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Analysis of Pre-service Mathematics Teachers' Representational Ability Regarding the Prime Numbers Concept

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Abstract: A deep understanding of prime numbers remains a significant challenge, particularly for pre-service mathematics teachers responsible for conveying this essential knowledge to their students. Prime numbers have an essential role in mathematics, including in factorization, proof, and cryptography. Therefore, this study aims to describe and analyze the representational abilities of prospective mathematics teacher students at Alkhairaat University, regarding the prime numbers in Number Theory. This study uses a descriptive exploratory method with a mixed-methods approach based on a sequential explanatory design. Data collection was carried out through diagnostic tests, semi-structured interviews, and documentation, which were then analyzed using descriptive statistics for quantitative data and the Miles & Huberman model for qualitative data. Triangulation of methods and time was used to ensure the validity of the data. The findings showed that out of 17 students, only 29.41% were able to provide accurate verbal representations related to the prime and composite numbers, and only 5.88% were able to link the two meaningfully. In addition, while students were able to mention definitions, most of them had difficulties applying the prime numbers in the abstract problem context, including examples in algebraic notation. This study concludes that there is still a significant gap in the representation ability of prime number concepts of pre-service mathematics teachers. The implications emphasize the need for integrating multidimensional representation learning strategies into the mathematics education curriculum so that prospective teachers are better prepared to teach prime numbers in a comprehensive and meaningful way.

Keywords: representation, prime numbers, number theory, pre-service mathematics teachers.

▪ INTRODUCTION

The Number Theory class speaks about a lot of subjects, such as prime numbers. This theory is very important in math and has a significant impact on a lot of hard fields, such as factorization, cryptography, and mathematical proofs (Curtis & Tularam, 2011; Zaman, 2024). Prime numbers are numbers that can only be divided by one and themselves (Dehmeche et al., 2024). They were originally taught in elementary school and are currently being studied in college (Gürefe & Aktaş, 2020). Even if prime numbers were introduced early on, they are still hard to fully understand, especially for pre-service mathematics teachers (Funeme & Lopez, 2022; Gürefe & Aktaş, 2020). Pre-service mathematics teachers need to understand many aspects of math to convey these ideas to children in a way that is both correct and useful (Gürefe & Aktaş, 2020; Li, 2020; Mainali, 2021). This is crucial because if students don't understand basic concepts like prime numbers, it can be more difficult for them to learn arithmetic in class, which can make it harder for them to understand other concepts (Piñeiro et al., 2021; Ramsingh, 2020).

One of the most important abilities for comprehension and expression of the concept of prime numbers is being able to represent and create mathematically (Gürefe & Aktaş, 2020; Zazkis & Liljedahl, 2004). Representation is about how well students can

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utilize symbols, graphs, pictures, notes, phrases, and other things to represent and explain math ideas (Mainali, 2021). This approach of talking about math makes it easier for individuals to understand ideas instinctively (Mainali, 2021; Zazkis & Liljedahl, 2004). According to Mainali (2021), there are four types of mathematical representations: 1) verbal, 2) pictorial, 3) algebraic, and 4) numerical representations. Verbal representation is the most important and crucial sort of representation when it comes to prime numbers since it requires a lot of knowledge about definitions, qualities, and the ability to communicate about things.

Although the representation aspect is essential, research related to the representation ability of prime number concepts among prospective mathematics teachers is still relatively limited. Previous research often focuses on procedural knowledge or surface-level conceptual knowledge without deeply analyzing the representational ability and reasoning patterns behind it (Loconsole & Regolin, 2022; Picado-Alfaro, 2021; Piñeiro et al., 2021). Aziz et al. (2019) investigated how to understand absolute value, Ganeeva & Anisimova (2020) examined arithmetic, Li (2020) studied teachers' knowledge in teaching arithmetic and numbers, Martin (2021) investigated how to reason about multiplication, and Akkurt & Durmuş (2022) investigated patterns of evidence in pre-service teachers. In fact, as prospective educators, they are not only expected to understand concepts but also to present them effectively so that students can easily understand and apply them (Gürefe & Aktaş, 2020; Li, 2020). Prospective teachers in various countries still experience difficulties in building conceptual thinking pathways (cognitive pathways) related to prime numbers, especially when connected to algebraic proofs or expressions (Zazkis & Liljedahl, 2004) and mathematical reasoning about prime numbers that is sometimes abductive (Hjelte et al., 2020; Jeannotte & Kieran, 2017). Such reasoning skills are essential for future teachers, who will ultimately develop logical and flexible mathematical thinking in students in schools (Brookhart, 2024).

Teaching and learning of prime numbers must also be contextualized within the Indonesian curriculum, which traditionally prioritizes procedural knowledge and correct answers, potentially limiting the growth of representational and reasoning competencies (Aziz et al., 2019; Nurdiana et al., 2021). In addition, as highlighted by Kong (2019), even elementary school students can develop computational thinking around prime numbers through creative pedagogies such as app development, which demonstrate how flexible representational models can be integrated into practice.

Considering the unique sociocultural and curriculum features in Indonesia, there is an urgent need for a study that explores how pre-service teachers construct, justify, and communicate the concept of prime numbers through representational and reasoning frameworks that are relevant to their future teaching practices (Akkurt & Durmuş, 2022; Brookhart, 2024). The mathematics curriculum in Indonesia, both in schools and universities, tends to emphasize procedural and memorization approaches, while conceptual representation aspects have not received much attention. In addition, a learning culture that emphasizes the “right” answer rather than the thinking process also affects the way students represent mathematical concepts flexibly and meaningfully (Aziz et al., 2019; Nurdiana et al., 2021). This highlights the need for contextual research to ensure that the study results align with the characteristics of Indonesian learning culture, curriculum, and pedagogy approaches.

While research related to prime number representation has been conducted in several countries, such as Ustunsoy et al. (2011) who examined the representational abilities of PGSD students in Canada, Zazkis & Liljedahl (2004) who examined students' problem-solving abilities in Türkiye, and Funeme & Lopez (2022) who explored teachers' knowledge of prime numbers through the didactic-mathematical knowledge model (DMK), the context of pre-service mathematics teacher students in Indonesia itself has not been studied in depth. In addition, previous studies used a descriptive qualitative approach (Zazkis & Liljedahl, 2004) or a quantitative descriptive approach (Ustunsoy et al., 2011). This approach is indeed useful, but it has limitations in revealing the deep relationship between quantitative data and students' reasoning narratives in an integrated manner. Therefore, this study chose a mixed-methods approach with a sequential explanatory design in order to be able to capture a more complete and comprehensive picture of student representation. Overseas studies have utilized mixed methods to combine performance data and cognitive interviews (Alcock et al., 2016). Quantitative data from diagnostic tests will provide a general portrait of students' representational abilities, while qualitative data from in-depth interviews will explore the thought processes, strategies, and misconceptions that occur. With this combination, the study is expected to be able to answer not only what students understand or do not understand, but also why they represent the concept of prime numbers in a certain way.

