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Conceptual and Procedural Knowledge of Prospective Mathematics Teacher in Solving Derivative Problems

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Abstract: Both procedural knowledge and conceptual knowledge are needed to solve derivative problems. This study aims to describe PMTs' conceptual and procedural knowledge in solving derivative problems. A qualitative with a case study as the research method was used. The research subjects were 63 prospective mathematics teacher (PMT) who attended Differential Calculus in the 2022-2023 academic year. The subject was chosen with purposive sampling. This study employed three problems from the derivative understanding test as the research instrument. The data analysis technique used in this study was the data analysis technique outlined by Miles & Huberman which began with data collection, then data reduction and drawing conclusions. The findings reveal that PMTs' lack of meaningful understanding of the definition of derivatives and their symbols may lead to algorithmic errors in finding the function f when the derivative of the function f at c is known. Of all the subjects, 82.5% found the derivative of a function without using the product rule. The procedural errors in finding the derivative of the product of functions stem from the subjects' misunderstanding of the rule that the derivative of a product is the product of their derivatives.Furthermore, 55.5% of the subjects determined the maximum and minimum values by first finding the stationary points. However, only 11% correctly found the minimum and maximum values. The results of this analysis highlight the importance of having a profound understanding of concepts when selecting and developing effective problem-solving procedures. Thus, the findings of this study are expected to assist lecturers in preparing teaching materials about derivatives effectively.

Keywords: conceptual knowledge, problem solving, procedural knowledge.

▪ INTRODUCTION

The concept of derivatives has a vital role (Haghjoo & Reyhani, 2021; Hamid et al., 2020; Hashemi et al., 2015) and broad implications, particularly for mathematics education students who will become prospective mathematics teachers. It enables students to solve problems in various fields of science, including mathematics, physics, economics, chemistry, engineering, and biology (Stanberry & Payne, 2023). With a profound understanding of derivatives, students can apply this concept effectively in various disciplines and teach it proficiently. Understanding the concept of derivatives not only helps students avoid mistakes in solving problems (Anugrah & Kusmayadi, 2019; Chikwanha et al., 2022; Rahardi & Lorenzo, 2021) but also strengthens their ability to tackle more complex problems (Mutawah et al., 2019; Ningrum et al., 2022).

Several studies have explored the challenges related to function derivatives. For instance, some research has focused on identifying the epistemological barriers faced by pre-service mathematics teachers regarding the basic concept of derivatives (Prihandhika et al., 2020). Other studies have investigated the understanding of derivative concepts and their representations (Rahardi & Hasanah, 2020) including the representation of

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derivatives (Prihandhika et al., 2022) as well as focusing on graphical representations (Moru, 2020), investigating four basic mental models of the derivative concept namely the local rate of change, tangent slope, local linearity and amplification factor (Greefrath et al., 2023), investigating the acquisition of knowledge about the derivative concept by applying the process-object framework (Litteck et al., 2024), and identifying and characterizing the level of development of derivative schemas using APOS theory (Fuentealba et al., 2018). Other studies also investigated the pedagogical content knowledge (PCK) of prospective professional teachers on the topic of derivatives (Rosnawati et al., 2020), compared the derivative knowledge of pre-service teachers and in-service teachers (Castro Gordillo & Pino-Fan, 2021), and revealed teachers' perceptions and meanings of derivative concepts (Mufidah et al., 2019). In addition, some studies describe the understanding of students with low and high levels of mathematics anxiety on the material of function derivatives based on APOS theory (Listiawati et al., 2023), describe the learning outcomes and difficulties faced by students in derivative material (Misdalina & Septiati, 2023) or explore students' errors and misconceptions when solving problems in differentiation (Chikwanha et al., 2022).

Conceptual and procedural knowledge which is crucial in developing mathematical understanding and problem-solving skills in mathematics (Ho, 2020; Mutawah et al., 2019; Yurniwati, 2018), including derivatives (Delastri et al., 2020; Ocal, 2017). A great integration between these two types of knowledge helps students understand not only the theory behind mathematical concepts but also how to apply them in various problem-solving contexts (Qetrani et al., 2021; Yurniwati, 2018). In solving derivative problems correctly, students need to comprehend basic concepts and procedures as a reference for having conceptual and procedural knowledge (Hurrell, 2021), so balance and interrelationship between them is highly required (Delastri et al., 2020; Ocal, 2017).

