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Analysis of Students' Mathematical Understanding using the Pirie-Kieren Lens

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Abstract: Creating a learning environment that supports students' understanding is an important notion in mathematics education (Carpenter & Lehrer, 1999). In this case, it is necessary to have a flow in the learning process. Learning trajectory will help teachers apply models, teaching material strategies and appropriate assessments according to students' thinking stages. The flow of student learning in the development of symbols and their meaning in mathematics can be done through technology and information (Bakker et al., 2003). The purpose of this study is to analyze the understanding of mathematical concepts in high school students using the Pirie-Kieren theory lens. Based on the results of the analysis of student worksheets for groups with visual, auditory, and kinesthetic learning styles, they have fulfilled the layer of Piere-Kieren understanding up to the stage/organizing layer. The difference in the results of the worksheets of the students in the three groups of learning styles can be seen in their process of finding ways to arrange the candies/boxes equally.

Keywords: mathematics learning, Piere-Kieren theory, high school students.

Abstrak: *Menciptakan lingkungan pembelajaran yang mendukung pemahaman siswa adalah gagasan penting dalam pendidikan matematika (Carpenter & Lehrer, 1999). Dalam hal ini, diperlukan adanya alur dalam proses pembelajaran. Learning trajectory akan membantu guru untuk menerapkan model, strategi bahan ajar dan penilaian yang tepat sesai dengan tahapan berpikir siswa. Alur belajar siswa dalam pengembangan simbol dan maknanya dalam matematika dapat dilakukan melalui teknologi dan informasi (Bakker et al., 2003). Tujuan dari penelitian ini adalah untuk analisis pemahaman konsep matematika pada siswa sekolah menengah dengan menggunakan lensa teori Pirie-Kieren. Berdasarkan hasil analisis lembar kerja peserta didik untuk kelompok dengan gaya belajar visual, auditori, dan kinestetik telah memenuhi lapisan pemahaman Piere-Kieren sampai pada tahap/lapisan organising. Perbedaan hasil lembar kerja peserta didik ketiga kelompok gaya belajar terlihat pada proses mereka mencari cara untuk menyusun permen/kotak menjadi sama rata.*

Kata kunci: *pembelajaran matematika, teori Piere-Kieren, siswa SMA.*

▪ INTRODUCTION

Creating a learning environment that supports students' understanding is an important notion in mathematics education (Carpenter & Lehrer, 1999). In this case, it is necessary to have a flow in the learning process. Learning trajectory is a flow of students' thinking skills and understanding that occurs in learning activities. Learning trajectory will help teachers apply models, teaching material strategies and appropriate assessments according to students' thinking stages. The flow of student learning in the development of symbols and their meanings in mathematics can be done through technology and information (Bakker et al., 2003). The mathematics education community suggests that one way to provide a learning environment that supports understanding of mathematics is to promote the effective use of multiple representations of mathematical ideas. Including some representation in an instructional setting to help students develop a deep

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understanding of mathematics has been highly recommended in the literature (Ainsworth et al., 1998; Gagatsis & Shiakalli, 2004; Kaput, 1998; Lesh et al., 1987; Ng & Lee, 2009). Representation is a kind of configuration process (Goldin & Kaput, 1996) and is a way of presenting something in another situation.

The basic idea behind the urge to deal with multiple representations is that each representation emphasizes some part of a mathematical object to the exclusion of others and students can develop a stronger understanding of these objects by taking advantage of the knowledge of each type of representation (Ainsworth, 1999). In this way, students build connections among representations of mathematical concepts to gain objectified/embodied and generalized mathematical knowledge of that concept (Goldin, 1987; Hiebert, 1988). But connecting several representations offered in a mathematics learning environment is not an easy task for students (Ainsworth et al., 1998). Representation in the teaching process is a tool to support students' mathematical understanding (Salkind, 2007) and help them to organize their thoughts (Cathcart et al., 2006).

Students only take advantage of the benefits of a multi-representational approach if they can build relationships between different representations (Dreher & Kuntze, 2015). The use of multiple teacher representations during instruction is not magic by itself; it is students' sense-making and reasoning activities that are important when they are learning math tasks that involve different representations (Flores et al., 2015).

