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Strategies to Minimize Students' Learning Obstacle in Fractions: A Grounded Theory

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Abstract: Fractions are a fundamental mathematical concept that is quite important for life. However, the existing facts prove that fraction concepts in mathematics are a problem that has become a learning obstacle for students in schools. Therefore, this study aims to find alternative strategies to minimize the learning obstacle. The researcher then used a grounded theory design to find this strategy. Researchers also combined research design with phenomenology to find the core categories. Researchers used systematic analysis (open, axial, and selective), combined with thematic analysis with NVivo-12. The analysis results revealed that ontogenic in the form of participants were unable to simplify, determine equivalent fractions, and carelessness was the most significant learning obstacle. The strategy obtained was the use of a didactic design based on a didactical situation in collaboration with didactical design research, which would minimize student learning barriers on fractions.

Keywords: fractions, learning obstacle, grounded theory, phenomenology, didactical situation.

Abstrak: *Pecahan merupakan konsep matematika dasar yang cukup penting bagi kehidupan. Namun, fakta yang ada membuktikan bahwa konsep pecahan dalam matematika merupakan masalah yang menjadi hambatan belajar siswa di sekolah. Oleh karena itu, penelitian ini bertujuan untuk mencari alternatif strategi untuk meminimalkan hambatan belajar. Peneliti kemudian menggunakan desain grounded theory untuk menemukan strategi tersebut. Peneliti juga menggabungkan desain penelitian dengan fenomenologi untuk menemukan kategori inti. Peneliti menggunakan analisis sistematis (terbuka, aksial, dan selektif), dikombinasikan dengan analisis tematik dengan NVivo-12. Hasil analisis menunjukkan bahwa siswa mengalami hambatan ontogenik, seperti siswa tidak dapat menyederhanakan, menentukan pecahan senilai, dan kecerobohan merupakan hambatan belajar yang paling signifikan. Strategi yang diperoleh adalah penggunaan desain didaktis berdasarkan situasi didaktis yang dikombinasikan dengan penelitian desain didaktis, yang akan meminimalkan hambatan belajar siswa dalam pembelajaran pecahan.*

Kata kunci: *pecahan, hambatan belajar, grounded theory, fenomenologi, situasi didaktis.*

▪ INTRODUCTION

The Fractions are one of the most important mathematical concepts (José Luis Cortina, Višnovská, & Zúñiga, 2015; Roni, Zulkardi, & Putri, 2017). Fractions have many benefits, not only for other mathematical concepts (Coetzee & Mammen, 2017; Gagani & Jr, 2019; Johnson & Kuennen, 2006; Lortie-Forgues, Tian, & Siegler, 2015) but also for other disciplines (Ballard & Johnson, 2004; Tian & Siegler, 2016). However, fractions in school are a problem for students. Most of the study participants tended to be unable to interpret fractions' meanings (Li & Kulm, 2008; Nguyen, Duong, & Phan, 2017). Joutsenlahti & Perkkila (2019) revealed that most of the University of Jyavasya and

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University of Tampere students who became participants could not make an illustrative model of a fraction. Besides, about 33.33% of the total participants could not make the illustration model on the number line correctly because they only interpreted fractions as part of the whole.

Solving problems involving fractional operations is also one of the students' problems (Coetzee & Mammen, 2017; Change & Bansilal, 2018). Gagani & Jr (2019) revealed that 100% of participants in the low math ability category, 75% of moderate category participants, and 50% of high category participants could not solve fraction operations problems. Yulianingsih, Febrian, & Dwinata (2018) then revealed that 80% of class VII A students at SMP Negeri 12 One Roof Tanjungpinang who became participants were unable to sort fractions properly. Several other studies also revealed similar results. In other words, fraction order is a problem for most study participants (Malone & Fuchs, 2017; Marmur, Yan, & Zazkis, 2019; Westenskow & Moyer-packenham, 2013).

Given the importance of the concept of fractions and the problems that arise when studying fractions, the researchers conclude that researching with a focus on studying fractions is essential, especially fractions in junior high school. Fraction material in SMP is more complex and has more to do with mathematics or other disciplines (Minister of Education and Culture of the Republic of Indonesia, 2018). When the problem occurs due to external factors, the problem is a learning obstacle (LO) (Brousseau, 2002). LO consists of three types: ontogenic, epistemological, and didactical (Aebi & Linde, 2015; Brousseau, 2002). Ontogenic is related to students' unpreparedness for learning. Students' unpreparedness is related to conceptual, psychological, and instrumental (Suryadi, 2019b). Epistemology is related to students' processes when learning a concept in class (Aebi & Linde, 2015). Didactical deals with didactic designs that teachers use, such as strategies or learning paths that teachers use. The teacher's ability regarding math content is one of the didactical LO indicators (Fauzi & Suryadi, 2020).