With conceptual knowledge, students can understand the reasons behind mathematical procedures, rather than merely memorizing formulas, with the emphasis being on transferring knowledge to new situations, thereby improving their problem-solving skills (Chirove & Ogbonnaya, 2021; Hussein & Csíkos, 2023) When students do not understand concepts properly, they tend to make mistakes in interpreting questions, using inappropriate procedures, or ignoring the connections between concepts (Hurrell, 2021; Jailani et al., 2020; Mutawah et al., 2019). Procedural knowledge, on the other hand, is essential in learning mathematics as it links the appropriate processes to problem situations, allowing the delivery of results through specific and sequential steps (Felia & Defitriani, 2021; Mutawah et al., 2019). Procedural fluency supports problem-solving and critical thinking skills by emphasizing mastery of rules, in-depth understanding, evaluation of results, and application of systematic steps (Hussein & Csíkos, 2023). To enable students to modify procedures according to the concepts they have mastered, conceptual and procedural mastery should align and complement each other (Rittle-Johnson et al., 2015; Yurniwati, 2018).

Several studies have examined conceptual and procedural knowledge, including describing conceptual understanding ability, procedural knowledge, and problem-solving skills in the context of numbers and operations, algebra, geometry, measurement, and data analysis and probability (Mutawah et al., 2019), successes and challenges of proficient students in applying conceptual understanding to solve mathematical problems (Ningrum et al., 2022), describe the use of conceptual and procedural knowledge in non-routine problem-solving (Braithwaite & Sprague, 2021) and characteristics of students who solve problems with conceptual and procedural understanding (Delastri et al., 2020). Other studies also develop conceptual and procedural fraction knowledge test instruments (Lenz et al., 2020) applied frameworks to develop proceptual knowledge in derivative materials (Barumbun & Kharisma, 2022), as well as the effects of applying multisensory approaches (Yurniwati, 2018), and geogebra (Ocal, 2017) on conceptual and procedural knowledge. There are also studies investigating the difference between the concept image and the formal concept definition of derived concepts based on procedural and conceptual understanding of cognitive structure (Prihandhika et al., 2022), examined students' relative performance on assessment items measuring conceptual knowledge, procedural knowledge and procedural flexibility in algebraic equation solving (Chan et al., 2023), evaluated the effect of instruction on a math concept and procedure (Rittle-Johnson et al., 2016) and the effect of conceptual understanding and procedural knowledge on students' achievement, anxiety, and attitude in mathematics (Hussein & Csíkos, 2023).

Based on the description above, it appears that no research focuses on the description of conceptual and procedural knowledge in problem-solving, especially on derivative material. To support the novelty of this research, the author presents the results of an analysis using VosViewer with a database extracted from Scopus of research articles with the keywords ‘Procedural and conceptual knowledge.’ The search results resulted in 1.610 articles published since 1984. Visualization of VosViewer output is presented in figure 1.