In reviewing related literature, it can be seen that studies on multiple representation are generally concentrated at the elementary level (Cai, 2004; Gagatsis & Elia, 2004). However, there are a number of undergraduate level studies (Delice & Sevimli, 2010; Even, 1998). In this context, when the study was carried out at the undergraduate level, it appears that the study was mainly concentrated on prospective mathematics teachers. For example, teacher candidates are not very successful in transitioning between representations as a result of his research in which he examined the knowledge of 152 math teacher candidates about subject functions and how the transformations of one representation work (Even, 1998). Another example, students experience many difficulties in changing algebraic representations into verbal representations (Gagatsis & Elia, 2004).

The use of various forms of mathematical representation is important to be the focus of attention in learning mathematics which can be derived from aspects of the process and evaluation of learning mathematics (Afriyani, Sa'dijah, Subanji, & Muksar, 2018). The recent studies mentioned above suggest that there is a need for greater clarification of how a learning environment, enriched with multiple representations of related concepts, can lead to students' understanding of mathematics. The material in mathematical activities is “not representations but their transformations” (Duval, 2006) and understanding of mathematical concepts “involves synergistic coordination of at least two registers of representation” (Duval, 2017). Research conducted by Gulkilik (2020) encourages examining different contexts of learning mathematics using a framework that includes treatment and conversion theory.

Analysis of one student's understanding of mathematics about mathematical concepts based on the main characteristics and features can use the Pirie-Kieren theory. This theory is a growth theory that helps us to analyze continuous processes rather than situational theory which limits understanding of mathematics to instrumental/relational or conceptual/procedural knowledge (Meel, 2003).

The diagram in Figure 1 shows the eight levels of the model as eight embedded circles that one traverses during the growth of mathematical understanding. This model emphasizes that each level contains all the previous levels, and is embedded in all subsequent levels. The circle in the Pirie-Kieren model represents the level of understanding of mathematics.

Primitive knowledge consists of all existing knowledge that a student previously brought into the learning environment except knowledge of tensile glue. At the Image Making level, students make distinctions in their prior knowledge and use it in new ways through mental or physical action. Students have a mental representation of transformation at the Image Having level. At the Noticing Property level, students examine and note differences or connections between images. By consciously reflecting on these traits, students are able to generalize and develop formal mathematical ideas at the Formalization level. At the Observational level, students think about their recent formal ideas and use them to create algorithms or theorems based on these ideas. Therefore, in this study an analysis of the understanding of mathematical concepts in high school students will be carried out using the lens of the Pirie-Kieren theory.

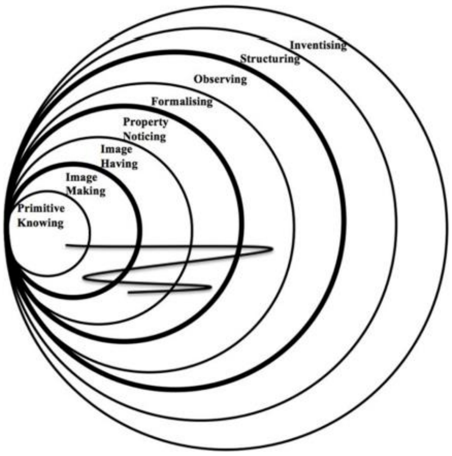

Figure 1. Pirie-Kieren Theory Diagram

▪ METHOD

According to Creswell & Creswell (2018, p.14) the case study research method is a research method that carries out an in-depth analysis of a case. Not infrequently about events, programs, activities, processes, or more than one individual. Researchers collect data with various ways of data collection. This case study method is suitable for use when the research question is a question of why and how. In the case study research method itself there are four types of designs, including: holistic single case design, holistic case design, intertwined single case design, and intertwined case design (Yin, 2014, p. 46). In this study the researcher will use a single case type case study approach.

Participants in this study were vocational school students at SMA N 1 Mantup, totaling 33 students. In this study, qualitative data collection techniques were carried out, namely learning style tests, observations or observations, learning process documents.

The learning style questionnaire uses an instrument consisting of 30 questions to determine auditory, visual, and kinesthetic learning styles. Observations were made to observe students' mathematical understanding by collecting field notes as a non-participant. The researcher is an outsider from the group being studied, witnesses and makes field notes without being directly involved with the group's activities. Meanwhile, the documents used are documents during the learning process in the form of video recordings of learning and the results of students’ worksheets work. This students’ worksheets was compiled by researchers and validated by experts in mathematics.