Knowing LO and its types is one of the things that is quite important in learning, including learning fractions. The teacher or researcher knows about the weaknesses of learning, contexts, or learning illustration models that easy for students to understand, what prerequisite materials are essential and have a significant effect on learning, and become one of the basics that teachers can use when making didactic designs (Çiltaş & Tatar, 2011; Wijaya, Retnawati, Setyaningrum, Aoyama, & Sugiman, 2019). Which type of LO was the most influential in learning fractions? What are the strategies that teachers can do to minimize the LO? The two questions then become a trigger for researchers in carrying out this research activity. Therefore, the purpose of this study was to determine the type of LO that was most influential when studying fractions and strategies to minimize the LO.

▪ METHOD

Research Design

The researchers used an approach qualitative with the phenomenology type of hermeneutics that the researcher combined with grounded theory. The paradigm that underlies this research is interpretive because it explores the meaning, the experience of meaning, and a theoretical description of strategies to minimize student LO on fraction material (Suryadi, 2019b). This research's primary instrument is the researcher with several additional instruments, such as fraction ability tests, interview guides,

documentation studies (audio, photos, and videos), and research journals (Creswell, 2017). Researchers have carried out this research during the Covid-19 pandemic to conduct research activities and interviews online.

Participants and Research Site

Participants in this study were a mathematics teacher for class VII-8 in the 2019/2020 school year and 35 students of class VIII-8 in the 2020/2021 school year. The researcher chose the class because the class had an average mathematics score of 29.7 or was below the minimum completeness criteria that the school set for mathematics subjects. Researchers use three types of ethics in this study because the participants who participate in research activities are humans. These ethics are informed consent (participant consent), anonymity (not mentioning real names), and confidentiality (confidentiality of the information). The location of this research is one of the junior high schools in West Nusa Tenggara Province. Indonesia's only province with an international motorcycle racing circuit and is trusted to host MotoGP 2022 and World Super Bike 2021. The school was chosen because it is classified as a favorite and is good in achievement, but is still having problems learning fractions.

Data Collection

Researchers as the main instrument require researchers to interact simultaneously with participants (Creswell, 2012). However, researchers compile additional instruments in collecting research data, like fraction ability tests, semi-structured interviews, documentation studies, and daily research journals. Researchers then carry out exams, analyze exam results, determine interview participants, conduct student and teacher interviews, and analyze interview results to confirm the type of LO students experienced.

Data Analysis

Researchers used systematic design analysis for grounded theory research with several steps, such as open, axial, and selective coding (Vollstedt & Rezat, 2019). When doing open coding, researchers used thematic analysis assisted by NVivo-12 software to make it easier for researchers to find themes or core categories. The thematic analysis stages are reading data repeatedly, compiling initial code, searching and reviewing themes, and naming themes according to their characteristics (Nowell, Norris, White, & Moules, 2017).

▪ RESULT AND DISSCUSSION

Which Type of LO was Most Influential in Learning Fractions?

Based on thematic analysis of the student answer sheet data, Table 1 shows the themes of fraction problem formation for all categories of the participants' initial mathematical abilities. The average score that the participants received on the exam was below 75, which was 30.46 that indicates the participants experienced problems or difficulties while learning. Does the mean score indicate participants experiencing LO? Before answering these questions, the researcher confirmed several things about the student participants. There are no participants who experienced pain during the exam. Participants were able to see and listen to the teacher's explanation properly while studying fractions at school. The participants' learning time was right while at school

(Rodriguez, 2012). If the difficulties that students experience are due to external factors, the researcher can conclude that they experience LO (Suryadi, 2019b).

Table 1. The Theme of Formation of Fraction Problems

No.	Case	Theme	References		
			Low	Medium	High
1	Meaning of fractions	Rational Numbers	14	28	19
		Benefits	4	6	2
2	Area model	Already able to illustrate, but wrong	7	4	2
		Unable to make illustrations	6	17	2
3	Number lines	Already able to illustrate, but wrong	6	3	2
		Unable to make illustrations	8	21	2
4	Group of objects	Unable to make illustrations	12	24	24
5	Sequence of fractions	Can not procedure	10	16	16
		Write down question information	2	0	0
		Can not prerequisite material	2	5	5
6	Addition of fractions with unlike denominator	Can not procedure	10	10	10
		Can not prerequisite material	3	9	9
7	Subtracting fractions involving mixed fractions	Can not procedure	8	10	10
		Can not prerequisite material	6	10	10
8	Multiplication of fractions involving decimals	Can not procedure	6	12	12
		Can not prerequisite material	8	5	5
9	Division fractions	Can not procedure	10	15	15
		Inadvertent	2	7	7
10	Solve problems related to sequences and multiplication fractions	Can not procedure	16	24	24
		Can not prerequisite material	2	5	5
		Can not make mathematical modeling	3	12	12
		Write down question information	2	2	2

Researchers ensure that no participant is sick by not asking participants who feel unwell to take the exam activity. Moreover, this is the time of the Covid-19 pandemic, so that the participants' health is the main thing. Researchers confirmed that the participants could see and hear the teacher's explanation well during the interview. All interview participants (informants) were able to see and hear the teacher's explanation well. Regarding the learning time, 7 out of 8 informants stated that the participants' learning time at home was sufficient. These results then indicate that the participants experienced LO while studying fractions at SMP Negeri 1 Narmada.