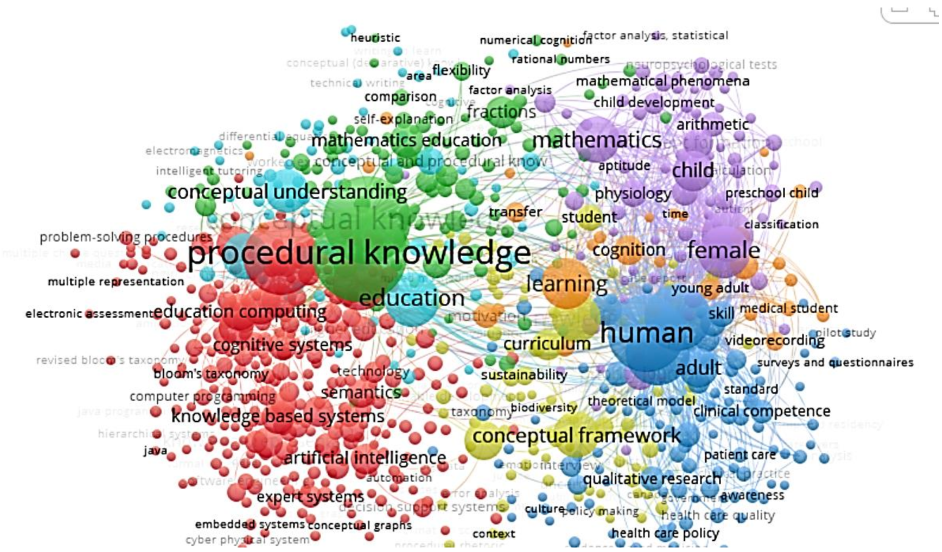

Figure 1. VosViewer view of conceptual and procedural knowledge articles

Based on figure 1, it appears that there are still minimal studies that examine problem-solving procedures and conceptual especially on derivative. Therefore, the purpose of this study is to describe prospective mathematics teachers conceptual and procedural knowledge in solving derivative routine and non-routine problems. This study can help lecturers prepare teaching materials that are more effective and relevant to students’ needs, thereby contributing to the development of the mathematics curriculum at the college level.

▪ METHOD

Research Design

In this study, research design used qualitative with case study approach to describing and analyzing the conceptual and procedural knowledge of prospective mathematics teachers in solving derivative problems. This design was chosen because it provides an ideal framework to explore the phenomenon of prospective mathematics teachers solving problems using conceptual and procedural knowledge.

Participants

The subject of this study is 63 prospective mathematics teacher (PMT) at Tadulako University attending Class B and Class D of the Differential Calculus Course in the 2022/2023 academic year. Subjects were selected using purposive sampling.

Instrument

Data on PMTs' conceptual and procedural knowledge in solving derivatives were collected by administering a test to research subjects. Data collection included observation, and document analysis. The instruments used in this study are three derivative problems that required PMTs' to explain the steps. The questions taken from Varberg & Purcell are as follows:

1. Given the derivative of the function f whose value at c is $f'(c) = \lim_{h\to 0} \frac{(4+h)^2-16}{h}$. Find $f(x)$ and the value of c.
2. For $f(x) = x\sqrt{\sin x}$, find the first derivative of the function f.
3. Find the local extreme values of $f(x) = \frac{1}{3}^{x^3-x^2-3x+4}$ at $(-\infty, +\infty)$.

Problem 1 is a non-routine question, while problems 2 and 3 are routine questions. Usually, the function and the value are known, and the question is about the slope of the tangent to the curve or the function's derivative. In Problem 1, the function's derivative is known, and the question is about the curve and the value. The questions were used to describe the conceptual and procedural knowledge of the PMTs. Before being used, the questions were first validated by experts

Data Analysis

The qualitative data analysis technique used in this study was the data analysis technique outlined by Miles & Huberman which began with data collection through various methods such as test, observation, and document analysis. After data collection, the next step was data reduction, where data were filtered and organized to identify relevant patterns, themes, or categories. Subsequently, the reduced data were presented visually or narratively using techniques such as tables, diagrams, or direct quotes to facilitate understanding and interpretation. The final step was drawing conclusions, where the researcher integrated the findings from the analysis to formulate comprehensive conclusions and provide insights into the researched phenomenon. Kilpatrick's theory was used to analyse PMTs conceptual and procedural knowledge (Salim Nahdi & Gilar Jatisunda, 2020). This technique provided a systematic and holistic approach to dealing with qualitative data, ensuring accuracy and reliability in the analysis. Checking the validity of the data is done using triangulation techniques.

▪ RESULT AND DISSCUSSION

Analysis of Answers to Problem 1

Subjects' answers in solving Problem 1 are grouped under 5 categories, with algorithmic problem-solving characteristics as displayed in the following table:

Table 1. Algorithmic problem-solving characteristics for problem 1

Category	Algorithmic problem-solving characteristics	Number of Subject
1	Did not answer the question.	3
2	The steps did not use the information in the question.	7
3	Solved the problem by determining the derivative of the function f at c or $f'(c)$ but did not continue to the process of finding the $f(x)$ and the value of c.	46
4	Solved the problem by determining $f'(c)$ and proceeded to find the value of c but did not find the function f.	1
5	Solved the problem by manipulating the algebraic form contained in the definition of derivative so that $f(x)$ and the value of c are found correctly.	6

The following are examples of the answers to Problem 1 from Category 3, Category 4, and Category 5 subjects.