This study will hopefully add to what already exists about mathematics knowledge of pre-service math teachers (Ball et al., 2008; Li, 2020) and how to teach it (Jeannotte & Kieran, 2017). The findings could also help schools teach basic math better by making teacher education programs better, especially in preparing new teachers to construct clear and useful idea maps.

This study aims to systematically describe and analyze the mathematical representational and reasoning abilities of pre-service mathematics teachers at Alkhairaat University related to the concept of prime numbers in number theory courses. Using a mixed-methods sequential explanatory design, this study will capture an overview through diagnostic tests and deeper insights through semi-structured interviews. This study aims to answer the following research questions: "How are the verbal and algebraic representational abilities of prospective mathematics teachers related to the concept of prime numbers and composite numbers?"

▪ METHOD

Participant

The population of this study was 17 pre-service mathematics teacher students at the Mathematics Education Study Program, Alkhairaat University, First semester of the 2023/2024 Academic Year. The selection of interview subjects (informants) was carried out using purposive sampling to explore their knowledge of prime and composite numbers. From the diagnostic test, five students (S1–S5) were selected who gave “interesting” answers, namely referring to responses that were different from the actual concept or the existence of unique misconceptions that could reveal cognitive pathways worth investigating. This criterion is intended to explain how prospective teachers construct their mathematical representations. In addition, the selection of informants was based on their ability to communicate (express their ideas) and represent answers that were almost the same as other research subjects.

Research Design

The purpose of this study is to explain and analyze how effectively pre-service math teachers can represent the concept of prime numbers in the Number Theory course. This study combines a mixed methods approach using a descriptive exploratory approach. The research's methodology is a sequential explanatory design, which implies that both quantitative and qualitative approaches are utilized one after the other (Bascones et al., 2024; Khalil et al., 2024).

The sequential explanatory technique is thought to be the best way to address this research issue since it may combine the strengths of both quantitative and qualitative data in a single analysis (Nabayra & Jr, 2022; Sánchez-Jiménez et al., 2025). The first step of the study was a quantitative diagnostic test to find out how well students could describe things in general. This gave an idea of how many students understood the representation of the idea of prime numbers. However, just looking at numbers is not enough to explain why certain students can develop representations of the idea and others cannot, or how their thought processes work.

Procedure

The research procedure was carried out through 6 (six) steps as follows (Suciati & Wahyuni, 2018) are 1) collecting quantitative data through diagnostic tests, 2) quantitative data analysis using descriptive statistics, 3) selection of subjects for interview based on diagnostic test results, 4) collecting qualitative data through interviews, 5) qualitative data analysis using the Miles & Huberman model, 6) combined interpretation of quantitative and qualitative data. The flowchart research procedure is presented in Figure 1.

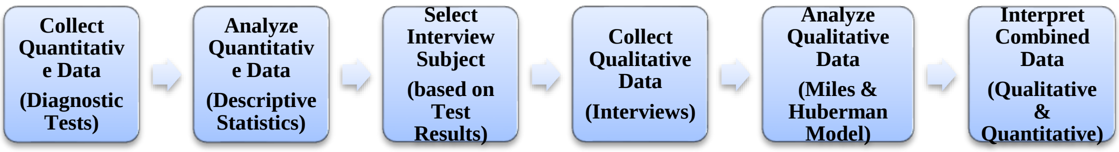


Figure 1. Flowchart research procedure

Instruments

Data collection was carried out through diagnostic tests, interviews, and documentation. The diagnostic test consists of 3 (three) questions adapted from Zazkis & Liljedahl (2004), which have also been used by Ustunsoy et al. (2011). These questions measure students' verbal representation abilities related to prime numbers and composite numbers concepts. The questions in the diagnostic test are as follows:

1. How to describe prime numbers and composite numbers? What is the relationship between the two? Try to explain.
2. Suppose F= 151 × 157. Is F a prime number? Circle (YES/NO). Explain your opinion.
3. Suppose m(2k+1), where m and k are integers. Is this a prime number? Can it be the best?

This diagnostic instrument was validated by experts, including two mathematics education lecturers who reviewed the content validity, linguistic clarity, and relevance to the indicators of mathematical representation. Their feedback helped improve the language and formulation of the questions so that they were in line with the traits of pre-service math teachers. The validated instrument is summarized in Table 1, which presents the diagnostic test grid along with the indicators being measured.

Table 1. Diagnostic test grid

No.	Indicators	Question	Representation Form
1.	Explaining the definition of prime numbers and composite numbers verbally	"How to describe prime numbers and composite numbers? What is the relationship between the two? Try to explain"	Verbal
2.	Analyzing the prime properties of the product of two prime numbers	"Suppose F= 151 × 157. Is F a prime number? Circle (YES/NO). Explain your opinion."	Verbal + numeric (check results)
3.	Analyze simple algebraic expressions involving prime and odd numbers.	Suppose m(2k+1), where m and k are integers. Is this a prime number? Can it be the best?	Verbal + algebra

Assessment of student answers uses an analytical rubric (Table 2) that covers four aspects: 1) Conceptual accuracy, 2) Suitability of the representation form to the context of the problem, 3) Clarity of reasoning or argument, and 4) Complexity of answer delivery. This rubric is used to code and quantify student answers, as well as being the basis for selecting subjects for interviews. Interviews were used to determine the suitability of data obtained from student answer sheets and oral responses related to prime and composite numbers. Interviews were used to determine the consistency and suitability of data obtained from student answer sheets and their oral responses regarding the concepts of prime and composite numbers. These interviews were conducted in a semi-structured manner. While giving students flexibility to elaborate on their ideas, the interviewer relied on a flexible interview guide drawn directly from their answers on the diagnostic tests. This ensured that questioning stayed relevant to the evidence of students' actual written thinking.

The percentage of incorrect answers in several categories from the quantitative phase had a direct impact on the questions asked during the interviews. Many students either got question 2 wrong or did not finish the explanation. The question asked them to work out if F=151×157 is a prime number. This pattern in numbers was a clue that interviews needed to go deeper. The semi-structured interview had probing questions that were meant to clear up or add to unclear written answers. For example:

- "Can you explain why you thought *F* was prime or not prime?"
- "How did you think about whether a product of two large numbers was prime?"
- "What other ways could you check if these numbers are prime?"