▪ RESULT AND DISSCUSSION

Analysis of Learning Style Test Results

This research was conducted on March 2, 2023, with a target of class X-1 students at SMA Negeri 1 Mantup with a total of 33 students. Researchers use process differentiation learning on statistics material. From the results of the learning style test analysis, the researcher divided students into 3 groups according to their learning styles, namely visual learning styles, auditory learning styles, and kinesthetic learning styles.

Each group gets a students’ worksheets that suits their learning style. For groups with visual and auditory learning styles, the questions on the students’ worksheets are the same while the groups with kinesthetic learning styles have several differences, because they adjust the learning media they get according to their respective learning styles. The group with the visual learning style was given reading material according to the material and Ms. Excel as their medium for completing students’ worksheets. Then the group with the auditory learning style was given media in the form of a learning video which contained material and questions related to their students’ worksheets. As well as groups with kinesthetic learning styles are given media in the form of visual aids to help them complete questions in students’ worksheets.

Student Worksheets (students’ worksheets) from the three groups based on learning styles (Visual, Auditory, and Kinesthetic) were analyzed to determine students' understanding of mathematics based on the Piere-Kieren Lens. According to Piere and Kieren (1994) Understanding Layer Indicators based on the Piere-Kieren Understanding Theory are as follows: 1) Primitive knowing; 2) Image making; 3) Image having; 4) Property notifications; 5) Formalizing; 6) Organizing; 7) Structuring; 8) Investing.

Analysis of the results of group students’ worksheets based on visual learning styles

The results of group student worksheet analysis based on layers of Piere-Kieren theory understanding are as follows:

Primitive knowing

Primitive knowing indicators appear in questions number 1, 6 and 7. In problem number 1 group 1 with this visual learning style arranges the candies in the following order: 10 chocolate candies, 7 orange candies, 3 matcha candies, 6 strawberry candies and 4 blueberry candies . It turned out that the arrangement that had been prepared by group 1 was in accordance with the information in the problem, both in terms of the number of candies and the order of flavors. Thus, group 1 can mention all the definitions of the terms found in the problem which are indicators of primitive knowing.

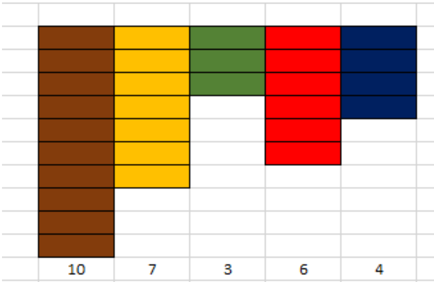

Figure 2. Results of completion of number 1 visual group

In question number 6, group 1 answered correctly according to the number of boxes and the information on the question, which was 10 candies. Thus, the auditory group can understand the new definition or present it which is the definition of primitive knowing.

In question number 7, the auditory group was asked to sort the boxes/candies according to color and number from smallest to largest. Here they have arranged the candies/boxes correctly (from smallest to biggest). The arrangement that the auditory group has arranged is in accordance with the information in the problem, both the number of candies and the order of taste. Thus, the auditory group has fulfilled the primitive knowing understanding layer. From the 3 questions given, the visual learning style group fulfills the primitvie knowing understanding layer.

Image Making

The indicator for understanding image making appears in question number 2. The visual learning style group writes down the number of boxes according to the pictures in Ecxel. Thus, the visual group can create understanding from prior knowledge and use it in new ways which is the very definition of image making.

Image Having

The third indicator of understanding is image having appearing in questions number 3 and 5. The visual group discussed with their group members, they tried several times and asked the model teacher what the question meant and finally after trying and discussing again they were able to answer by generalizing the number of candies. becomes 6. Thus, the visual group is able to get ideas or images that will be used in solving problems which is an indicator of image having.

In problem number 5, the visual group arranged the boxes according to their initial arrangement. Thus, group 1 already has an idea of a topic and makes a mental picture of that topic which is the definition of image having, so the visual group fulfills the layer of understanding image having.

Property Notifications

The fourth indicator appears in question number 8. The visual group can easily find the middle value of the boxes because the amount of data/the number of types of candies is an odd number, so the middle value is in red with a total of 6 (strawberry candies).