Based on Table 1, the type of LO that most participants experience in the meaning of fractions is that participants cannot express the meaning of fractions in their entirety. Participants are only able to interpret fractions as rational numbers. Fractions have five meanings that participants must master, namely fractions as a ratio, operator, part of the whole, quotient, and size (Frudenthal, 2002; Kennedy, Tipps, & Johnson, 2008; Kilic, 2015; Roy & Roy, 2008). In this context, the researcher assumed that the type of LO that participants experienced was epistemological. However, after the researcher conducted interviews with several student participants and the researcher teacher obtained information, the student participants could not know the meaning of fractions because the

teacher participants did not know the meaning of fractions. Therefore, the researcher concluded that the type of LO that participants experienced was didactical.

Didactical in the form of limited knowledge of mathematics teachers regarding a mathematical concept is one type of LO which is quite influential in learning. These types of obstacles lead to differences in understanding the concept of fractions between students and mathematicians or what they should be. Besides, low mastery of the teacher's concept will break the knowledge transposition chain (Bosch, Chevallard, & Gascon, 2005; Quessada & Clément, 2007). Mastery of concepts is one of the competencies that teachers must have (Bertschy, Künzli, & Lehmann, 2013; Boyd & Ash, 2018).

In the case of the illustration model, most of the participants were unable to make illustrations. Based on the interviews with several student participants, researchers obtained information that participants tended not to understand the questions' meaning because they rarely got these questions from the teacher. The teacher participants then confirmed this during the interview. In other words, participants experience problems in terms of limited context or models when studying fractions. Researchers categorize this type of LO as epistemological (Brousseau, 2002).

The use of illustrative models in studying mathematics, including studying fractions, is essential (Clinton & Walkington, 2019). The illustration model is a component of the world of mathematics. This component is a bridge between conceptual-embodied and axiomatic-formal (Tall, 2004, 2005, 2008). Some illustrative models that can be an alternative for teachers when learning fractions are area models, object collections, and number lines (Duzenli-Gokalp & Sharma, 2010; Greenberg, 1996).

In the case of fraction sequences, the prerequisite material was a problem for some participants. The prerequisite material includes participants who cannot find equivalent fractions, so that they have difficulty sorting fractions. The fraction equivalence procedure is the main procedure when sorting fractions (Jose Luis Cortina, Zuniga, & Visnovska, 2014; Simon, Placa, Kara, & Avitzur, 2018). Based on the results of interviews with several student participants, the researcher obtained information that participants tended not to find equivalent fractions because they could not become an LCM. The results of interviews with teacher participants also confirmed this information. The teacher participant revealed that most of the participants had difficulty finding the LCM from an integer. Most LO types are conceptual ontogenic. Because they relate to the participant's prerequisite material's unpreparedness when participating in learning activities (Brousseau, 2002).

There are exciting things for researchers when interviewing one of the participants in the high category. The participant clears the procedure for sorting fractions on the answer sheet. At the time of the interview, the participants were able to describe the procedure for sorting fractions correctly. Participants reveal that participants forget when writing procedures. In other words, these participants tend to be careless when working on problems. This type of LO is ontogenic psychology (Brousseau, 2002).

The fraction equivalence problem then reappears in the case of the addition of unequal fractions. In this case, most of the participants experienced difficulties because they could not find equivalent fractions, simplify fractions, and perform integer multiplication operations properly. Finding the equivalent fraction and simplifying fractions then has a strong connection in learning fractions. Simplifying fractions is one example of applying the equivalent fraction procedure (Greenberg, 1996; Iulia & Gugoiu,

2006). Therefore, researchers conclude that the type of LO that most participants experience is conceptually ontogenic (Brousseau, 2002).

In this study, the researcher obtained information that most participants, when adding two dissimilar fractions, tended to add the numerator with the denominator and the denominator with the two available fractions' denominator. This trend is also quite frequent by researchers in several previous studies (Coetzee & Mammen, 2017; Duzenli-Gokalp & Sharma, 2010; Ojose, 2015). Most of the study participants tended to experience the same errors as some participants in previous studies.

In the case of subtracting fractions involving mixed numbers, the problem of simplifying the fraction also arises. Although, there are some other prerequisite materials that participants also experience, such as being unable to convert a mixed number to a common fraction and perform the subtraction operation well. The problem of the participants being careless also reappeared in this case. One of the high category participants forgot to write down the steps to solve the problem and described these steps well at the time of the interview. The types of LO that students experience are conceptual and ontogenic psychology (Brousseau, 2002).