These probing questions were changed in real time based on how students answered, but they always tried to find out how students built, defended, or even went against their earlier replies. This process created a cause-and-effect link between the two research phases: results from the diagnostic tests (quantitative) not only identified gaps in students' understanding but also generated focused, meaningful follow-up questions for the qualitative interviews. This ensured that the qualitative exploration was not random, but systematically derived from patterns of student misconceptions and incomplete reasoning identified earlier, thus strengthening the coherence and validity of the mixed-methods sequential explanatory approach.

Another tool employed in this study was documentation, which was used to collect and organize research data in the form of student answer sheets. These answer sheets indicated how participants responded to the questions on the diagnostic examinations in writing. They showed how they explained their comprehension of prime and composite numbers in their own words. The documentation also helped the researcher make sure that what the students wrote was the same as what they said in interviews later. Also, the answer sheets from the students were used as a reference to find any mistakes, misunderstandings, or holes in their logic. This information was subsequently utilized to select subjects to interview and arrive at probing questions. This documentation maintained a clear record of the research process while making sure the information could be observed and followed throughout every step of the investigation.

Table 2. Analytical rubric

Aspects	Score 4 (Very Good)	Score 3 (Good)	Score 2 (Enough)	Score 1 (Less)
Conceptual Accuracy	The concept is correct and complete, according to the academic definition, without misconceptions.	The general concept is correct; there are just a few flaws.	The concept is partly correct but incomplete; there are minor misconceptions.	The concept is fundamentally wrong or completely inappropriate.
Suitability of Representation Form	The form of representation is very appropriate and fits the context of the question, supporting the answer in its entirety.	The form of representation is quite precise and relevant, although not yet fully optimal.	The representation chosen is less appropriate to the context, but is still understandable.	Representation is not in context or does not support understanding of the question.
Clarity of Reasoning/ Argument	The flow of reasoning is clear, logical, and coherent, and supports the argument.	The reasoning is clear, partly logical, although less coherent.	The reasoning is quite vague, and there are some illogical parts.	The reasoning is unclear, illogical, or does not support the answer.
Complexity of Answer Delivery	In-depth answers, displaying critical thinking and rich elaboration.	Quite in-depth answer, there is an attempt to explain more than just a definition.	Simple answer, just answers the surface without elaboration.	The answers are very shallow, just copying the definition, or even completely wrong.

Data Analysis

The collected data was then analyzed using two techniques. The first analysis concerned quantitative data to determine the percentage of student answers that describe, using descriptive statistics. The second analysis related to qualitative data, which was analyzed using the Miles & Huberman Model, consisting of data collection, data reduction, data display, and conclusion drawing (Uyar, 2023).

At this stage, data were obtained from the results of diagnostic tests (17 students), documentation of answer sheets, and semi-structured interviews with five selected students. The interviews then explored further how students explained or justified these answers. In the data reduction stage, all interview transcripts were first transcribed verbatim from audio recordings. Then, the researcher conducted open coding, in which meaningful segments were identified and labeled with initial codes reflecting the students' ideas, reasoning patterns, or misconceptions. For Example:

- The quote "F = 151 × 157 is prime because 151 and 157 are primes" was coded as a misconception of prime product.
- The quote "prime numbers have only two factors" is coded as a correct conceptual definition.

Axial coding arrived after open coding. This is the process of grouping similar first codes into larger topics. Some of the things that came up during this process were not fully understanding what prime numbers are, having inaccurate beliefs about how to factor, and having problems utilizing symbols or mathematics to explain ideas. After that, the coded data was shown in tables and tales that described each person who answered. The graphic depiction of the data helped researchers uncover spots where people were more likely to have wrong ideas and observe where there were gaps in representation between verbal and symbolic representations.

The results of the diagnostic test are evaluated to determine the number of correct and incorrect answers of students to reach a conclusion. After that, the interview data was used to find out the reasons answers were given and whether they were based on a good understanding of the ideas or were inaccurate. The students' answer sheets are evaluated to determine how closely their written responses align with the oral explanations they provided during interviews. This consistency is a key indicator of how stable students' conceptual frameworks are.

In addition, to obtain data validity, triangulation was carried out based on method and time (Hajerina et al., 2022; Suciati & Wahyuni, 2018). Method triangulation is done to check the validity of various data collection methods. In this case, it is the results of diagnostic tests and interviews. Time triangulation is used to check the data validity taken at different times. In this case, it is related to the time of the diagnostic test and the time of the interview.

▪ RESULT AND DISSCUSSION

Prime numbers are one of the main topics in Number Theory, which cannot be separated from the relationship between the multiplication of natural numbers, namely factors, multiples, composite numbers, and divisibility (Zazkis & Liljedahl, 2004). A student is said to have understood the concept of prime numbers if at least he knows that:

1. Natural numbers greater than 1 are prime or composite numbers (able to explain the definition of prime numbers).
2. If a number is represented as a product, the number is composite unless its factors are 1 and the number itself (a prime number).

3. Composite numbers have a unique prime decomposition and an infinite number of primes. (Ustunsoy et al., 2011; Zazkis & Liljedahl, 2004)

For question 1, "What do you think about prime numbers and composite numbers? What is the relationship between prime numbers and composite numbers? Try to explain". In question number 1 there are three important points, namely prime numbers, composite numbers, and the relationship between the two. Therefore, the description of the answer will be divided into three tables, where Table 3 is about prime numbers, Table 4 is about composite numbers, and Table 5 is about the relationship between prime and composite numbers.

Table 3 is the result of grouping the answers of 17 students based on the definition of prime numbers. A total of 41.18% of students answered that a prime number is a number that can only be divided by 1 and itself. Although almost all students answered the concept of prime numbers like that, in reality, the sentence "can only be divided by 1 and itself" causes ambiguity in considering the priority of the number 1, because in mathematical conventions the number 1 is not a prime number. Therefore, the most accurate answer related to the indicator of the nature of prime numbers is "has exactly two factors" (Ustunsoy et al., 2011; Zazkis & Liljedahl, 2004). Based on the diagnostic test, only 29.41% of students mentioned the indicator of the nature of prime numbers, namely "having exactly two factors". The answer "divisible by the number itself" was answered by 11.76% of students. This answer is not wrong because prime numbers can be divisible by the number itself. However, this answer gives the impression that prime numbers only have 1 factor, when prime numbers have two factors, namely the number 1 and themselves. In addition, 5.88% of students answered "In prime numbers, there are even numbers, namely two, and there are also odd numbers". This answer is also not wrong, because in reality, in prime numbers, there are even numbers, namely 2, and the others are odd numbers. However, this statement does not describe the indicator of prime numbers. In addition, not all odd numbers are prime numbers. Only 11.76% did not answer the question given because students had forgotten the concept of prime and composite numbers. From the description in Table 1, it can be seen that only a handful of students can describe the definition of prime numbers accurately and precisely; the others can describe the properties of prime numbers well but not precisely.

