforced by the results of interviews with the subject.

- Researcher : What do you think, can a function be made from the two sets? And explain the reasons!
- Subject : Yes, because elements from set A can be related to elements from set B to form a function.
- Researcher : What is the relationship like?
- Subject : The functions are (a,p),(b,q),and (c,q) too (while writing).
- Researcher : Why is it called a function?
- Subject : Because all domain elements are related to the exact codomain elements. Each domain element is mapped exactly one to the codomain element

Subjects with an integrated mathematical thinking style

Based on the data in Figure 6, the subject provides examples of functions in two forms of representation, namely in the form of arrow diagrams and ordered pair sets. Both representations are appropriate representations; even from the images and ordered pair sets that are made, it appears that there are codomain members that have two pairs in the domain, and this is not a problem from the concept of function.

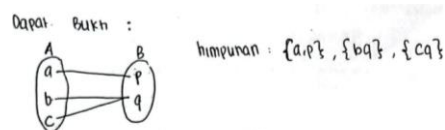


Figure 6. Example of the function concept described by subject 3

This data is also reinforced by the results of interviews with the subject.

- Researcher : Do you understand the question?
- Subject : Yes.
- Researcher : It is known from the question that set A has members a,b,c. While set B has members p,q. Can this create a function?
- Subject : How can a function be created or not?

The above findings show that the subject with a visual mathematical thinking style provides examples of concepts in the form of images. The subject with an analytic mathematical thinking style provides examples of concepts in analytical form, expressing examples of concepts in a form rich in mathematical symbols and variables that describe the concept precisely. The subject with an integrated mathematical thinking style provides examples of concepts in two forms of representation, namely in the form of diagrams and analytical forms rich in mathematical symbols. In substance, in providing examples of concepts, all three subjects have done well.

This is apparent because the three subjects understand the definition of a function well and emphasize 2 characteristics of a function, namely mapping every member of the domain to exactly one member of the codomain. This is in accordance with the theory of Tall et al., that in mathematics, all theorems, proofs, and procedures are built from

definitions. If the definition is not understood correctly, then logical reasoning and mathematical argumentation will be flawed (Gray et al., 1999) (Tall & Vinner, 1981). This finding is in line with the results of research conducted by Rupnow et al., which highlighted the importance of understanding concepts and definitions (Rupnow & Fukawa-Connelly, 2023).

Classifying

Subjects with a visual mathematical thinking style

Based on the data in Figure 7, the subject classifies the concept by drawing a table first, each column is named, and then writing the objects that he classifies one by one into the prepared column based on the similarity of their characteristics. From the accuracy in answering, the subject answered carefully and correctly. The subject is very proficient in the types of functions and their characteristics.

F. Kuadrat	F. Linier	F. Rasional	F. Polino
a. $x^2 + 2x - 3$	d. $5 - 2x$	e. $\frac{x-3}{2x-9}$	g. $(x-1)$
c. $-6x + x^2 + 5$			
h. $(x-1)(x-1)$ $= x^2 - 2x + 1$			
F. Eksponen			
b. $5x - 1$	f. $4x - 1$		

Figure 7. Subject's way of classifying concepts

This data is also reinforced by the results of interviews with the subject.

Researcher : Okay, let's look at the next question, there are several functions. Do these functions have differences?

Subject : Yes

Researcher : Try to classify them based on their types.

Subject : $f(x) = x^2 + 2x - 3$, and then $h(x) = -6x + x^2 + 5$, and $m(x) = (x - 1)(x - 1)$ it's a quadratic function.

Researcher : Why is it called a quadratic function?

Subject : Because the highest order is 2. And then $k(x) = 5 - 2x$ linear function because has order 1.

Researcher : Are there any other functions?

Subject : Rational function because in the form $\frac{f(x)}{g(x)}$. There is polynom function $d(x) = (x - 1)^4$, because the order more than 2. There is an exponential function, $(x) = 5^x - 1$ and $c(x) = 4^{x-1}$, because the order is variable.

Subjects with an analytic mathematical thinking style

Based on the data in Figure 8, the subject classified the concept by directly writing down the objects he classified one by one based on their characteristic similarities without separating them with lines or tables. From the accuracy in answering, the subject answered carefully and correctly. The subject is very proficient in the types of functions and their characteristics.

• fungsi kuadrat
 a. $f(x) = x^2 + 2x - 3$
 c. $h(x) = -6x + x^2 + 5$
 h. $m(x) = (x-1)(x-1)$

• fungsi linear
 d. $k(x) = 5 - 2x$

• fungsi eksponen
 b. $g(x) = 5^x - 1$
 f. $c(x) = 4^{x-1}$

• fungsi 'polinum
 g. $d(x) = (x-1)^4$

• fungsi rasional
 e. $j(x) = \frac{x-3}{2x-8}$

Figure 8. The subject's way of classifying concepts

This data is also reinforced by the results of interviews with the subject.