The problem of simplifying fractions and being careless also re-emerged in multiplying fractions involving decimals. Another prerequisite that becomes a problem, in this case, is that some participants cannot convert decimal numbers to regular fractions and change mixed fractions to regular fractions. In this case, converting decimal fractions to regular fractions should also get more attention from researchers. The concept of decimal numbers is essential because someone often uses it when studying mathematical concepts or other disciplines and everyday life (Morais & Serrazina, 2017). The researcher then concluded that most of the type of LO experienced was conceptual and ontogenic psychology (Brousseau, 2002).

In splitting the fraction, the problem of simplifying the fraction and being careless reappears. The carelessness of the participants, in this case, was quite significant. Some of the participants in the low and high categories were wrong when writing questions. Even though the participants are correct in the procedure, the participants' final result will still be wrong when they write the questions wrong. This type of LO is conceptual and ontogenic psychology (Brousseau, 2002). Apart from this research, participants who tend to be careless have also appeared in several previous studies regarding fractions or other mathematical concepts (Fitriani, Turmudi, & Prabawanto, 2018). The problem should get attention in learning or didactic design of fractions in the future.

Inadvertent participants also arose in solving problems of everyday life related to the sequence and multiplication of fractions. Some of the participants wrote wrong questions again and forgot to write down the steps for the problem. Some of the problems that became obstacles for some participants were that they could not make good mathematical modeling and some prerequisite materials. The prerequisite material includes participants who cannot convert ordinary fractions to mixed numbers, change mixed fractions to decimal numbers, and incorrectly multiply.

In this case, there is one exciting problem: most participants tend not to do mathematical modeling well. Only a participant was able to answer the story question correctly. However, after conducting interviews with the participants, the researcher obtained information that the participants only guessed when they answered and could

not correctly describe the steps that the participants wrote to the researcher. In other words, the participant experienced obstacles when working on the problem.

During the interview with several other student participants, the researcher obtained information that some participants could not solve the story problems because they did not understand the questions' meaning. Besides, some participants also revealed that participants rarely get story questions from math teachers. In this context, the researchers concluded that most LO that most participants experienced were conceptual, psychological, and instrumental (Brousseau, 2002).

The researcher then confirmed this during the teacher interview. Teacher participants revealed that teachers tended to use the lecture method, group discussions and rarely gave story questions or daily life problems as the primary input during fraction learning. What tends to cause problems during learning because basically, the essence of learning mathematics is to start learning activities from daily life problems to discovering mathematical concepts through a series of students' ways of thinking (Harel, 2008; Suryadi, 2019a; Tall, 2004, 2006, 2008). Based on the interview results, the researcher concluded that the type of LO was didactical (Brousseau, 2002). At the interview, teacher participants also revealed that the teacher rarely used an illustration model, and only a few procedures used an illustration model when learning fractions. Teachers must use an illustrated model as a link between the contextual world and the formal world of mathematics (Tall, 2004, 2006, 2008). Furthermore, there are several illustrative models that mathematics teachers can use when learning fractions, such as area models, number lines, and collections of objects (Greenberg, 1996; Petit, Laird, & Marsden, 2010).

Based on the description above, the researcher concluded that the type of LO that had the most influence on participants when studying fractions at SMP Negeri 1 Narmada was ontogenic. Most of the cases that researchers have disclosed contain ontogenic LO. However, the most LO type of the participants experienced was didactical for the meaning of fractions. In contrast, the illustrated model was an epistemological LO.

What are the Strategies that Teachers can do to Minimize the LO?

Researchers use systematic analysis in grounded theory to determine strategies that researchers can recommend to minimize LO. Figure 1 presents the results of the axial analysis coding. In the previous description, one of the most common problems for some participants for most of the cases in this study was that participants tended to be unable to simplify fractions and determine equivalent fractions. We are not careful when working on problems. Based on this, the researcher decided to make the problem a core category when doing axial coding. One category of conditions that affects some participants having problems simplifying fractions is the participant's ability to determine the GCD of a number. Based on the interview results, some participants revealed that the participants tended to be less able to find the GCD of the numerator and denominator, so they had difficulty simplifying fractions. This phenomenon is ontogenic (Moru, 2007). Besides, at the interview time, several participants also revealed that the participants experienced problems when forming two equivalent fractions because they could not find the LCM of the two denominators of known fractions. Researchers conclude that one of the causal conditions directly affected the core categories participants' ability to seek GCD and LCM.

One of the strategies teachers can use to solve problems or minimize LO in learning fractions is a didactic design. Based on the theory of didactical situations (TDS) (Brousseau, 2002) and didactical design research (DDR) (Suryadi, 2019a). The didactic design must consist of several components: LO, hypothetical learning trajectory (HLT), didactical situation, and didactical contract (Suryadi, 2019b). The HLT contained in the didactic design must then contain three main components, namely learning objectives, student activities, and a series of student thinking responses during learning (Clements & Sarama, 2004; José Luis Cortina et al., 2015).