Formalizing

The fifth indicator appears in question number 4, the visual group arranges the candies/boxes equally. But the visual group hasn't written down the steps for assembling the candies/boxes in detail, they just described the steps in general. The visual group can find their own concepts and use the concepts found to solve a given problem which is an indicator of formalizing.

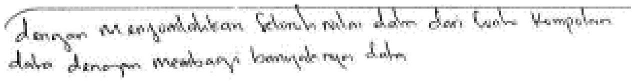

Figure 4. Results of solving problem number 4 visual group

Organizing

The sixth indicator appears in question number 3. As a result of the visual group discussion with their group members, they tried several times and asked the model teacher the meaning of the question and finally after trying and discussing again they were able to answer by generalizing the number of candies to 6. Thus, visual groups can find structured patterns from concepts to solve a given problem. This is an indicator of organization. Of the eight indicators of understanding the Pirie-Kieren theory the visual group raised 6 indicators and 2 other indicators did not appear.

Analysis of the results of group students' worksheets based on auditory learning styles

The results of group student worksheet analysis based on layers of Piere-Kieren theory understanding are as follows:

Primitive Knowing

The first indicator appears on questions 1, 6 and 7. The Auditory Group answers in the order of the boxes: 10 chocolate candies (pink), 7 orange candies (yellow), 3 matcha candies (green), 6 candies strawberry (red color) and 4 blueberry candies (blue color). It turned out that the answers from the auditory group matched the information in the video, it's just that they didn't immediately describe the arrangement of the boxes/candy. Thus, the auditory group fulfills the primitive knowing understanding layer.

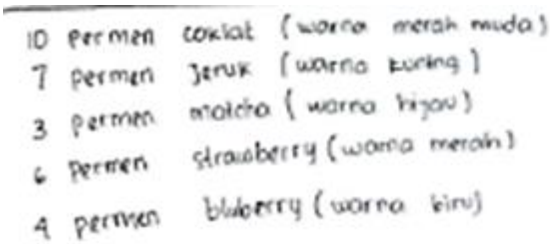

Figure 3. Results of solving problem number 1 auditory group

In question number 6, the auditory group answered correctly, that the color of the box that took the most was brown with a total of 10 candies. Thus, group 2 can understand the new definition or present it which is the definition of primitive knowing.

In question number 7, the auditory group arranged the candies/boxes correctly (from smallest to largest). Thus, group 2 can understand new definitions, bring previous knowledge to the next level of understanding, through actions that involve definitions, or represent definitions. These things are definitions of primitive knowing.

Image Making

The second indicator appears in question number 2. The auditory group draws a box according to the order. Thus, the auditory group can make an understanding of previous knowledge and use it in a new way which is the definition of image making.

Image Having

The third indicator appears in questions 3 and 5. The auditory group discusses with their group members, they generalize the number of candies to 6. The auditory group can arrange the candies equally. Thus, the auditory group is able to get ideas or images that will be used in solving problems which is an indicator of image having. In problem number 5, the auditory group arranges the boxes according to their initial arrangement. Thus, the auditory group already has an idea of a topic and makes a mental picture of the topic which is the definition of image having.

Property Notifications

The fourth indicator appears in question number 8. The auditory group is sorted from smallest to largest. Thus, the auditory group can realize that there is a relationship between the definitions understood and can verify the relationship between these definitions which is an indicator at the property noticing stage.

Formalizing

The fifth indicator appears in question number 4. The results of the auditory group discussion are dividing the candies equally by adding up all the candies first. So that the result is 6. Thus, the auditory group can find their own concept and use the concept found to solve the given problem which is an indicator of formalizing.

Figure 4. Results of solving problem number 4 in the auditory group

Organizing

The sixth indicator appears in question number 3. From the results discussed by group 2 with their group members, they generalized the number of candies to 6 by adding up all the candies and dividing it by the number of candies (type of candies). Thus, group 2 can find a structured pattern of concepts to solve the given problem so group 2 fulfills the organizational layer. Of the eight indicators of understanding the Pirie-Kieren theory the visual group raised 6 indicators and 2 other indicators did not appear.