Table 3. Description of student answers related to prime numbers

Answers	Students Number	Percentage (%)
Can only be divided by 1 and itself	7	41.18
Has exactly two factors	5	29.41
Divisible by the number itself	2	11.76
There are even numbers (2), and there are odd ones	1	5.88
No answer	2	11.76

For answers related to composite numbers, it can be seen in Table 4 that 29.41% of students answered that composite numbers “have factors other than one and themselves". This answer can also cause ambiguity, which means that composite numbers have other factors besides one and the number itself, where the view that can be raised is that the number 1 and itself are not factors of composite numbers.

Of course, this causes confusion, which impacts the following answer that composite numbers are numbers other than prime numbers or non-prime numbers, which also causes ambiguity in terms of number factors or terms of number properties. The answer was answered by 17.65% of students. If we pay attention to the language aspect, of course, the sentence is not wrong, but it is not right. If we pay attention to the number factor aspect, we will see that the factor is the opposite of a prime number. While in terms of number properties, it certainly does not describe the definition of a composite number. Only 29.41% of students answered that composite numbers "have more than two factors". This statement is the most appropriate and accurate answer in describing the concept of composite numbers. Where it has been given a limitation that a composite number is a natural number more than 1 and has more than 2 (two) factors, namely, there are other factors besides one and itself, and can be broken down into a unique prime number without limitations (Ustunsoy et al., 2011; Zazkis & Liljedahl, 2004). Furthermore, only 5.88% answered that composite numbers “have more than two divisors”. Of course, this sentence raises almost the same ambiguity as the previous answer about the definition of prime numbers that can be “divided by 1 and itself”, namely, regarding the primacy of the number 1. Moreover, 17.65% did not answer the description of composite numbers. Based on the results described in Table 4, it can be seen that most students are still mistaken in understanding the concept of composite numbers, but the essence of composite numbers can be understood well.

Table 4. Description of student answers related to composite numbers

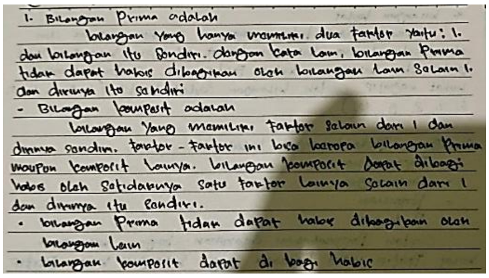
Answers	Students Number	Percentage (%)
Not a prime number	3	17.65
Have more than 2 factors	5	29.41
Has other factors besides 1 and itself	5	29.41
Have more than 2 dividers	1	5.88
No answer	3	17.65

Based on the results of the answers in Table 5, it can be seen that only 5.88% can show the relationship between composite numbers and prime numbers accurately and precisely, namely "Composite numbers can be broken down into prime factors". Students who answered "composite numbers are the result of combining prime factors" were not wrong; it just caused ambiguity that "combination" can be interpreted as "addition", whereas the meaning of the answer is the multiplication of prime factors. Meanwhile, the answer "prime numbers are the factorization of composite numbers" was answered by 5.88%, and this answer is certainly wrong. However, if we pay attention, the direction of the answer meant by the student is "composite numbers are the factorization of prime numbers". This error is caused by students who do not understand the meaning of "factorization," so it causes a misperception regarding the relationship between the two numbers (prime and composite numbers). Another answer is "composite numbers are not prime numbers". As many as 11.76% of students answered with this perception. Logically, this statement is not wrong; it just does not describe the properties or concepts that link the relationship between the two. The sentence only describes the group or grouping of numbers that cannot be owned by a number simultaneously. The answer is almost the same as the statement "if a number is not a prime number, then the number is a composite number, and vice versa", but in a negative perception. The statement was answered by 5.88% of students. In addition, 11.76% of students answered "can be divided", 5.88% of students answered "both have factors", 5.88% answered "both have factor 1", and 5.88% answered that the combination of composite numbers and prime numbers could be an ordered number. These answers do not show the nature or concept of prime and composite numbers and their relationship. However, the answer is not wrong, it is seen from another perspective. Of the 17 students, the majority, namely 35.29%, did not respond to the relationship between prime numbers and composite numbers. This shows that most students have not been able to link the relationship between prime numbers and composite numbers based on the nature and definition of the numbers.

Table 5. Description of student answers regarding the relationship between prime numbers and composite numbers

Answers	Students Number	Percentage (%)
Composite numbers are the result of combining prime factors.	1	5.88
Composite numbers can be decomposed into prime factors.	1	5.88
Prime numbers are the factorization of composite numbers.	1	5.88
Composite numbers are not prime numbers	2	11.76
Can be divided	2	11.76
Both have factors	1	5.88
Both have factor 1	1	5.88
If a number is not a prime number, it is a composite number. Vice versa.	1	5.88
If prime numbers and composite numbers are combined, they produce an ordered number.	1	5.88
No answer	6	35.29

For the interview, 5 (five) respondents were selected who had interesting answers. Therefore, the researcher conducted a document analysis (answer sheets) of the students' answers for each number. Here are some answer sheets of respondents who have interesting answers.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bilangan lain
• bilangan komposit dapat di bagi habis

Figure 2. S1 Answer sheet for question number 1

From the answer sheet, it can be seen that S1 defines a prime number, namely a number that has two factors. This answer is certainly correct, but there is a supporting sentence that causes ambiguity, namely, "prime numbers cannot be divisible by other numbers other than 1 and themselves". This sentence causes ambiguity regarding the primacy of 1 which has been explained previously which is not a prime number.

Next, for the answer related to the definition of composite numbers, S1 answered that composite numbers are numbers that have factors other than 1 and themselves. Of course, this answer also raises meaningful ambiguity that composite numbers have different factors from prime number factors. Composite number factors are also found in prime numbers, namely 1 and the number itself. However, it is added with other factors that support the number. Therefore, composite numbers should have more than 2 factors (Ustunsoy et al., 2011; Zazkis & Liljedahl, 2004).