Researcher : Classify the function based on the type.

Subject : This is the quadratic function $f(x) = x^2 + 2x - 3$, and then $h(x) = -6x + x^2 + 5$, and $m(x) = (x-1)(x-1)$.

Researcher : Why is it called a quadratic function?

Subject : Because the highest power is power 2.

Researcher : Are there any other types of functions?

Subject : Yes, linear functions are $k(x) = 5 - 2x$ Then.

Researcher : Why is it called a linear function?

Subject : Because it is power 1.

Researcher : Are there any other types of functions, besides those mentioned earlier?

Subject : There is the exponential function, which is included in the exponent. $g(x) = 5^x - 1$ and $c(x) = 4^{x-1}$.

Researcher : Why is it called an exponential function?

Subject : Because the exponent is a variable.

Researcher : Are there any other types of functions?

Subject : Next, the polynomial function is $d(x) = (x-1)^4$

Researcher : Why is the function called a polynomial function?

Subject : Because the polynomial or exponent is more than 2.

Researcher : Are there any other types of functions?

Subject : The last one is a rational function, which is a rational function (indicates a rational function).

Researcher : Why is it called a rational function?

Subject : Because the rational function is in the form of $f(x)/g(x)$.

Subjects with an integrated mathematical thinking style

Based on the data in Figure 9, the subject classifies the concept by drawing a dividing line first, then writing down the objects that he classified one by one based on the similarity of their characteristics into the space that has been prepared. From the accuracy in answering, the subject answered carefully and correctly. The subject is very proficient in the types of functions and their characteristics.

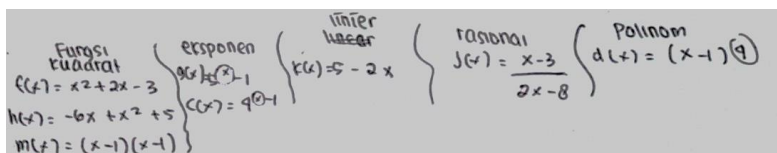


Figure 9. Subject 3's method of classifying concepts

This data is also reinforced by the results of interviews with the subject.

Subject : How to group several types of functions?

Researcher : Grouping means making a table. This is a quadratic function because it is squared $ax^2 + bx + c = 0$. This (pointing to the answer of the first quadratic function) x^2 , this is b, this is c. This (pointing to the answer of the second quadratic function) can (pointing to x^2) be placed in front.

Subject : As for the others, what about them?

Researcher : This (pointing to the answer of the exponential function) exponent, because the characteristic of the exponent is exponent (pointing to the exponent of x in the exponent answer).

Subject : What is x?

Researcher : The exponent, the exponent is not yet known. If there is a question like that, it means maybe you are asked to find x.

Subject : So what is the name of x?

Researcher : Oh yeah, it's a variable; the exponent is a variable.

Subject : Are there other types of functions?

Researcher : Linear, the characteristic is $ax + b$. This means ax, this is b (pointing to the answer of the linear function). If rational, the function is fractional. So every number that has a fraction like this is rational because this is a special characteristic that distinguishes it from other numbers (pointing to the answer to the rational function). Now this is a polynomial, this polynomial is a polynomial proven by its exponent which is more than two. If for example the exponent is two, it means it is a square, but because it is more than two, it means it is a polynomial (pointing to the answer to the polynomial function).

The subject with a visual mathematical thinking style classifies the concept by drawing a table first, each column is named, then writes the objects that are classified one by one into the prepared column based on the similarity of the characteristics of the classified concept. The subject with an analytic mathematical thinking style classifies the concept by directly writing down the objects that he/she classifies one by one based on their characteristic similarities without separating them with lines or tables. The subject with an integrated mathematical thinking style classifies the concept by drawing a dividing line first, then writing down the objects that he/she classifies one by one based on their characteristic similarities into the prepared space.

It appears that all three subjects engaged in a process of abstraction, extracting the underlying structure, pattern, or nature of a mathematical concept, removing any dependence on real-world objects with which it might have been originally associated, and generalizing it so that it has broader application or congruence among equivalent abstract descriptions of phenomena. This process allows the classification of

mathematical objects based on their essential properties, without relying on their concrete representations (Dreyfus, 2020). This finding is in line with research conducted by Nur Hasanah et al. that the abstraction process that occurs is dominated by empirical abstraction which emphasizes the aspect of eliminating the characteristics of objects that are manipulated or imagined during the recognition and development process (Nurhasanah et al., 2017). The process of abstraction and classification of mathematical objects are two processes that support each other in developing mathematical understanding. Abstraction allows us to identify the general properties that underlie various mathematical objects, while classification helps us to organize these objects into more organized and easily understood categories. Both enrich the process of learning mathematics by allowing deeper understanding, as well as broader applications in various mathematical contexts and problems (Shivhare & Kumar, 2016).