The didactic design should facilitate an action, formulation, validation, and institutionalization during fractions learning (Brousseau, 2002; Smaniego & Barrera, 1999). Action situations and formulations aim to shape ways of thinking (WoT), while situations of validation and institutionalization aim to shape ways of understanding (WoU) and the application of WoU in different contexts (Harel, 2008).

One type of didactical contract that can facilitate students in forming didactic situations is a potential a-didactical contract. Type of contract provides students with more opportunities to solve problems (Chacón, 2005). The mathematics teacher did not directly answer the students' questions but instead asked the students to discuss in the milieu so that there is retroaction between students during learning activities (Sollervall & Iglesia, 2016). All the TDS components that the researcher had previously described are in a didactic hypothesis design (prospective analysis). The didactic design of the hypothesis continued with implemented design in learning fractions to minimize the LO students' experience.

When using a potential a-didactical contract, teachers usually use scaffolding with the help of an illustration model, such as the area model. The use of illustrations has many benefits when learning mathematics, such as developing students' mathematical representation abilities, especially visual representations (Salkind & Hjalmarson, 2007). Students' ability to think geometry will be better with this illustration model (Rizkianto, Zulkardi, & Darmawijaya, 2013). Some special conditions affect the didactic design that the teacher or researcher uses, such as the teacher's knowledge of fraction material, learning methods, and illustration models (researchers have previously described). Teachers' knowledge about fraction concepts and procedures (taught knowledge) should not be different from the knowledge of mathematicians (scholarly knowledge) to minimize the difference in knowledge that students have during fractions learning (Bosch et al., 2005).

Teachers' knowledge of learning methods will also affect the didactic design that teachers use. This knowledge is one of the knowledge teachers must master during learning (Galante, 2014; Nessipbayeva, 2012). No matter how good the didactic design the teacher uses, when the math teacher cannot manage the class and interact well with students, the didactic design will not work well (Turmudi, Kusumah, Juandi, & Mulyana, 2014).

Furthermore, some general contextual conditions (intervention conditions) affect didactic design in fraction learning, such as student characteristics and fraction material, student interactions, and student learning time. During learning, student motivation will usually determine the success of a didactic or learning design (Tokan & Imakulata, 2019). Likewise with students' attitudes towards mathematics. Students who have the right motivation and attitudes tend to succeed during learning (Brezavšček, Jerebic, Rus, &

Žnidaršič, 2020). The material characteristics of fractions should also receive attention when composing didactic designs. All fractions' meanings should be contained in a didactic design to make it easier for students to operate fractions and solve daily life problems involving fractions (Frudenthal, 2002; Roy & Roy, 2008). The material's order should get attention because it is related to HLT (Gunawan, Putri, & Zulkardi, 2017; Simon et al., 2018).

Interaction between students must also get attention when implementing the design. Maximizing interaction between students will make it easier for students to solve problems so that mathematical concepts or procedures will be more straightforward for students to construct (Topciu & Myftiu, 2015; Verenikina, 2003). Study time will also influence didactic design. A suitable time allocation and good study time will also help teachers implement didactic designs in the classroom. Students tend to understand a concept more easily when class time is not interrupted by other school activities and occurs at appropriate times, such as in the morning (Lestari, 2014).