Next, there is a supporting sentence: "These factors can be several prime numbers or other composites". The problem is the sentence "...can be other composite numbers". Whereas, based on the limitations that have been described previously, composite numbers are a unique and unlimited decomposition of prime numbers. Then, there is a supporting sentence: "These factors can be several prime numbers or other composites". The problem is the sentence "...can be other composite numbers". Based on the limitations that have been described previously, composite numbers are a unique and unlimited decomposition of prime numbers. Based on these answers, an interview was then conducted with S1 regarding the answers given, with the following conclusions:

S1: *Prime numbers only have two factors, namely 1 and the number itself. While composite numbers have more than 2 factors. For the relationship between prime numbers and composite numbers, I understand that composite numbers can have prime number factors or composite number factors.*

Based on the results of the interview, it shows that S1 understands the concept of prime numbers and composite numbers, only in explaining the definition, S1 is not good and does not understand the meaning of the sentence written by S1. Meanwhile, the relationship between composite numbers and prime numbers can be understood by S1 regarding factors. However, S1 does not yet understand the meaning of the prime factors that make up a number.

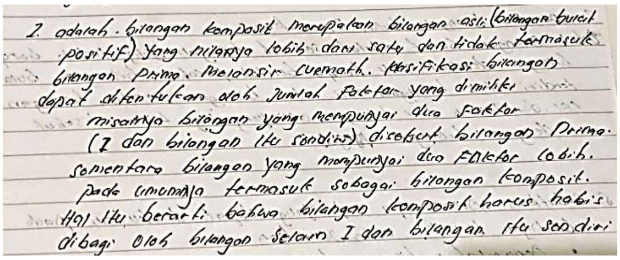

2. adalah bilangan komposit merupakan bilangan asli (bilangan bulat positif) yang nilainya lebih dari satu dan tidak termasuk bilangan prima. Melansir cuemath. klasifikasi bilangan dapat ditentukan oleh jumlah faktor yang dimiliki misalnya bilangan yang mempunyai dua faktor (1 dan bilangan itu sendiri) disebut bilangan prima. sementara bilangan yang mempunyai dua faktor lebih. pada umumnya termasuk sebagai bilangan komposit. Hal itu berarti bahwa bilangan komposit harus habis dibagi oleh bilangan selain 1 dan bilangan itu sendiri

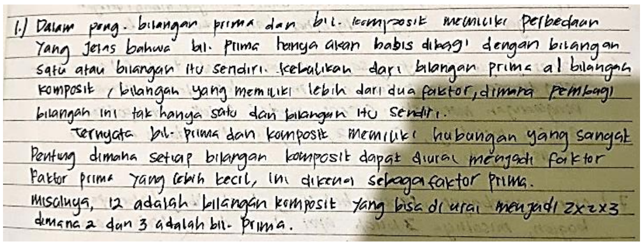
Figure 3. S2 Answer sheet for question number 1

In Figure 2, S2 explains that "composite numbers are natural numbers whose value is more than 1 and are not prime numbers". This answer is interesting because S2 adds the property of composite numbers, which are natural numbers more than 1. Furthermore, S2 adds that "prime numbers have more than 2 factors". Then, S2 defines prime numbers as numbers that have 2 factors. The definition given by S2 is the correct answer. Although S2 does not write what the relationship is between composite numbers and prime numbers, it is implied that the relationship between the two is related to the difference in the number of factors they have. The conclusions from the interview results conducted with S2 are:

S2: *Prime numbers have two factors (1 and the number itself). Meanwhile, composite numbers have more than 2 factors. The relationship between the two is that they are both natural numbers whose value is more than 1. However, the difference is the number of factors they have.*

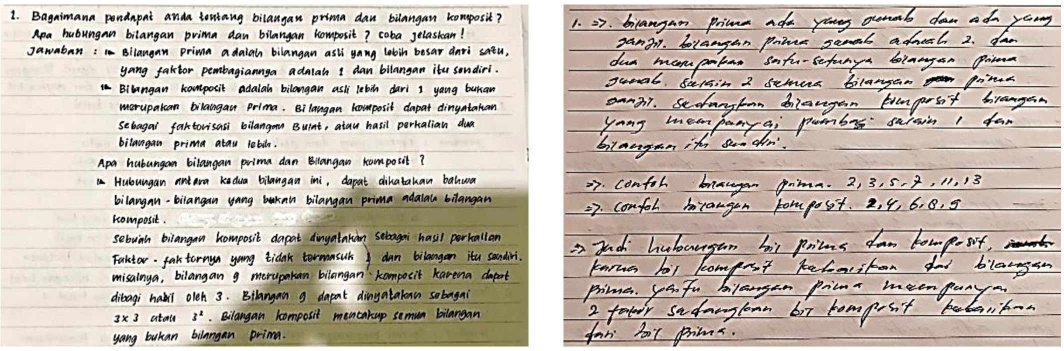
Based on the results of the interview with S2, it is seen that S2 has understood the concept of prime numbers and composite numbers by examining the factors they have. Almost the same as S1, S2 also did not write explicitly about the relationship between prime and composite numbers, but implicitly explained the relationship with the difference in the number of factors they have.

S3's answer (Figure 4) reveals that "prime numbers are divisible by 1 and the number itself", which is different from the previous answers that revealed the factors that make up the number, and also different from S4's answer that revealed the divisor (Figure 4). Different from S5, which only mentions the types of prime numbers and composite numbers, rather than explaining the meaning.



1.) Dalam peng. bilangan prima dan bil. komposit memiliki perbedaan yang jelas bahwa bil. prima hanya akan habis dibagi dengan bilangan satu atau bilangan itu sendiri. kebalikan dari bilangan prima al bilangan komposit, bilangan yang memiliki lebih dari dua faktor, dimana pembagi bilangan ini tak hanya satu dan bilangan itu sendiri.
Ternyata bil. prima dan komposit memiliki hubungan yang sangat penting dimana setiap bilangan komposit dapat diurai menjadi faktor faktor prima yang lebih kecil, ini dikenal sebagai faktor prima. Misalnya, 12 adalah bilangan komposit yang bisa di urai menjadi 2x2x3 dimana 2 dan 3 adalah bil. prima.