Summarizing

Subjects with a visual mathematical thinking style

Based on the data figure 10, the subject summarizes the properties of the function using images. The subject describes the properties of the function based on its type which describes the characteristics of each type of function presented.

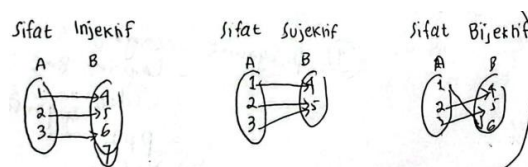


Figure 10. Summarizing

This data is also reinforced by the results of interviews with the subject.

Researcher : So what are the properties of a function?

Subject : The properties of a function are surjective (maps domain to codomain and codomain may be ambiguous), injective (domain maps codomain one-to-one), and bijective (a combination of surjective and injective)

Researcher : What is the difference between injective and bijective properties?

Subject : Injective is where the domain must map one to the codomain, but the codomain may have a remainder. Surjective is where all domains must be mapped to the codomain and may be ambiguous, but the members of the codomain must be exhausted. Bijective is a combination of surjective and injective or is called one-to-one correspondence.

Subjects with an analytic mathematical thinking style

Based on the data in Figure 11, the subject summarizes the properties of the function using verbal sentences without being given Figure illustrations. The subject describes the properties of the function based on its type which describes the characteristics of each type of function presented using verbal sentences.

- 1) Injektif, fungsi dimana setiap elemen kodomain tidak boleh berelasi lebih dari satu dengan elemen domain.
- 2) Surjektif, fungsi dimana setiap elemen kodomain harus berelasi dengan elemen domain.
- 3) Bijektif, fungsi dimana memiliki sifat injektif dan surjektif.

Figure 11. Summarizing

This data is also reinforced by the results of interviews with the subject.

- Researcher : What do you think are the properties of a function?
 Subject : Injective.
 Researcher : What is an injective function like?
 Subject : An injective function is a function where each codomain element must not be related to more than one domain element.
 Researcher : Are there any more?
 Subject : The second property is surjective. The surjective property is a function where each codomain element must be related to a domain element or must not be paired
 Researcher : So, for example, there are 4 domain elements and there are 3 codomain elements, is it okay if there are codomain elements that are branched?
 Subject : Yes, as long as there are none that are not related, all codomain elements must be related.
 Researcher : Are there any more?
 Subject : Bijective function
 Researcher : So bijective is a function that fulfills the injective and surjective properties.
 Subject : What do you mean?
 Researcher : So, each codomain element must be paired and must not be related to more than one.
 Subject : So, for example, if there are pairs, for example, the codomain has more than one pair, what kind of function is that?
 Researcher : If there is more than one.
 Subject : What is the function called?
 Researcher : If there is more than one, it is surjective.
 Subject : If there is an empty codomain?
 Researcher : That means it is injective.
 Subject : So if there is no empty one and no branching one, what function is it?

Subjects with an integrated mathematical thinking style

Based on the data in Figure 12, the subject summarizes the properties of the function using two forms of representation, namely verbal sentences and arrow diagrams. The subject describes the properties of the function based on its type, which describes the characteristics of each type of function presented. The subject even provides non-examples of the type of function as a comparison.

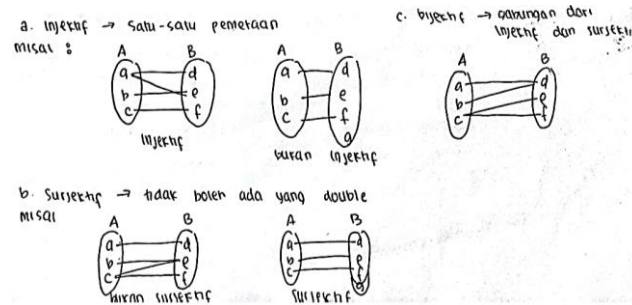


Figure 12. Summarizing made by subject 3

This data is also reinforced by the results of interviews with the subject.

Subject : Based on the examples of functions above, try to explain what kind of function properties are there?

Researcher : (working on the problem while talking) if it is injective, it cannot be alone, but it can be double. Well, this means it is not injective (showing the second arrow diagram).

Subject : Why?