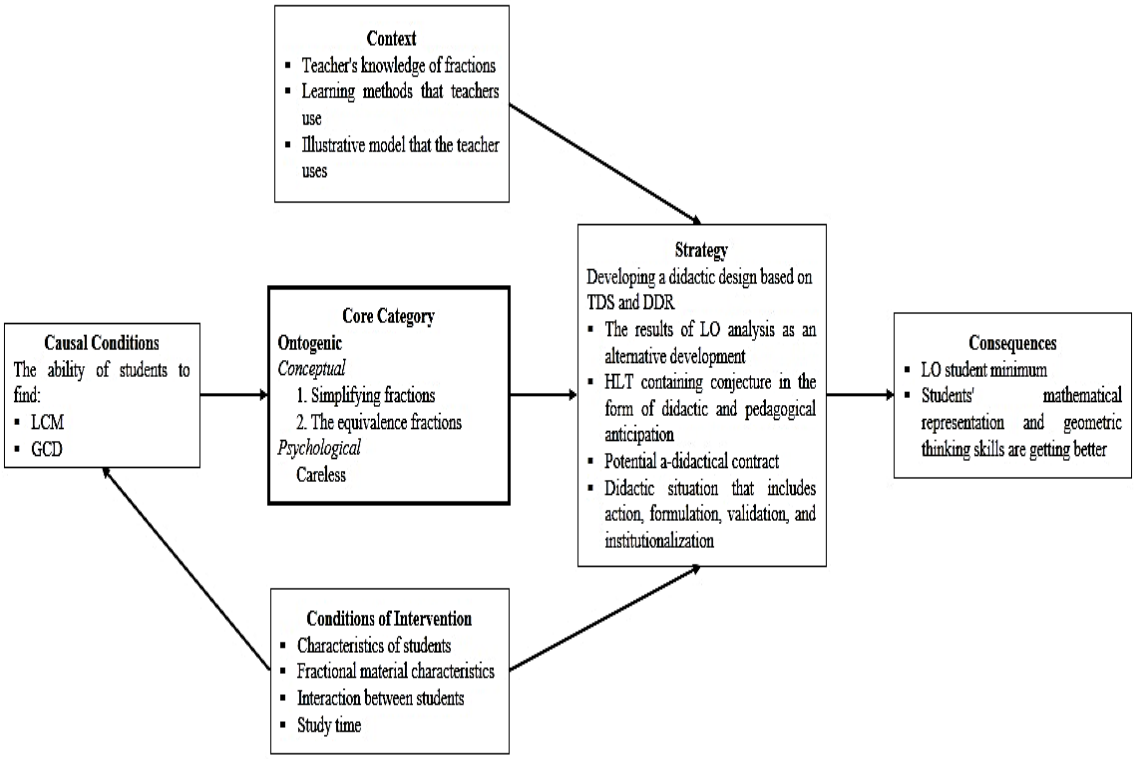

Figure 1. Results of axial coding analysis

▪ CONCLUSION

Researchers conclude that several types of LO occurred when learning fractions. Didactical in the meaning of fractions case. Epistemological in illustration model case. Ontogenic in operations and solving everyday life problems involving fractions cases. However, of the three types of LO, ontogenic is one of the most common problems in most cases. Ontogenic is related to the participants' unpreparedness in terms of prerequisite (conceptual) material and psychology. The prerequisite material relates to some participants being unable to simplify fractions and determine some equivalent

fractions, whereas psychology deals with some participants being careless when working on problems. A theory is formed: "didactic design based on TDS and DDR will be able to minimize the LO that students experience and develop visual representation and geometric thinking skills of students at fraction material." This theory then requires verification by conducting some qualitative or quantitative research (in subsequent studies).

▪ REFERENCES

Aebi, M. F., & Linde, A. (2015). The epistemological obstacles in comparative criminology: A special issue introduction. *European Journal of Criminology*, *12*(4), 381–385.

Ballard, C. L., & Johnson, M. F. (2004). Basic math skills and performance in an introductory economics class. *Journal of Economic Education*, *35*(1), 3–23.

Bertschy, F., Künzli, C., & Lehmann, M. (2013). Teachers' competencies for the implementation of educational offers in the field of education for sustainable development. *Sustainability (Switzerland)*, *5*(12), 5067–5080.

Bosch, M., Chevallard, Y., & Gascon, J. (2005). *Science or magic? The use of models and theories in didactics of mathematics*. Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education, Universitat Ramon Llull, 17-21 February 2005. Sant Feliu de Guixols, Spain: ERME.

Boyd, P., & Ash, A. (2018). Mastery mathematics: Changing teacher beliefs around in-class grouping and mindset. *Teaching and Teacher Education*, *75*, 214–223.

Brezavšček, A., Jerebic, J., Rus, G., & Žnidaršič, A. (2020). Factors influencing mathematics achievement of university students of social sciences. *Mathematics*, *8*(12), 1–24.

Brousseau, G. (2002). *Theory of didactical situations in mathematics*. New York: Kluwer Academic Publishers.

Chacón, A. A. M. (2005). *Difficulties found by the students during the study of substraction of integer numbers.* Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education, Universitat Ramon Llull, 17-21 February 2005. Sant Feliu de Guixols, Spain: ERME.

Çiltaş, A., & Tatar, E. (2011). Diagnosing learning difficulties related to the equation and inequality that contain terms with absolute value. *International Online Journal of Educational Sciences*, *3*(2), 461–473.

Clements, D. H., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning*, *6*(2), 81–89.

Clinton, V., & Walkington, C. (2019). Interest-enhancing approaches to mathematics curriculum design: Illustrations and personalization. *Journal of Educational Research*, *112*(4), 495–511.

Coetzee, J., & Mammen, K. J. (2017). Science and engineering students' difficulties with fractions at entry-level to university. *IEJME*, *12*(3), 281–310.

Cortina, José Luis, Višnovská, J., & Zúñiga, C. (2015). An alternative starting point for fraction instruction. *International Journal for Mathematics Teaching and Learning*, 1–28.