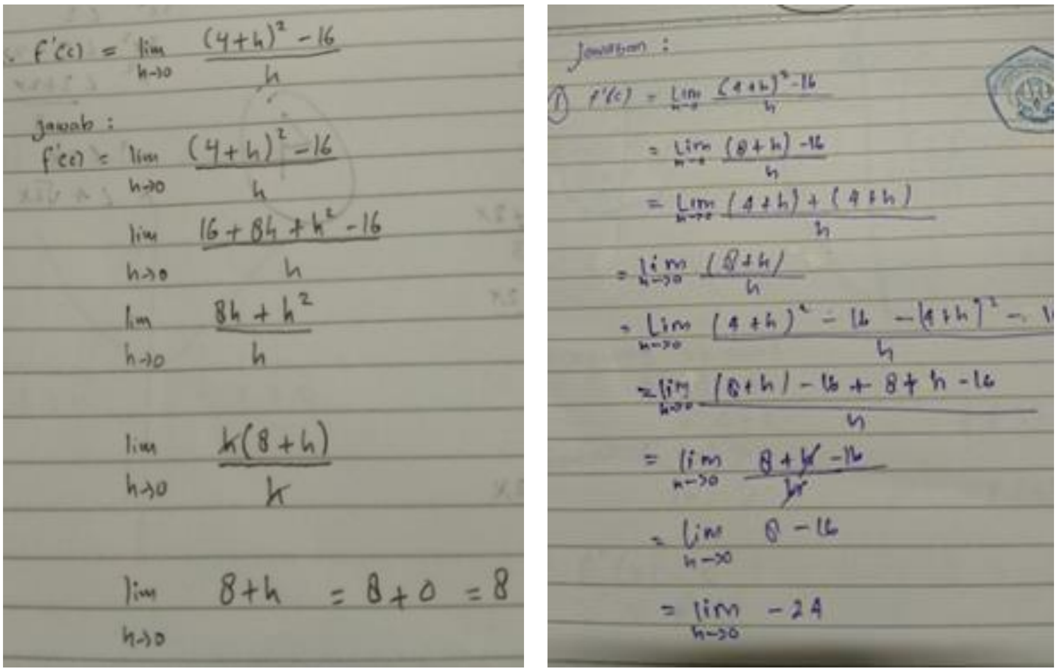

Figure 2. Example of a category 3 subject's answer for problem 1

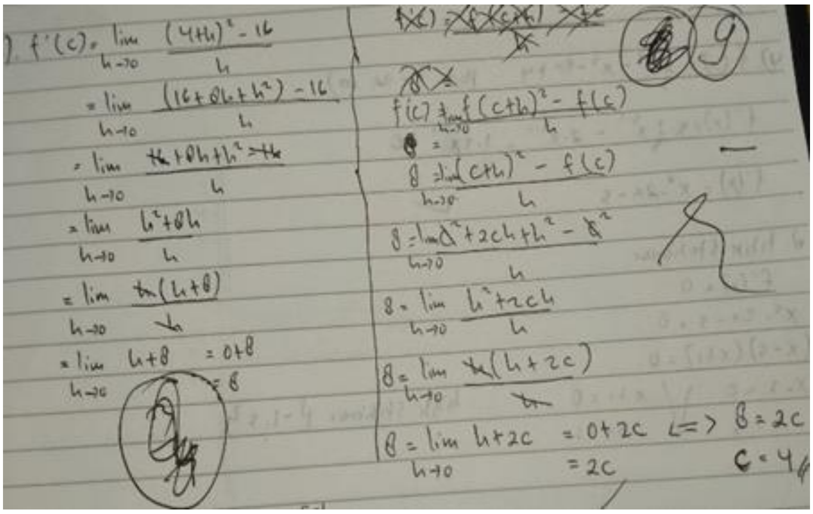

Figure 3. Example of a category 4 subject's answer for problem 1

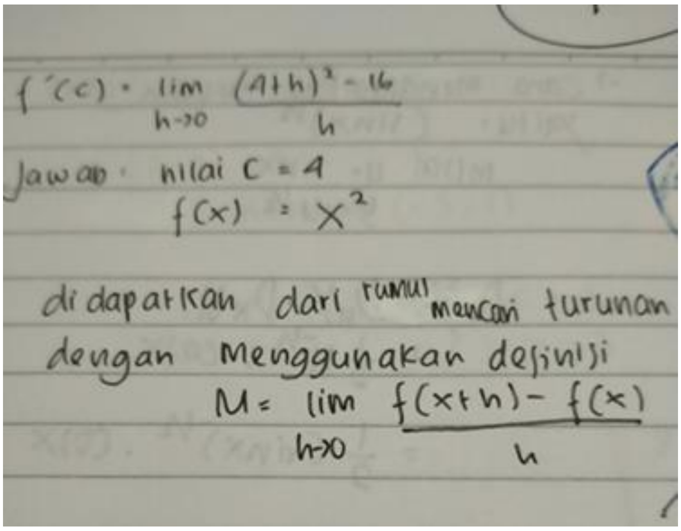

Figure 4. Example of a category 5 subject's answer for problem 1

As seen in Table 1, 74,6% of PMTs solved the problem by finding the derivative of the function f at c denoted by $f'(c)$. Figure 1 shows that the subject went through the process of finding the derivative of the function without realizing that the question was about finding the $f(x)$ and the value of c, and not the $f'(c)$. In Figure 2, the subject applied the steps to find the $f'(c)$ and used the result to find the value of c, but failed to find the $f(x)$, meaning that this PMTs' made an algorithmic error in finding the $f(x)$ and the value of c. The PMTs' did not use the definition of the derivative of the function f at c to find the $f(x)$ and the value of c. Such algorithmic errors can occur due to PMTs' inability to understand the definition of derivatives and the connection between the symbol $f'(c)$ and $f(x)$ as well as the value of c.

In general, the errors made by PMTs' are caused by a weak understanding of the concept of derivatives because they cannot apply the definition of derivatives appropriately in the context of the given problem. The PMTs' do not understand the context of the problem and how the concept of derivative is applied in the