Researcher : Because it stands alone (pointing to member g in set B), this is one-to-one (pointing to the mapping ad, be, cg). So it has a one-to-one partner. But it can also be double but it cannot stand alone (pointing to member g in set B). Because it stands alone, it is not injective. Then there are surjective ones. If it is surjective, it means there cannot be any double ones (drawing a non-surjective and surjective arrow diagram). Well, surjective is the opposite of injective. If this is not allowed to be alone (pointing to the non-injective arrow diagram). If this is allowed to be alone, but the condition is that there cannot be any double ones (pointing to the surjective arrow diagram). For example, a to d, b to e, c to e and f means it is not surjective, meaning it is okay if for example c is not combined with f so it is alone like this, that is called surjective (pointing to the arrow diagram not surjective). Then one less, bijective. If bijective is a combination of injective and surjective (drawing a bijective arrow diagram). Earlier here there could be double (pointing to the injective arrow diagram) and alone, but there cannot be alone codomain.

Subject with a visual mathematical thinking style summarize the properties of concepts using images. Subjects describe the properties of concepts based on their types that describe the characteristics or similarities of each type of concept presented. The subject with an analytic mathematical thinking style summarizes the properties of the concept using verbal sentences without being given a Figure illustration. The subject with an integrated mathematical thinking style summarizes the properties of the concept using two forms of representation, namely verbal sentences and images. The subject describes the properties of the concept based on its type that describes the characteristics or similarities of each type of concept presented. All three subjects performed summarizing well.

This finding is in accordance with the theory of constructivism, that the ability to construct concepts explicitly through language or verbal statements is influenced by an active process in which individuals construct new knowledge based on previous

experiences (Beth & Piaget, 1974). This finding is also in line with the results of research by Falani et al., that students with high mathematical abilities are able to write descriptions of mathematical concepts clearly and structuredly (Falani et al., 2023).

Inferring and Comparing

Subjects with a visual mathematical thinking style

Based on the data in Figure 13, the subject makes an important emphasis on a concept that characterizes the concept. The emphasis is made using image representation. The emphasis presented illustrates the important requirements of the function concept.

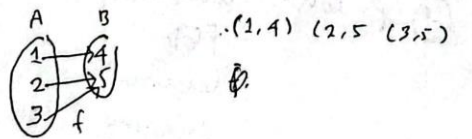


Figure 13. Inferring

This data is also reinforced by the results of interviews with the subject.

Researcher : What are the characteristics of a function that can distinguish it from other concepts? What concepts are similar to the concept of a function?

Subject : The concept of a relation. What distinguishes a relation from a function is that a relation is a friendly relationship between a domain and a codomain (drawing a relation). While a function is a domain and a codomain that are related to each other (drawing a function).

Researcher : Relation and function mean both are relationships, right?

subject : Yes, but if this has a remainder. So the remainder is not a function but a relation.

Researcher : Bijective.

Based on the data in Figure 14, the subject compares the concept of function with relation in the form of a Figure. There are important requirements of the function that are understood so that the example given is correct not as a function but only a relation. This means that the subject emphasizes that the requirements of the function are stricter than the requirements of a relation.

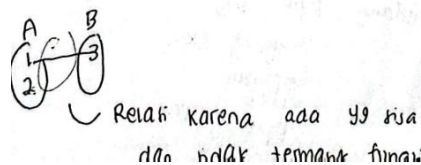


Figure 14. Subject method of comparing the concept of function with relation

Subjects with an analytic mathematical thinking style

Based on the data in Figure 15, the subject makes an important emphasis of a concept that characterizes the concept. The emphasis made is represented in the form of a verbal sentence. However, the emphasis presented does not yet describe the important

requirements of the function concept. Through the same answer, the subject compares the function concept with the relation represented in the verbal sentence as well.

relasi = hubungan antara 2 himpunan
Fungsi : pemetaan elemen dalam domain ke elemen dalam kodomain.

Figure 15. Inferring and comparing by subject 1

This data is also reinforced by the results of interviews with the subject.

Researcher : Based on the example above, what distinguishes a function from other concepts?

subject : Basically, a function is a mapping from elements in the domain to elements in the codomain.

Researcher : When compared to the concept of a relation, what is the difference?

Researcher : A relation is a relationship between set A and set B; there are no conditions whatsoever.

subject : What about a function?

Researcher : A function has a condition, that a domain member must have a partner and the partner must be exactly one in the codomain element.

Subjects with an integrated mathematical thinking style

Based on the data in Figure 16, the subject makes an important emphasis of a concept through the type of concept, not from the definition of a concept. The emphasis made uses a representation in the form of verbal sentences. The emphasis presented does not yet describe the important requirements of the concept of function.

Ciri khas :
a. Memiliki sifat injektif, surjektif, bijektif
b. di dalam nya terdapat relasi, domain, kodomain, range

Figure 16. Inference made by subject 3

This data is also reinforced by the results of interviews with the subject.

Subject : So what are the characteristics of a function?

Researcher : The characteristics of this function are that it has injective, surjective, bijective properties, it also has relations.