Cortina, Jose Luis, Zuniga, C., & Visnovska, J. (2014). Equipartition as a didactical obstacle in fraction instruction. *Acta Didactica Universitatis Comenianae*

Mathematics, *14*, 1–18.
Creswell, J. W. (2017). *Research design: Pendekatan metode kualitatif, kuantitatif, dan campuran [Research design: Qualitative, quantitative and mixed methods approach]*. Yogyakarta: Pustaka Pelajar.
Creswell, J. W. (2012). Educational research: Planning, conducting, and evaluating quantitative and qualitative research (4th ed.). Boston: Pearson.
Duzenli-Gokalp, N., & Sharma, M. D. (2010). A study on addition and subtraction of fractions: The use of Pirie and Kieren model and hands-on activities. *Procedia-Social and Behavioral Sciences*, *2*(2), 5168–5171.
Fauzi, I., & Suryadi, D. (2020). Learning obstacle the addition and subtraction of fraction in grade 5 Elementary Schools. *MUDARRISA: Jurnal Kajian Pendidikan Islam*, *12*(1), 51–68.
Fitriani, H. N., Turmudi, T., & Prabawanto, S. (2018). Analysis of students error in mathematical problem solving based on Newman's error analysis. *International Conference on Mathematics and Science Education*, *3*, 791–796.
Frudenthal, H. (2002). *Didactical phenomenology of mathematical structurs*. New York: Kluwer Academic Publishers.
Gagani, R. F. M., & Jr, F. M. D. (2019). Characterizing the difficulty in fraction operation. *International Journal of Advanced Research and Publications*, *3*(6), 168–174.
Galante, D. (2014). The use of the history of mathematics in the teaching of preservice teachers. *Redimat*, *3*(2), 110–120.
Greenberg, D. (1996). *Funny & fabulous fraction stories: 30 reproducible math tales and problems to reinforce important fraction skills*. New York: Scholastic Professional Books.
Gunawan, M. S., Putri, R. I. I., & Zulkardi. (2017). Learning fractions through swimming context for elementary school students. *Advances in Social Science, Education and Humanities Research*, *100*, 61–65.
Harel, G. (2008). What is mathematics? A pedagogical answer to a philosophical question. In Bonnie Gold & R. A. Simons (Eds.), *Proof and Other Dilemmas: Mathematics and Philosophy* (pp. 265–290). Washington: The Mathematical Association of America, Inc.
Iulia, & Gugoiu, T. (2006). *The book of fractions*. Mississauga: La Citadelle.
Johnson, M., & Kuennen, E. (2006). Basic math skills and performance in an introductory statistics course. *Journal of Statistics Education*, *14*(2), 1–15.
Joutsenlahti, J., & Perkkila, P. (2019). development in mathematics education-A case study of what kind of meanings do prospective class teachers find for the mathematical symbol "2/3"? *Sustainability*, *11*(457), 1–15.
Kennedy, L. M., Tipps, S., & Johnson, A. (2008). *Guiding children's learning of mathematics* (11th.). Belmont: Thomson Wadsworth.
Kilic, C. (2015). Analyzing pre-service primary teachers' fraction knowledge structures through problem posing. *Eurasia Journal of Mathematics, Science and Technology Education*, *11*(6), 1603–1619.
Lestari, I. (2014). Pengaruh waktu belajar dan minat belajar terhadap hasil belajar matematika [The effect of study time and interest in learning on mathematics learning outcomes]. *Jurnal Formatif*, *3*(2), 115–125.
Li, Y., & Kulm, G. (2008). Knowledge and confidence of pre-service mathematics