Figure 4. S3 Answer sheet for question number 1



1. Bagaimana pendapat anda tentang bilangan prima dan bilangan komposit? Apa hubungan bilangan prima dan bilangan komposit? coba jelaskan!
Jawaban : Bilangan prima adalah bilangan asli yang lebih besar dari satu, yang faktor pembagiannya adalah 1 dan bilangan itu sendiri.
Bilangan komposit adalah bilangan asli lebih dari 1 yang bukan merupakan bilangan prima. Bilangan komposit dapat dinyatakan sebagai faktorisasi bilangan bulat, atau hasil perkalian dua bilangan prima atau lebih.
Apa hubungan bilangan prima dan bilangan komposit?
Hubungan antara kedua bilangan ini, dapat dikatakan bahwa bilangan-bilangan yang bukan bilangan prima adalah bilangan komposit.
Sebuah bilangan komposit dapat dinyatakan sebagai hasil perkalian faktor-faktornya yang tidak termasuk 1 dan bilangan itu sendiri. misalnya, bilangan 9 merupakan bilangan komposit karena dapat dibagi habis oleh 3. Bilangan 9 dapat dinyatakan sebagai 3 x 3 atau 3^2. Bilangan komposit mencakup semua bilangan yang bukan bilangan prima.

1. => bilangan prima ada yang genap dan ada yang ganjil. bilangan prima genap adalah 2. dan dua merupakan satu-satunya bilangan prima genap. selain 2 semua bilangan prima ganjil. sedangkan bilangan komposit bilangan yang mempunyai pembagi selain 1 dan bilangan itu sendiri.
=> contoh bilangan prima. 2, 3, 5, 7, 11, 13
=> contoh bilangan komposit. 2, 4, 6, 8, 9
=> Jadi hubungan bil prima dan komposit, karena bil komposit kebalikan dari bilangan prima. yaitu bilangan prima mempunyai 2 faktor sedangkan bil komposit kebalikan dari bil prima.

Figure 5. S4 and S5 answer sheets for question number 1

Based on the document analysis and interviews related to the definition of prime numbers, it can be seen that S1, S2, and S4 have understood the concept of prime numbers, namely having 2 factors. However, S1 added a negative explanation related to the nature of prime numbers: prime numbers are not divisible by numbers other than 1 and themselves. S4 has also added the nature of prime numbers, which are natural numbers greater than 1. S3 understands the concept of prime numbers from the perspective of divisors of prime numbers, namely 1 and the number itself. This differs from S5, which mentions the types of prime numbers, namely even prime and odd prime numbers. This answer is certainly not the nature of prime numbers that researchers expect to be answered by research subjects.

S1, S2, and S4 have comprehended the concept of composite numbers and their relationship with prime numbers. However, S4 is more precise in explaining the relationship between prime and composite numbers. Regarding S3, while his explanation of prime and composite numbers remains incorrect, he is able to clarify the relationship between them. This is different from S5 which is seen to be able to explain the concept of prime numbers and composite numbers and the relationship between the two. For question number 2, namely "Suppose F = 151 x 157. Is F a prime number? (Yes/No)? Please explain why you chose that answer?" The students' answers' description is presented in the following Table 6.

Table 6. Description of student answers to question number 2

Correct Answer (Justification)	Students Number	Percentage (%)
True Claims (Not)		
Definition of Prime Numbers	5	29.41
Definition of Composite Numbers	3	17.65
Implementation of Algorithm	1	5.88
Total	9	52.94
Wrong Answer (Justification)	**Students Number**	**Percentage (%)**
False Claims (Yes)		
The product of prime numbers is a prime number	5	29.41
Incorrect implementation of the algorithm	2	11.76
Total	7	41.18
No answer	1	5.88

Out of the 17 students, 52.94% responded with “No” to the question; however, not all students provided the appropriate reasons for their answers. As many as 29.41% of students used the definition of a prime number which stated that F is not a prime number because it has other factors besides 1 and itself. Meanwhile, 17.65% of students reason that F is a composite number because 151 and 157 are factors of F other than 1 and itself. Meanwhile, 5.88% of students answered using an algorithm, by finding the value of F and proving that F is not a prime number. Among the three types of reasons students give, using the definitions of a prime number and a composite number is more efficient and effective than employing an algorithm. Because using an algorithm, besides being rather troublesome and more difficult to implement, cannot conclude that F is a composite number, even though it considers its representation as a product (Ustunsoy et al., 2011; Zazkis & Liljedahl, 2004). The following are excerpts from interviews with research subjects (S1, S2, S3, S4, and S5) based on question number 2:

S1: *F = 151 x 157 is a prime number, because 151 is a prime number and 157 is a prime number, so it is said that F is a prime number.*

S2: *F = 151 x 157 is a prime number because it has only two factors. All the given numbers are prime numbers.*
151 = 1 x 151, there are only two factors, namely 1 and 151
157 = 1 x 157. There are only two factors, namely 1 and 157.

S3: *F is the result of multiplying 151 and 157. In this problem, f is not a prime number because it has more than two factors, namely 1, 151, and 157. Which means F is a composite number and not a prime number.*

S4: *F is a prime number because a prime number is a number that can only be divided by 1 and itself without any other factors. F = 151 x 157, which means F is the result of multiplying two prime numbers, namely 151 x 157, which cannot be divided by any other number except 1 and itself. Therefore, F can be categorized as a prime number.*

S5: *I don't know, so I didn't write down the answer.*

Based on interview excerpts from several respondents, it can be seen that only S3 answered correctly that F is a composite number by considering the factors of F without creating an algorithm. While S1, S2, and S4 use the definition of prime numbers, they do not yet understand the meaning of the definition. They only focus on the constituent elements, namely 151 and 157, which are prime numbers, thus concluding that F is also a prime number.

For question number 3, namely "Suppose m(2k+1), where m and k are integers. Is this a prime number? Can it be a prime number?". Question number 3 also requires almost the same analysis as question number 2, namely related to a product of prime numbers. It is just that question number 3 is presented in the form of algebraic notation, which is different from question number 2. The description of the answers of 17 students is presented in Table 7 below.

Table 7. Description of student answers to question number 3

Answers	Students Number	Percentage (%)
Varies, depending on the values of m and k	9	52.94
Prime Numbers	4	23.53
Not a prime number	3	17.65
No answer	1	5.88

There are two important points in answering the question above, namely: 1) paying attention to the definition of prime numbers and composite numbers, and 2) providing examples that strengthen the answer. The existence of examples can provide strength to convince the answers and arguments given. Examples can provide a different perspective on a mathematical problem. Examples are an empirical inductive proof scheme (Zazkis & Liljedahl, 2004). If we pay attention to the factors, namely 1, m, (2k + 1), and m (2k + 1), then the numbers are composite. However, if we take m = 1 and k = 1, then m (2k + 1) = 3 which is a prime number. Only 52.94% of students answered that the answer to question number 3 can vary, namely, prime numbers or composite numbers, depending on the values of m and k. If students only give 1 example related to the problem, then they will tend to one of the answers, either prime numbers or composite numbers. Therefore, 23.53% of students answered prime numbers, and 17.65% answered composite numbers. Some students also only answered questions related to the definition of prime numbers, such as S2 (Figure 6).