Analysis of the results of group students' worksheets based on Kinesthetic learning styles

The results of group student worksheet analysis based on layers of Piere-Kieren theory understanding are as follows:

Primitive Knowing

The first indicator appears in question numbers 1, 6 and 7. Group 3 writes down the number of boxes they have taken randomly and according to the number and different color for each box, with the following details: 8 orange boxes, 6 pink boxes, 4 green boxes, 2 blue boxes, and 5 brown boxes. Thus, group 3 can mention all the definitions of the terms found in the problem which are indicators of primitive knowing.

Figure 5. Results of solving problem number 1 for the kinesthetic group

In question number 6, the kinesthetic group answered that the color of the squares that was picked the most was orange with a total of 8 squares. Thus, the kinesthetic group can understand the new definition or present it which is the definition of primitive knowing.

In problem number 7, the kinesthetic group arranged the squares correctly (from smallest to largest). The composition of the kinesthetic group, arrange the number according to the number of boxes on the media that has been provided.

Thus, the kinesthetic group can understand new definitions, bring previous knowledge to the next level of understanding, through actions that involve definitions, or represent definitions. These things are definitions of primitive knowing.

Image Making

The second indicator appears in question number 2. The kinesthetic group describes the number of boxes according to what they have taken from the learning media provided. Thus, the kinesthetic group can make understanding of previous knowledge and use it in a new way which is the definition of image making.

Image Having

The third indicator appears in questions number 3 and 5. The kinesthetic group discussed with their group members, they generalized the number of squares to 5 using the media they got. Thus, the kinesthetic group is able to get ideas or images that will be used in solving problems which is an indicator of image having.

In problem number 5, the kinesthetic group was asked to rearrange the boxes that had been arranged equally into the initial arrangement of boxes according to the color and number. The kinesthetic group arranges the boxes according to orders. Thus, the kinesthetic group already has an idea of a topic and makes a mental picture of that topic which is the definition of image having.

Property Notifications

The fourth indicator appears in question number 8. The kinesthetic group sorts the boxes and gets the result that the middle value is in brown with a total of 5 (brown). Thus, the kinesthetic group fulfills the indicators on the noticing property.

Formalizing

The fifth indicator appears in question number 4. The results of the kinesthetic group discussion are by adding up all then dividing by the number of boxes/candies, the result is 5. Thus, the kinesthetic group can find their own concepts and use the concepts found to solve the given problem which is formalization indicator.

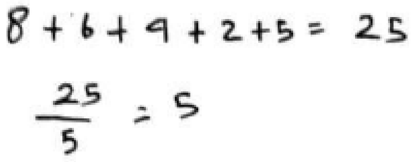

Figure 6. Results of solving problem number 4 for the kinesthetic group

Organizing

The sixth indicator appears in problem number 3. From the results that the kinesthetic group has discussed with their group members, they generalize the number of squares to 5 using the media they get. So the kinesthetic group fulfills the layer of organizational understanding. Of the eight indicators of understanding the Pirie-Kieren theory the visual group raised 6 indicators and 2 other indicators did not appear.

▪ CONCLUSION

Based on the results of the analysis of student worksheets for groups with visual, auditory, and kinesthetic learning styles, they have fulfilled the layer of Piere-Kieren understanding up to the stage/organizing layer. The difference in the results of the worksheets of the students in the three groups of learning styles can be seen in their process of finding ways to arrange the candies/boxes equally. The group with the visual learning style found a way to make it equal by discussing and trying several times on Ms. Excel until it finds the right or equal arrangement. The group with the auditory learning style found a way to make it equal by discussing and adding up all the candy first. After that they divided the number of candies by the number of candies (kinds of candies). The result of this calculation is the number of candy arrangements that can be arranged equally. Finally, the group with the kinesthetic learning style found a way to make it equal, namely by discussing and they immediately compiled the results of the discussion on the learning media that had been provided. They experimented several times to arrange the learning media so that the boxes they took were equally distributed.

▪ REFERENCES

Ainsworth, S. E., Bibby, P. A., & Wood, D. J. (1998). Analysing the costs and benefits of multi-representational learning environments. In P. R. H. P. A. B. & T. de J. (Eds.). M. W. van Someren (Ed.), Learning with multiple representations (pp. 120–134). Oxford: Elsevier Science Ltd.

Ainsworth, S. (1999). The functions of multiple representations. www.elsevier.com/locate/compedu

Bakker, A., Doorman, M., & Drijvers, P. (2003). Design research on IT Design research on how IT may support the development of symbols and meaning in mathematics education.