Subject : What is the difference with relations?

Researcher : So earlier it had injective, surjective, and bijective properties that were here one by one (pointing to the injective arrow diagram), it cannot be double (pointing to the surjective arrow diagram) with a combination of injective and surjective (pointing to the bijective arrow diagram). Then, the second one, it has relations, relations means not mapping. If mapping is a function, it means only the relationship from the domain to the codomain but it has not yet become a function like this (pointing to the arrow diagram in answer number 1) the domain is the area of origin, the codomain is the area of results, and the results are in the codomain.

Based on the data in Figure 17, the subject compares the concept of function with relation in the form of verbal sentences and images. The subject understands that relation does not provide any conditions, while function has stricter conditions.

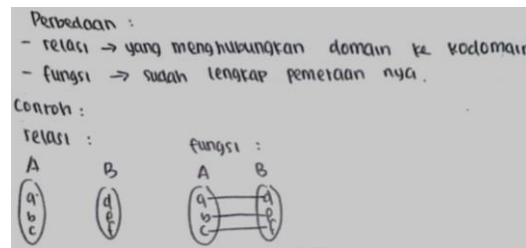


Figure 17. Subject 3's method of comparing the concept of function with relation

This data is also reinforced by the results of interviews with the subject.

Researcher : What is the difference between a function and a relation?

Subject : A relation that connects the domain to the codomain, meaning that later set A will go to set B (pointing to the first image in the answer), but we haven't written the line yet. If, for example, we have drawn the relation, it means that it goes into the function because in that function there is a relation (pointing to the line connecting set A to set B in the arrow diagram image in the answer).

The subject with a visual mathematical thinking style makes an important emphasis of a concept that characterizes the concept. The emphasis is made using pictorial representation. While subjects with an analytical and integrated mathematical thinking style use representations in the form of verbal sentences. For the comparing indicator, subjects with a visual mathematical thinking style compare concepts with other concepts in the form of images. Subjects with an analytical mathematical thinking style compare concepts with other concepts in the form of verbal sentences. While subjects with an integrated mathematical thinking style compare concepts with other concepts in the form of images and verbal sentences. All three subjects mention the similarities and differences of the two concepts being compared well.

In making inferences, students construct the meaning of two mathematical concepts based on previous experiences and interactions. Previous knowledge or interactions are related to their understanding of the definition, determining examples and non-examples to classify the concept. This is in line with the research of (Uegatani et al., 2023), who said that conceptual difficulties often arise not because of a lack of knowledge but because of a lack of inferential relationships between mathematical concepts. Inference plays an important role in constructing knowledge, according to research conducted by Noorloos et al. (Noorloos et al., 2017).

Furthermore, the findings show that in comparing two concepts, subjects identify similarities and differences between the two concepts. This is in line with research conducted by Corral et al., that when comparing two mathematical concepts, students activate relevant schemas for each and then identify similarities and differences (Corral et al., 2020).

Explaining

Subjects with a visual mathematical thinking style

Based on the data in Figure 18, the subject explains the concept of solving problems using graphic image representation, without first mentioning the information in the problem. Mathematical models are a reference for calculations in making representations in graphic form.

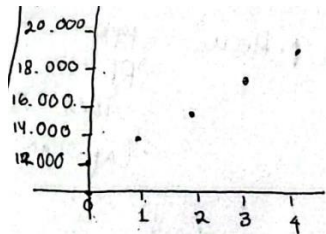


Figure 18. The subject's way of explaining concepts in solving real problems

This data is also reinforced by the results of interviews with the subject.

- Researcher : How to solve the question?
 Subject : Draw a graph (Figure 7).
 Researcher : Where is the number x entered?
 Subject : Entered into this function (function in the question)
 Researcher : So if x is 1, what does it mean?
 Subject : So the additional side dish is 1
 Researcher : If no side dish is added, how much is the price?
 Subject : 12,000 (Drawn with a graph)
 Researcher : If you add 1 side dish, how much is it?
 Subject : 14,000 (Drawn with a graph)
 Researcher : If you add 2 side dishes, how much is it?
 Subject : 16,000 (Drawn with a graph)
 Researcher : If you add 3 side dishes, how much is it?
 Subject : 18,000
 Researcher : If you add 4 side dishes, how much is it?
 Subject : 20,000 (Drawn with a graph)
 Researcher : You made a graph, which one is x and which one is y?
 Subject : The $x=1,2,3$, and 4 and the $y=12,000,14,000,16,000,18,000,20,000$

Subjects with an analytic mathematical thinking style

Based on the data in Figure 19, the subject explains the concept to solve the problem using a fairly good representation, starting by mentioning the information in the problem, creating a mathematical model until finding the results as an answer to the problem. However, there is no connection used, related to the concept of function.