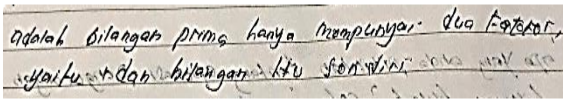

adalah bilangan prima hanya mempunyai dua faktor, yaitu 1 dan bilangan itu sendiri.

Figure 6. S2 Answer sheet for question number 3

The following are excerpts from interviews with research subjects (S1, S2, S3, S4, and S5):

S1: *Yes, because the values of integers can be included in the values of prime numbers, then it can be said that integers are prime numbers. Because, prime numbers are integers that have two factors, namely 1 and the number itself.*

S2: *I don't know whether the number can be a prime number or not because I don't have any idea about the example.*

S3: *m(2k+1) can be converted into a prime number, but we have to choose the right value to be able to produce a prime number through m(2k+1).*
For example, if we take m = 2 and k = 4, it will produce a value of 18, which is not a prime number because it consists of factors 1, 2, 3, 6, 9, and 18. This does not follow the definition of a prime number which has 2 divisors, namely 1 and itself. However, if we take the right value, for example, m = 2 and k = 0 which will produce a value of 2, where 2 is a prime number, then m (2k + 1) can be changed into a prime number.

S4: *It cannot be concluded that m (2k + 1) is an integer because m (2k + 1) can produce prime numbers, depending on the values of m and k.*

S5: *I don't know, so I didn't write the answer.*

Based on the results of the analysis of student answers (documents and interviews), only S3 was able to provide the right description and example related to the question in question number 3. While S1, S2, and S3 only provided an understanding of prime numbers but could not provide examples related to the numbers. While S5 did not write anything and could not provide an answer even through an interview.

After descriptively analyzing the percentage of responses from 17 prospective mathematics teacher students to three main questions related to the concepts of prime and composite numbers, it was apparent that there was still a significant gap between the expected conceptual understanding and the representations shown in their answers. The results of this quantitative study showed that only a limited number of pupils could correctly define prime numbers, explain composite numbers, and logically connect the two. Also, when students had to answer questions that involved algebraic expressions or multiplying two prime numbers, many of them made mistakes when trying to figure out what kind of number they got. Therefore, to further explore the causes of the errors, ambiguities, and misconceptions recorded in the written answers, a qualitative analysis was conducted through data triangulation in the form of semi-structured interviews and documentation of answer sheets. This process aimed to identify patterns of students' understanding and representation in greater depth based on themes emerging from interview transcripts and written documents, thereby providing a more comprehensive picture of the characteristics of the conceptual difficulties they experienced.

Incorrect Conceptual Understanding of Prime Numbers Definition

The analysis results show that most prospective mathematics teacher students still have an inaccurate understanding of the definition of prime numbers. Although 41.18% of them answered that prime numbers are divisible only by 1 and themselves, only 29.41% used the more precise term, namely "has exactly two factors." The phrase "can only be divided by 1 and itself" might be confusing because it makes people think that the number 1 is a prime number, even if it is not by mathematical convention. This finding supports the research of Zazkis & Liljedahl (2004) and Ustunsoy et al. (2011) which showed that many prospective teachers fail to convey the formal definition of prime numbers precisely, and that teaching practices in the classroom tend to reinforce incomplete informal definitions. This indicates a weak link between students' declarative knowledge and the representational needs in conceptual learning.

Misconceptions about the Concept of Composite Numbers and Their Relationship with Prime Numbers

The second topic that came up was misunderstandings about how to converse about composite numbers and how they are logically related to prime numbers. The best way to describe a composite number is that it has more than two elements. Only 29.41% of students stated this. Some people said, "a number that is not prime" or "has factors other than 1 and itself," which is correct grammar but not very clear. Only 5.88% of people correctly responded that a composite number can be split down into prime elements when asked about the relationship between prime and composite numbers. The majority of students (35.29%) were unable to identify this relationship at all. This difficulty reinforces the view that student teachers are not fully equipped to connect basic concepts logically and still approach concepts only as separate categories, rather than as interconnected systems, as reviewed by Piñeiro et al. (2021).

Misrepresentation of the Multiplication Form of Prime Numbers

The third interesting theme was the rise of misunderstandings when encountering situations involving the multiplication of two prime numbers, like $F = 151 \times 157$. A total of 41.18% of students said that F is a prime number just because its factors (151 and 157) are prime. By definition, however, the product of two prime integers is always a composite number. This study supports what Barbarani (2021) stated, that a lot of future teachers have issues connecting the method of factorization to what composite numbers represent. Others were able to state "no," but they did not always give a good reason for why they thought that way. They did not tie their use of algorithms or explicit computations to the idea of factorization.

Difficulties in Algebraic Representation: General Form m(2k+1)

Over fifty percent of the students (52.94%) who employed the algebraic representation ($m(2k+1)$) in the third issue understood that the form could function for either a prime or a composite number, depending on the parameters of m and k. While they had not looked closely at all of its fragments, 23.53% of them thought the form was certain to produce a prime number, while 17.65% claimed it would create a composite number. This indicates that students still have difficulty using algebraic forms to test the properties of numbers. Zazkis & Liljedahl (2004) emphasized the importance of the ability to use symbolic representations in understanding and explaining mathematical concepts. Students' failure to provide varied and logical examples also demonstrates their lack of experience with empirical evidence through inductive examples (empirical-inductive proofs) as revealed in the research of Funeme & Lopez (2022).

Inconsistencies between Comprehension and Oral-Written Representation

Triangulated data from documents, diagnostic tests, and interviews revealed inconsistencies between students' conceptual understanding and their verbal or written representations. For example, while some students (S1, S2, S4) appeared to understand the concept of prime numbers in general, their explanations contained inaccuracies or additional misconceptions. On the contrary, students like S3 were able to construct logical arguments about how $F = 151 \times 157$ is made subject to other numbers, even though they did not know as much about prime numbers. This implies that in order to represent something, you need to not only know what it is, but also be able to explain mathematical ideas clearly and logically in various manners (Mainali, 2021; Zazkis & Liljedahl, 2004).