situation. They only memorize the derivative procedure or formula without understanding the mathematical meaning behind it. Observation results show that in the learning process, PMTs' struggle to deal with problems involving function limits.

This is in line with other findings that there are still cognitive conflicts in understanding the concept of derivatives (Prihandhika et al., 2022). Although teachers have conveyed concepts in the correct order, the relationship between concepts has not been emphasized (Rosnawati et al., 2020). The ability to understand thoroughly depends on knowledge and relationships between concepts (Zou et al., 2023). Learners who need help understanding concepts well tend to make mistakes in interpreting problems, use inappropriate procedures, or ignore the relationship between concepts (Hurrell, 2021; Jailani et al., 2020; Mutawah et al., 2019).

Analysis of Answers to Problem 2

Subjects' answers in solving Problem 2 fall into 4 categories, with algorithmic problem-solving characteristics as shown in Table 2. The following are examples of the answers to Problem 2 from Category 2 and Category 3 subjects.

Table 2. Algorithmic problem-solving characteristics for problem 2

Category	Algorithmic problem-solving characteristics	Number of Subject
1	Did not answer the question.	2
2	The steps did not follow the Product Rule.	52
3	The steps followed the Product Rule but did not apply the Chain Rule.	6
4	The steps used both the Product Rule and the Chain Rule.	3