teachers: The case of fraction division. *ZDM*, *40*, 833–843.
Lortie-Forgues, H., Tian, J., & Siegler, R. S. (2015). Fraction and decimal arithmetic. *Development Review*, *38*, 201–221.
Malone, A. S., & Fuchs, L. S. (2017). Error paterns in ordering fractions among at-risk fourth-grade students. *Journal of Learning Disabilities*, *176*(5), 139–148.
Marmur, O., Yan, X., & Zazkis, R. (2019). Research in mathematics education fraction images: The case of six and a half. *Research in Mathematics Education*, 1–26.
Morais, C., & Serrazina, L. (2017). Decimal numbers: A potential bridge between rational numbers representations. *Journal of Mathematics Education*, *10*(1).
Moru, E. K. (2007). Talking with the literature on epistemological obstacles. *For the Learning Mathematics*, *27*(3), 34–37.
Nessipbayeva, O. (2012). The competencies of the modern teacher. *International Perspectives on Education*, 148–154.
Nguyen, P. L., Duong, H. T., & Phan, T. C. (2017). Identifying the concept fraction of primary school students: The investigation in Vietnam. *Educational Research and Reviews*, *12*(8), 531–539.
Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic analysis: Striving to meet the trustworthiness criteria. *International Journal of Qualitative Methods*, *16*, 1–13.
Ojose, B. (2015). Students' misconceptions in mathematics: Analysis of remedies and what research says. *Ohio Journal of School Mathematics*, *72*, 30–34.
Petit, M. M., Laird, R. E., & Marsden, E. L. (2010). *A focus on fractions: Bringing research to the classroom*. New York: Routledge.
Quessada, M. P., & Clément, P. (2007). An epistemological approach to French syllabi on human origins during the 19th and 20th centuries. *Science and Education*, *16*(9–10), 991–1006.
Rizkianto, I., Zulkardi, & Darmawijaya. (2013). Constructing geometric properties of rectangle, square, and triangle in the third grade of Indonesian primary schools. *Journal on Mathematics Education*, *4*(2), 160–171.
Rodriguez, M. (2012). The importance teaching of listening and speaking skill. Madrid: Universitas Complvtensis Matritensis.
Roni, A., Zulkardi, Z., & Putri, R. I. I. (2017). Learning divisions of fractions through sprint running pictures. *Journal of Education and Learning*, *11*(4), 381.
Roy, J. R., & Roy, G. (2008). *Holiday fractions (math all around)*. New York: Marshal Cavendish Benchmark.
Salkind, G. M., & Hjalmarson, M. (2007). *Mathematical representations*. Virginia: George Mason University.
Simon, M. A., Placa, N., Kara, M., & Avitzur, A. (2018). Empirically-based hypothetical learning trajectories for fraction concepts: Products of the learning through activity research program. *Journal of Mathematical Behavior*, 1–13.
Smaniego, A. H. F., & Barrera, S. V. (1999). Brousseau in action: Didactical situation for learning how to graph functions. *ATCM*, 1–9.
Sollervall, H., & Iglesia, D. G. de la. (2016). *Designing a didactical situation with mobile and web technologies.* Proceedings of 9th Congress of European Research in Mathematics Education, Charles University in Prague, 4-8 February 2015. Prague: Charles University in Prague.

Suryadi, D. (2019a). *Landasan filosofis penelitian desain didaktis (DDR) [Philosophical foundations of didactical design research (DDR)]*. Bandung: Pusat Pengembangan DDR Indonesia.

Suryadi, D. (2019b). *Penelitian desain didaktis (DDR) dan implementasinya [Didactical design research (DDR) and its implementation]*. Bandung: Gapura Press.

Tall, D. (2004). Building theories: The three worlds of mathematics. *For the Learning of Mathematics*, 24(1), 29–32.

Tall, D. (2006). A theory of mathematical growth through embodiment, symbolism and proof. *Annales de Didactique et de Sciences Cognitive*, 11, 195-215.

Tall, D. (2008). The transition to formal thinking in mathematics. *Mathematics Education Research Journal*, *20*(2), 5–24.

Tian, J., & Siegler, R. S. (2016). Fractions learning in children with mathematics difficulties. *Journal of Learning Disabilities*, 1–24.

Tokan, M. K., & Imakulata, M. M. (2019). The effect of motivation and learning behaviour on student achievement. *South African Journal of Education*, *39*(1), 1–8.

Topciu, M., & Myftiu, J. (2015). Vygotsky theory on social interaction and its influence on the development of pre-school. *European Journal of Social Science Education and Research*, *2*(3), 172–179.

Turmudi, T., Kusumah, Y., Juandi, D., & Mulyana, E. (2014). Development of didactical design of mathematics pedagogy through professional program of mathematics teacher education. *Jurnal Pendidikan dan Pembelajaran Universitas Negeri Malang*, *21*(1), 10–23.

Ubah, I. J. A., & Bansilal, S. (2018). Pre-service primary mathematics teachers' understanding of fractions: An action-process-object-schema perspective. *South African Journal of Childhood Education*, *8*(2), 1–12.

Verenikina, I. (2003). Vygotsky's socio-cultural theory and the zone of proximal development. In E. H. Hasan, E. Gould, & I. Verenikina (Eds.), *Information Systems and Activity Theory: Expanding the Horizon* (pp. 1–8). Wollongong: University of Wollongong Press.

Vollstedt, M., & Rezat, S. (2019). *An introduction to grounded theory with a special focus on axial coding and the coding paradigm*. Hamburg: Springer Open.

Westenskow, A., & Moyer-packenham, P. (2013). Using an iceberg intervention model to understand equivalent fraction learning when students with mathematical learning difficulties use different manipulatives. *International Journal of Technology in Mathematics Education, 23*(2), 45–62.

Wijaya, A., Retnawati, H., Setyaningrum, W., Aoyama, K., & Sugiman. (2019). Diagnosing students' learning difficulties in the eyes of Indonesian mathematics teachers. *Journal on Mathematics Education*, *10*(3), 357–364.

Yulianingsih, A., Febrian, & Dwinata, A. (2018). Analisis kesalahan konsep pecahan pada siswa kelas vii a SMP Negeri 13 Satu Atap Tanjungpinang [Analysis of the error concept of fractions in class VII a SMP Negeri 13 one roof Tanjungpinang]. *Mosharafa: Jurnal Pendidikan Matematika*, *7*(2), 199–206.