Implications for the Learning and Education of Pre-service Teachers

In general, these results support the idea that understanding algorithms and procedures well enough is sufficient to enable future math teachers to teach basic concepts like prime numbers. The fact that teachers at LPTKs can't explain ideas clearly, build arguments with examples, or employ algebraic forms as instruments for understanding indicates that the way they teach needs to change. Li (2020) and Gürefe & Aktaş (2020) say that we should cease implementing curricula that are excessively focused on procedures and do not delve enough into representations. Instead, we should adopt methods that put more emphasis on holding concepts deeply and cross-representation.

The results above illustrate several key themes that resonate with prior literature on conceptual understanding and representation among pre-service mathematics teachers. Mathematical concepts are abstract things related to mental objects in a person's mind. Representation and construction help in learning a mathematical concept. A lack of representation skills can hinder students from building mental objects (Mainali, 2021; Zazkis & Liljedahl, 2004). This can be seen from the ability of pre-service mathematics teachers to represent prime numbers and composite numbers. Where students are only able to explain the meaning of prime numbers and composite numbers, but when given a problem related to the application of the concept, students begin to have difficulty in representing it. Especially if given in the form of algebraic notation (Barbarani, 2021; Funeme & Lopez, 2022; Ustunsoy et al., 2011; Zazkis & Liljedahl, 2004).

The present study emphasizes this challenge among pre-service mathematics teachers, who often can only restate definitions, such as “a prime is divisible by 1 and itself,” but begin to struggle when asked to apply these ideas to problem-solving contexts (Funeme & Lopez, 2022; Ustunsoy et al., 2011; Zazkis & Liljedahl, 2004). Piñeiro et al. (2021) remarked that teacher applicants commonly fail to fully understand the representational construction needed to teach mathematical reasoning since they combine topic knowledge with procedural fluency. This is exactly what they said.

Zazkis & Liljedahl (2004) claim that using ambiguous phrases like "divisible by 1 and itself" rather than "exactly two factors" certainly creates things less clear. However, it also opens the door to big mistakes, such as letting the number 1 prime in. In the

Indonesian context, this linguistic imprecision is further reinforced by didactic traditions that emphasize memorization and informal phrasing over rigorous definition-building.

Second, the fact that many pre-service teachers fail to understand how composite numbers are related to prime numbers structurally supports what Ustunsoy et al. (2011) and Piñeiro et al. (2021) determined that that many of them treat those concepts separately despite being connected. If students lack it understand that composite numbers are made up of more than one factor, such as fragmenting them into various primes, it makes it more difficult for them to teach factorization and primality.

Third, the fact that the result of two prime numbers remains prime indicates that you fail to comprehend how multiplication impacts how we perceive numbers. As Barbarani (2021) says that this mistake gets worse since individuals present fail to connect procedural multiplication with conceptual reasoning regarding factor structures. Students know that 151 and 157 are prime numbers; however, they fail to understand that the answer has to include more than two factors, which goes against the definition of primality.

Also, triangulated showing (student tests, interviews, and document analysis) suggested that these incorrect predictions are not heading away. For example, many students indicated that $F = 151 \times 157$ was "prime" on the test, and stated that detailing this was accurate during interviews. This illustrates how deeply these lies are rooted (Jeannotte & Kieran, 2017; Piñeiro et al., 2021). Kong (2019), Hjelte et al. (2020), and Martin (2021) all say that teacher training does not give teachers enough chances to talk about, test, and update their mental models. This consistency indicates how essential these chances are.

The implications of these findings are significant. A mathematics curriculum that is dominated by procedural or rote methods cannot sufficiently prepare pre-service teachers to develop flexible, robust representational skills (Gürefe & Aktaş, 2020; Piñeiro et al., 2021). When future teachers are just taught to memorize prime lists or do standard exercises, they fail to clarify, reason, and defend math concepts in different ways, such as algebraic, graphical, and verbal (Mainali, 2021; Martin, 2021).

This disconnect can make it difficult for teachers to educate pre-service teachers on how to communicate issues well. Given this, they might have issues teaching their future students correct concepts, especially those on prime numbers (Li, 2020). So, it is necessary to provide pre-service teachers a lot of different math problems to work on to help them enhance their ability to communicate issues (Piñeiro et al., 2021). At the same time, it can improve the ability to understand the concepts communicated and empirical inductive proof (David et al., 2020). Pre-service mathematics teachers need to have skills in mathematics content and pedagogical knowledge to teach that mathematics content (Ramsingh, 2020). Therefore, mathematics teacher education programs should explicitly integrate assignments and learning experiences to (Gürefe & Aktaş, 2020; Li, 2020): 1) encourage precise, student-generated definitions using formal mathematical language, 2) foster the creation and testing of counterexamples to stress logical consistency, 3)apply primality tests not only in numeric but also in general symbolic contexts, and 4) develop graphical models of primes and factor structures, such as factor trees or area diagrams. Such multi-representational tasks will help build deeper conceptual understanding and reduce language-driven or procedural misconceptions (Loconsole & Regolin, 2022; Picado-Alfaro, 2021; Piñeiro et al., 2021).

Using many diverse ways to educate can assist future teachers learn more about the subject they are teaching and how to teach it (Ball et al., 2008; Ramsingh, 2020). This would help them convey ideas in a way that works for everyone and is clear. These skills are particularly crucial for teaching and giving students the tools they need to accomplish actual math (Hjelte et al., 2020; Jeannotte & Kieran, 2017).

In short, the results show that Indonesian math classes should stop focusing on methods and examples and start openly encouraging students to think in more than one way. Teacher candidates will still have poor and incomplete ideas about prime and composite numbers if this does not happen. This will make it harder for them to help future generations learn arithmetic better (Barbarani, 2021; Martin, 2021; Piñeiro et al., 2021).

▪ LIMITATIONS

This study has several limitations. First, the sample was small (N=17) and drawn from a single university, which limits the generalizability of the findings. Second, the study primarily focused on verbal and simple algebraic representations of prime numbers and did not explore other important forms of mathematical representation, such as graphical or visual models. Future research with a larger, more diverse sample and a broader focus on multiple representations is needed to confirm and extend these results.

▪ CONCLUSION

This study contributes evidence that pre-service mathematics teacher still demonstrate significant gaps in their verbal and algebraic representation of prime numbers and composite numbers, highlighting persistent misconceptions that may stem from language habits and a curriculum overly focused on numeric procedures. These findings underscore the importance of explicitly supporting representational flexibility in teacher education. Future research should consider conducting longitudinal studies to monitor how pre-service teachers' representational skills develop over time, as well as experimental studies to test the effectiveness of pedagogical interventions designed to enhance flexible, multi-representational thinking in prime number concepts.

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