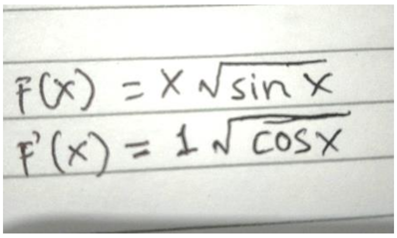

Figure 5. Example of a category 2 subject's answer for problem 2

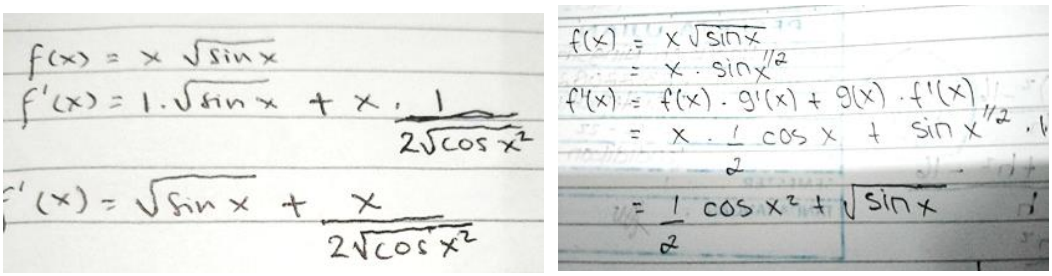

Figure 5. Example of a category 3 subject's answer for problem 2

Table 2 shows that 82.5 % of PMTs' solved the problem without following the Product Rule. As displayed in Figure 4, the subject found the derivative without realizing that the question was about the multiplication of two functions. This subject only used the Rank Rule and did not apply both the Product Rule and the Chain Rule to find the derivative of the product of two functions, meaning that the subject made a procedural error due to the comprehension of the rule that the derivative of the product of functions is the multiplication of its derivatives. Figure 6 reveals that the subject has knowledge of the procedure for finding the derivative of the product of functions and applies the Product Rule. However, this subject did not use the Chain Rule procedure, resulting in an incorrect final result. Such error can occur because the subject does not fully understand the Chain Rule.

Based on the description provided, the errors made by PMTs can be categorized as procedural errors in applying the differentiation rules. PMTs do not use the proper rules to solve the derivative problem of the product of two functions. In addition, PMTs do not choose procedures that are appropriate to the context of the situation and do not follow the steps necessary to solve the problem completely.

In mathematics learning, procedural knowledge supports conceptual knowledge and vice versa (Rittle-Johnson et al., 2015). If PMTs' do not have procedural knowledge or conceptual knowledge, errors will occur when solving mathematical problems. The application of procedural knowledge, which relates to various problem-solving techniques including the development and manipulation of procedures, requires conceptual knowledge that is rich in conceptual interconnections. Conceptual knowledge facilitates the acquisition of procedural knowledge (Braithwaite & Sprague, 2021). In assisting the selection and development of problem-solving procedures, conceptual knowledge deals with definitions, rules, and principles. When carrying out the process of solving problems, the utilization of basic knowledge, which is procedural knowledge, also affects the final result.

Analysis of Answers to Problem 3

Subjects' answers in solving Problem 3 are divided into 5 categories, with algorithmic problem-solving characteristics as presented in Table 3.

Table 3. Algorithmic problem-solving characteristics for problem 3

Category	Algorithmic problem-solving characteristics	Number of Subject
1	Did not answer the question or solved the problem with steps that did not use derivatives and did not obtain a final answer.	13
2	Solved the problem simply by finding the derivative of the function.	6
3	Solved the problem by looking for stationary points but did not use the steps to find maximum and minimum values.	9
4	Solved the problem by looking for stationary points and proceeded to find maximum and minimum values.	28
5	Solved the problem by looking for stationary points and proceeded to find maximum and minimum values with the correct final answer.	7

The following are examples of the answers to Problem 3 from Category 3 and Category 4 subjects.