7. Diketahui: $h(x) = 12.000 + 2.000x$
 $x = \{x \mid 0 \leq x \leq 4, x \in \text{bilangan bulat}\}$

$f(0) = (12.000 + 2.000(0))$
 $= 12.000$

$f(1) = (12.000 + 2.000(1))$
 $= 14.000$

$f(2) = (12.000 + 2.000(2))$
 $= 16.000$

$f(3) = (12.000 + 2.000(3))$
 $= 18.000$

$f(4) = (12.000 + 2.000(4))$
 $= 20.000$

Himpunan pasangan: $\{(0, 12.000), (1, 14.000), (2, 16.000), (3, 18.000), (4, 20.000)\}$

Figure 13. Subject 2's method of explaining concepts in solving real problems.

This data is also reinforced by the results of interviews with the subject.

Subject : How do you solve the problem?

Researcher : (writing a mathematical model in the form of an equation, and substituting the values 0,1,2,3 and 4 into the problem to get the solution)

Subject : Can you explain what this means?

Researcher : If x is 0, the price is 12,000, if x is 1, the price is 14,000, and so on.

Subject with integrated mathematics thinking style

Based on the data in Figure 20, the subject explains the concept of solving problems using a very complete representation, starting by mentioning all the information in the problem, creating a mathematical model, to a representation in the form of a graph. The subject appears to be very thorough in solving problems and making connections with the representation of a concept in question.

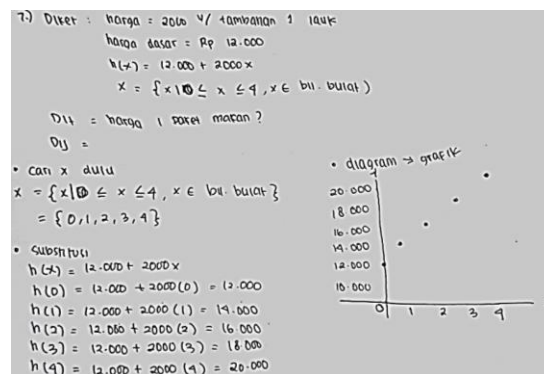


Figure 20. Subject 1's method of explaining concepts in solving real problems.

This data is also reinforced by the results of interviews with the subject.

Researcher : Okay, for the last problem.

Subject : It is known that the price is Rp. 2,000 for an additional side dish. So for example, if someone orders but wants to add one side dish, it means adding Rp. 2,000 per side dish. Well, the basic price for one plate of rice and vegetables is Rp. 12,000, if you want to add it, you just add Rp. 2,000 with h (x) of Rp. 12,000 plus Rp. 2,000. Here there is a provision that x is 10 less than or equal to and here 4 is also less than or equal to. Oh this, 0 (replacing 10 with 0). This is 4 with an integer. Integers mean

that it can start from 0,1,2,3,4. It is known that $h(x)$ is this much, meaning we enter it from the x we are looking for. $h(x)$ is the initial formula. $h(0)$ means $x=0$ means 12,000 because what is multiplied by 0 results in 0. Then this $h(1)$ is still 2,000, meaning $12,000+2,000=14,000$. $h(2)$ is $4,000+12,000=16,000$. $h(3)$ is $6,000+12,000=18,000$. $h(4)$ is $8,000+12,000=20,000$ (pointing to the answer). Now if we have found the graph, the diagram, this is the graph diagram. This is x , this is y . The x means we enter this x earlier (pointing to the graph image).

Researcher : What does the x mean?

Subject : The x is the number of additional side dishes if this is the price (pointing to the y line). So if for example (looking at the answer again)

Researcher : if we don't add side dishes, what is the price?

Subject : 12,000 because x is 0. But if, for example, you add side dishes, you add 2,000 which means 14,000. This adds another 2, this is 16,000. if 3, 18,000. if 4, 20,000.

Subjects with a visual mathematical thinking style explain the concept of solving contextual problems using graphical image representations, without first mentioning the information in the problem. Meanwhile, subjects with an analytical mathematical thinking style solve contextual problems using fairly good representations, starting with mentioning information in the problem, creating a mathematical model, and finding the results as the answer to the problem. Meanwhile, subjects with an integrated mathematical thinking style solve contextual problems using very complete representations, starting from mentioning all the information in the problem and creating a mathematical model to representation in graphic form.

All three subjects are very good at applying concepts to solve real problems. This is in accordance with the principle of RME (realistic mathematics education) that students can apply mathematical concepts if they see a direct relationship between the concept and real life (Freudenthal, 1968) (Freudenthal, 1972). In addition, students are able to apply concepts because they have a structured framework of understanding, not just mastery of formulas (Cook & Ausubel, 1970).