Cai, J. (2004). Why do U.S. and Chinese students think differently in mathematical problem solving? Impact of early algebra learning and teachers' beliefs. Journal of Mathematical Behavior, 23(2), 135–167.

Carpenter, T. P., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema, & T. A. Romberg (Eds.). Mathematics Classrooms That Promote Understanding, 19–32.

Cathcart, W. G., Pothier, Y. M., Vance, J. H., & Bezuk, N. S. (2006). Learning mathematics in elementary and middle Schools. (4th ed.). N.J.: Merrill/Prentice Hall.

Chuang- Yih Chen. (2002). A hypothetical learning trajectory of arguing statements about geometric figures.

Clements, D. H., & Sarama, J. (2020). Learning and teaching early math: the learning trajectories approach (3rd ed.). Routledge. .

Delice, A., & Sevimli, E. (2010). Ogretmen adaylarinin coklu temsil kullanma becerilerinin problem cozme basarilari yonuyle incelenmesi: Belirli integral ornegi. Educational Sciences: Theory & Practice, 10(1), 111–149.

Dreher, A., & Kuntze, S. (2015). Teachers' professional knowledge and noticing: The case of multiple representations in the mathematics classroom. Educational Studies in Mathematics, 88(1), 89–114.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. In Educational Studies in Mathematics (Vol. 61, Issues 1–2, pp. 103–131).

Duval, R. (2017). Understanding the mathematical way of thinking - The registers of semiotic representations. In Understanding the Mathematical Way of Thinking - The Registers of Semiotic Representations. Springer International Publishing.

Even, R. (1998). Factors involved in linking representations of functions. Journal of Mathematical Behavior, 17(1), 105–121.

Flores, R., Koontz, E., Inan, F. A., & Alagic, M. (2015). Multiple representation instruction first versus traditional algorithmic instruction first: Impact in middle school mathematics classrooms. Educational Studies in Mathematics, 89(2), 267–281.

Gagatsis, A., & Elia, I. (2004). The effects of different modes of representation on mathematical problem solving "Ability to use multiple representations in Functions and Geometry: The Transition from Middle to High school"-Research Promotion Foundation View project.

Gagatsis, A., & Shiakalli, M. (2004). Ability to translate from one representation of the concept of function to another and mathematical problem solving. In Educational Psychology (Vol. 24, Issue 5, pp. 645–657).

Goldin, G. A. (1987). Cognitive representational systems for mathematical problem solving. In Problems of representation in the teaching and learning of mathematics (pp. 125–145). Hillsdale, NJ: Lawrence Erlbaum Associates.

Goldin, Gerald A., & Kaput, J. J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. In Theory of Mathematical learning (1st ed., pp. 397–430). Routledge.

Gravemeijer, K., & Cobb, P. (2006). “Design Research from a Learning Perpective. Routledge.

Hiebert, J. (1988). Theory of developing competence with written mathematical symbols. Educational Studies in Mathematics, 19(3), 333–355.

Kaput, J. J. (1998). Direct all correspondence to. Jmb Journal Of Mathematical Behavior, 17(2), 265–261.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. Problems of Representation in the Teaching and Learning of Mathematics, 21, 33–40.

Meel, D. E. (2003). Models and theories of mathematical understanding: Comparing Pirie and Kieren’s model of the growth of mathematical understanding and APOS theory. USA: American Mathematical Society., 12, 132–182.

Ng, S. F., & Lee, K. (2009). The model method: singapore children’s tool for representing and solving algebraic word. In Source: Journal for Research in Mathematics Education (Vol. 40, Issue 3).

Presmeg, N. C., & Balderas-Cañas, P. E. (2001). Visualization and affect in nonroutine problem solving. Mathematical Thinking and Learning, 3(4), 289–313.

Salkind, G. M. (2007). Mathematical presentation. http://mason.gmu.edu/~gsalkind/portfolio/products/857LitReview.pdf

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. Journal for Research in Mathematics Education, 26(2), 114–145.

Van de Walle, J. A. (2007). Elementary and middle school mathematics: teaching developmentally (6th ed.). Johns Hopkins University.

Yin, R. K. (2014). *Case study research: Design and methods (applied social research methods)* (p. 46). Thousand Oaks, CA: Sage publications.