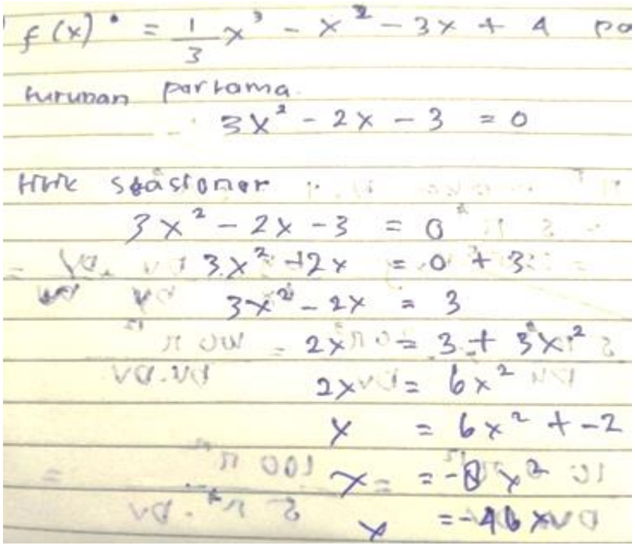

Figure 7. Example of a category 3 subject's answer for problem 3

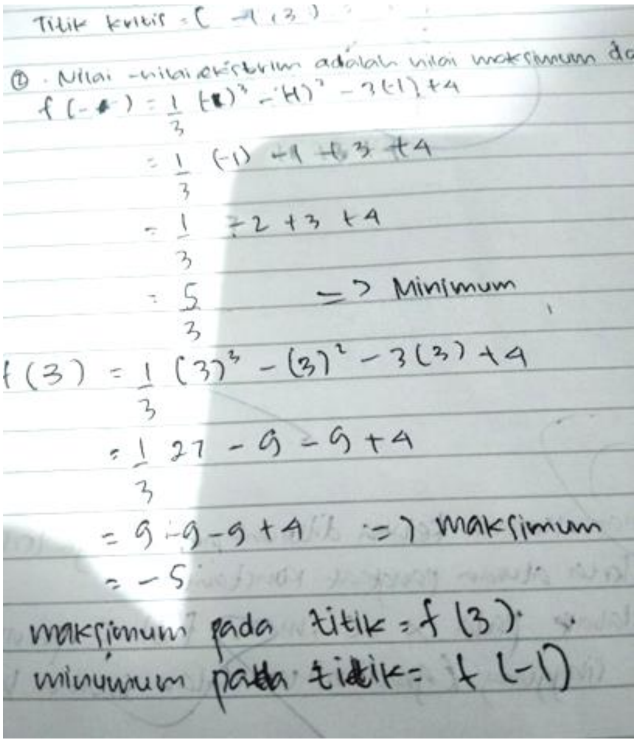

Figure 8. Example of a category 4 subject's answer for problem 3

As presented in Table 3, 55.5 % of PMTs' solved the problem by first finding the stationary points and continuing the steps to find the maximum and minimum values. Figure 7 shows that the subject solved the problem by finding the stationary points using the concept of derivatives but did not proceed to the steps of finding the maximum and minimum values. In applying the concept of derivatives, this subject is not skilled at finding derivatives of functions, as seen in the incorrect answer to the problem. Thus, it can be concluded that this subject has inadequate basic knowledge and understanding of algorithms for finding maximum and minimum values.

PMTs fail to complete the next step after finding the stationary point due to a lack of understanding of the complete algorithm for finding maximum and minimum values, as well as a lack of knowledge of the significance of the stationary point. A stationary point is a point that causes the first derivative of a function to be equal to zero. A stationary point is not necessarily a maximum or minimum point. PMT may not understand that stationary points are only candidates for obtaining maximum or minimum function values and need to be tested further. PMTs only understand the optimization process (searching for stationary points) partially, so they believe that the explanation is correct after finding the stationary point, without realizing that further steps are needed.

Figure 8 indicates that the Category 4 subject determined the maximum and minimum values by using the correct procedure but making calculation errors and mistakes related to the criteria of the maximum and minimum values. This signifies that the subject has procedural knowledge in solving problems but makes mistakes during the addition process and the determination of the criteria of the maximum and minimum values. Such errors may occur because the subject does not properly understand the definition of maximum and minimum values.

▪ CONCLUSION

This study discovered that algorithmic errors in finding the function when the derivative of this function at is known may occur due to PMTs' lack of understanding of the definition of derivatives and the symbols used. Of all subjects, 82.5 % found the derivative of the product of functions without using the Product Rule. The procedural error in finding the derivative of the product of functions is caused by the subjects' comprehension of the rule that the derivative of the product of functions is the multiplication of its derivatives. Furthermore, 55.5% of the subjects found the maximum and minimum values by first determining the stationary points, but only 11% of them found the correct minimum and maximum values. In conclusion, it is crucial for PMTs' to have profound conceptual knowledge for effective selection and development of problem-solving procedures

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