This finding is also in accordance with the results of research conducted by Powell that students who are taught using schemas have a better understanding of the structure of the problem and can transfer their knowledge to new situations (Powell, 2011). This is apparent because the other indicators of conceptual understanding have been well understood by students so that they are already in their cognitive scheme.

Next, we will analyze the differences in how subjects build conceptual understanding in terms of their different mathematical thinking styles. Before discussing it, it will be easy to read if presented in Table 2.

Table 2. Conceptual Mathematical Understanding

Indicator	Subject with Visual Mathematics Thinking Style	Subject with Analytic Mathematics Thinking Style	Subject with Integrated Mathematics Thinking Style
interpreting	Subjects interpret concepts in 2 different forms of representation, namely in verbal form and	The subject interprets the concept in one form of representation, namely in verbal form. In	The subject interprets concepts in two different forms of representation, namely in verbal form and in pictorial

	in pictorial form. In understanding the definition of a concept, subjects emphasize the components contained in the concept. Meanwhile. The compulsory requirement from a concept explained verbally. Judging from the level of understanding of the concept, the subject understands it very well.	understanding the definition of the concept, the subject emphasizes the components of the concept. While the mandatory requirements of a concept are stated verbally. Judging from the level of understanding of the concept, the subject is understood well.	form. In understanding the definition of a concept, subjects emphasize the components of the concept more. While the mandatory requirements of a concept are stated verbally. Judging from the level of understanding of the concept, the subject understands it very well.
exemplifying	The subject provides examples of concepts in the form of images. Examples of concepts expressed in the form of images also describe the concept accurately.	The subject provides examples of concepts in analytical form, expressing examples of concepts in a form rich in mathematical symbols and variables that describe the concept precisely.	The subject provides examples of concepts in two forms of representation, namely in the form of diagrams and analytical forms rich in mathematical symbols. Both representations are appropriate representations. The purpose of the subject is to create examples in other forms of representation to make them more convincing.
Classifying	The subject classifies the concept by drawing a table first, each column is named, and then writes the objects that are classified one by one into the prepared column based on the similarity of the characteristics of the classified concept. The subject answers carefully and correctly, the subject is very familiar with the characteristics of the classified concept.	The subject classifies the concept by directly writing down the objects that he/she classifies one by one based on their characteristic similarities without separating them with lines or tables. From the accuracy in answering, the subject answers carefully and correctly. The subject is very proficient in the types of concepts based on their characteristics.	The subject classifies the concept by drawing a dividing line first, then writing down the objects that he/she classifies one by one based on their characteristic similarities into the prepared space. From the accuracy in answering, the subject answers carefully and correctly. The subject is very proficient in the types of concepts based on their characteristics.
summarizing	The subject summarizes the properties of concepts using images. Subjects describe the properties of concepts based on their types that describe the characteristics or similarities of each type of concept presented. Subjects are able to summarize accurately.	The subject summarizes the properties of the concept using verbal sentences without being given a Figure illustration. The subject describes the properties of the concept based on its type that describes the characteristics or similarities of each type of concept presented using verbal sentences. The subject is able to summarize correctly.	The subject summarizes the properties of the concept using two forms of representation, namely verbal sentences and images. The subject describes the properties of the concept based on its type that describes the characteristics or similarities of each type of concept presented. The subject even provides non-examples of the type of concept as a comparison. The subject is able to summarize correctly.

inferring	The subject makes an important emphasis of a concept that characterizes the concept. The emphasis is made using pictorial representation. The emphasis presented illustrates the important requirements of the concept.	The subject makes an important emphasis of a concept that characterizes the concept. The emphasis made is represented in the form of a verbal sentence. However, the emphasis presented does not yet describe the important requirements of the concept.	The subject makes an important emphasis of a concept through the type of concept, not from the definition of a concept. The emphasis was made using representation in the form of verbal sentences. The emphasis presented does not yet describe the important requirements of the concept.
Comparing	The subject compares concepts with other concepts in the form of images. There are important requirements for the concept to be understood so that the examples given by the subject are correct. The subject can mention the similarities and differences between one concept and another concept correctly.	Through the same answer, the subject compares the concept with other concepts represented in verbal sentences as well. The subject can mention the similarities and differences between one concept and another concept correctly.	Subjects compare concepts with other concepts in the form of verbal sentences and images. Subjects can mention the similarities and differences between one concept and another concept correctly.
explaining	The subject explains the concept of solving contextual problems using graphic image representation without first mentioning the information in the problem. Mathematical models are used as a reference for calculations in making representations in graphic form.	The subject explains the concept of solving contextual problems using fairly good representation, starting by mentioning the information in the problem, creating a mathematical model, until finding the result as an answer to the problem. However, there is no connection used related to the concept.	The subject explains the concept of solving contextual problems using a very complete representation, starting by mentioning all the information in the problem, creating a mathematical model, and to a representation in graphic form. The subject appears to be very thorough in solving problems and making connections with the representation of a concept in question.

From the results of this study, it can be generally concluded that subjects with a visual mathematical thinking style tend to use strategies in understanding concepts using images. The subject with an analytic mathematical thinking style interprets the concept in one form of representation, namely in verbal form. The subjects with an integrated thinking style interpret concepts in two different forms of representation, namely in verbal form and in pictorial form. Through images, subjects can explain their previous understanding of concepts. This finding is in accordance with the theory developed by Krutetskii et al., that visual thinking relies on mental images and visual representations, while verbal-logical (analytical) thinking relies on formal logic, symbols, and deductive steps, and integrated thinking style is a combination of the two (Krutetskii et al., 1977).

This finding is also in accordance with research conducted by Rita Borromeo Ferri (Ferri, 2004), that students with this style tend to use visual representations such as

pictures, diagrams, and graphs to understand mathematical concepts. They find it easier to understand information presented visually and often rely on mental images in their thinking process. Students with an analytical thinking style focus more on logical and systematic processes in solving problems. They tend to prefer a structured and sequential approach and are more comfortable with formal mathematical symbols and notations. An integrated thinking style is a combination of visual and analytical approaches. Students with this style are able to combine visual representations with logical analysis to understand and solve mathematical problems effectively (Ferri, 2015). In addition, this thinking style influences the way individuals understand and solve mathematical problems (Ferri, 2015).

Meanwhile, differences in mathematical thinking styles, whether visual, analytical, or integrated, do not significantly affect the level of subject understanding of a mathematical concept. Conceptually, all indicators of conceptual understanding of the three subjects can be understood well.

The results of this study differ from the study conducted by Huincahue et al. Huincahue studied 275 16-year-old students in Chile, and the results showed that students who had an analytical mathematical thinking style showed a significant positive correlation with academic grades. Students with this style tended to have higher grades compared to students who were more dominant in using visual or integrated styles (Huincahue et al., 2021). This study also concluded that students with an analytical thinking style have an advantage in the current education system, especially because mathematics assessments often value formal and symbolic approaches.

The differences in the results of this study may be influenced by differences in the type of research, the method of determining the subject, or the test instrument used. The research was conducted by Huincahue with a quantitative research type; the sample was determined using a purposive sampling technique from 10 classes from 5 schools. The instrument used by Huincahue to measure students' academic mathematics performance was not specifically developed but used the final mathematics scores given by the sample. Meanwhile, this study was a qualitative study using 3 subjects in 1 school with similar high-level mathematics abilities. The school is also one of the schools in the superior school category. The instrument to measure conceptual understanding uses mathematics test questions developed by the researchers by taking function material with a moderate level of difficulty. However, it should be remembered that thinking style is not an indicator of ability, but rather an individual's preference in processing mathematical information. Therefore, it is important to recognize and appreciate different thinking styles in mathematics education to create an inclusive and effective learning environment.

▪ CONCLUSION

From the results of the study, it can be concluded that there are differences in the presentation of conceptual understanding built by subjects starting from indicators of interpreting, giving examples, classifying objects, summarizing, concluding, comparing, and explaining a concept. Subjects with a visual mathematical thinking style tend to use strategies in understanding concepts using pictures, diagrams, and graphs. They find it easier to understand information presented visually and often rely on mental images in their thinking process. Students with an analytical thinking style focus more on logical and systematic processes in solving problems. They tend to prefer a structured and

sequential approach and are more comfortable with formal mathematical symbols and notations. An integrated thinking style is a combination of visual and analytical approaches. Students with this style are able to combine visual representation with logical analysis to understand and solve mathematical problems effectively. However, if viewed from the substance and truth of the understanding of the concept, the three subjects understand the concept well. This is because the three subjects have the same mathematical ability at a high level.

So the final conclusion of the results of this study is that differences in mathematical thinking styles, whether visual, analytical, or integrated, do not significantly affect the level of subject understanding of a mathematical concept but significantly affect the conceptual understanding strategies they build.

However, as educators, we can make adjustments in designing learning by accommodating these three differences. This will provide meaningful support and different learning variations followed by students. It is also important for educators to be aware of these differences in thinking styles and adjust their teaching methods to be more inclusive. Integrating various mathematical representations and understanding students' cultural backgrounds can improve their understanding and engagement in learning mathematics (Akçakın & Kaya, 2020). Teachers can accommodate differences in mathematical thinking styles in the classroom by presenting information in various forms of symbols, images, graphs, or diagrams. This is very helpful when comparing two concepts visually and verbally. This is in accordance with the Multiple Representations (Dual Coding Theory) theory by Allan Paivio (Paivio, 2008).

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