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Can Concreteness Fading and Multi-Representational Learning Enhance Students' Understanding of Geometrical Optics?

Theo Jhoni Hartanto[1,*], Pri Ariadi Cahya Dinata[1], Luqman Hakim[2], & Suhartono[1]
[1]Department of Physics Education Program, Universitas Palangka Raya, Indonesia
[2]Department of Physics Program, Universitas Palangka Raya, Indonesia

Abstract: This study aims to analyze the effect of learning based on concreteness fading and multi-representation on prospective teacher students' understanding of geometric optics concepts. The research sample consisted of 40 prospective physics teacher students, selected using the total sampling technique, with 20 students in each experimental and control group. The experimental group received learning with concreteness fading and multi-representation approaches, while the control group received conventional learning. The research instrument was a concept understanding test in the form of descriptions, which was tested for content validity and reliability using the inter-rater reliability method, yielding a Cohen's Kappa coefficient of 0.68. Data analysis techniques included independent samples t-test, ANCOVA, and N-gain. The results of the independent samples t-test showed that, in the posttest, there was a significant difference ($p = 0.008 < 0.05$) with the average score of the experimental group ($M = 21.55$) being higher than that of the control group ($M = 19.10$). ANCOVA test results showed that learning with concreteness fading-multi representation significantly affected students' concept understanding after controlling for the pretest score ($p = 0.009 < 0.05$). Additionally, the N-Gain test results indicated increased concept understanding in the experimental class (0.75, high category) and the control class (0.56, medium category). Initially, many students struggled and relied solely on one form of representation to explain geometric optics problems. However, after learning, they began to utilize various interrelated representations, including diagrams, texts, and mathematical equations. The findings in this study confirm that learning with concreteness fading and multi-representation approaches is effective in improving understanding of geometric optics concepts.

Keywords: conceptual understanding, concreteness fading, geometric optics, multi-representation, physics learning.

▪ INTRODUCTION

Many studies in physics require various representations to understand them (Aregehagn et al., 2023; Gao et al., 2022). Meltzer (2005) states that there are four forms of representation found in physics concepts, namely verbal, diagrams, mathematics, and graphics. Munfaridah et al. (2021) stated that the representation format contained in a concept can be verbal, graphic, and numerical. Some references use the term external representation, such as manipulatives, pictures, diagrams, graphs, and equations (Corradi et al., 2012; Opfermann et al., 2017).

Geometric optics is one of the topics that involves various forms of representation (multi-representation), such as diagrams, physical models, and mathematical equations (Müller et al., 2017). Ray diagrams are used to depict the passage of light visually. Physical models refer to the use of real objects or experimental devices to demonstrate optical phenomena, such as observing shadows produced by a convex lens on a screen. Meanwhile, mathematical equations are used to calculate the distance of an object, the distance of its shadow, or the focal length, and to determine the characteristics of the shadow formed. The use of multi-representations helps convey information more fully

Theo Jhoni Hartanto
*Email: theo@fkip.upr.ac.id
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and supports students' knowledge construction (Opfermann et al., 2017). Tippett (2016) analyzed the literature on science representations and found that the ability to construct and interpret representations can improve students' understanding of science concepts.

Various studies have shown that students struggle to understand geometric optics concepts because they are abstract, difficult to visualize, and not directly observable (Ciobanu et al., 2023; Gacovska Barandovska et al., 2023; Sutarto et al., 2018). Geometric optics concepts, such as the formation of shadows by lenses or mirrors, cannot be observed directly but must be understood through visual models, experiments, and mathematical equations. The process of shadow formation involves the passage of light rays, which can only be represented through diagrams; therefore, students must be able to visualize how the rays move. In addition, the quantitative relationship between object distance, shadow distance, and focal length is more often presented in the form of mathematical equations that are symbolic and abstract. This abstractness makes it difficult for students to build connections between concrete experiences from experiments, visual images in diagrams, and mathematical relationships that explain these phenomena. This means that students need to be trained to connect concrete to abstract information (Donovan & Fyfe, 2022).

Difficulty understanding geometric optics concepts is also thought to occur because each representation presents information differently, so students often understand concepts separately. For example, ray diagrams are used to illustrate the passage of light and the position of the shadow, but they do not explain how the shadow position is calculated. Experiments with real lenses allow students to observe the formation of shadows on the screen directly. However, they are often viewed as a practical activity without an understanding of the underlying mathematical relationship. Meanwhile, the lens equation helps determine shadow distance and magnification, but its abstract form makes it difficult for students to relate it to observations and ray diagrams. This difference in presentation makes it difficult for students to see the connection between representations, resulting in partial and fragmented understanding (Kokkonen et al., 2022). According to Ainsworth (2006), in the framework of multiple representations theory, the difference in the way information is presented gives each representation a unique cognitive function. If students are not accustomed to connecting information from multiple representations, they tend to understand each representation in isolation without seeing the conceptual linkages between them. As a result, there is limited flexibility in representation, where individuals tend to get stuck in one particular form of representation without being able to convert it to other forms.

Students' difficulties in transitioning from one representation to another are often caused by learning patterns that have only emphasized one form of information presentation, such as formulas or text. Learning that focuses too much on how to calculate without linking it to images, graphs, or visual models results in students understanding procedures without truly grasping concepts (Hansen & Richland, 2020). As a result, when asked to explain concepts in other forms, students are confused because they are not used to seeing relationships between representations. This condition is exacerbated by the lack of explicit instruction on how to connect the various representations. As a result, students may understand concepts partially but are unable to transfer them to new situations because they lack the skills to link the various representations. In the context of geometric optics, if teachers do not explicitly teach the relationship between ray drawings, mathematical formulas, and their applications, students will struggle to transfer their understanding to various other contexts. The lack of explicit explanations from teachers about how each representation is interconnected further reinforces fragmented learning habits (Kokkonen et al., 2022). This condition causes students to tend to stick to one approach only, making it difficult for them to transfer their understanding when the form of information presentation changes (Roth, 2006).

To overcome this problem, a learning approach is needed that can help students understand the transition from concrete representations (empirical, obtained through experiments) to abstract representations (mathematical equations) systematically. One solution that can be applied is a combination of concreteness fading and multi-representation strategies. Concreteness fading is a stepwise approach that begins with concrete representations, then transitions to semi-concrete representations, and ultimately reaches abstract representations (Fyfe & Nathan, 2019). This strategy aims to bridge the gap between understanding gained through direct experience and symbolic thinking (Skulmowski, 2023), thereby enabling students to internalize physics concepts more easily. In the context of geometric optics, concreteness fading can begin with experiments using real lenses and mirrors, followed by the visualization of ray paths through diagrams, and ultimately connected to mathematical representations for deeper analysis.

Meanwhile, multi-representation plays a role in providing various forms of concept representation to strengthen students' understanding (Sutopo & Waldrip, 2014). By using multi-representations, students can more easily see the connections between different concepts and develop a more comprehensive understanding. In this way, students can connect different forms of representation and reduce the possibility of misconceptions. In discussing the role of multiple representations in physics learning, Opfermann et al. (2017) suggested that multiple representations have great potential in supporting students' learning of physics concepts because students learn more easily when problems contain multiple representations; therefore, the use of multiple representations can maximize the results of the student learning process.

The combination of concreteness fading and multi-representation has excellent potential for improving students' conceptual understanding, as these two strategies complement each other. Concreteness fading helps students to translate from concrete experiences to abstract understanding. At the same time, multi-representation provides various cognitive pathways that allow students to connect and understand concepts more effectively (Lichtenberger et al., 2024). The main principles of multirepresentation, according to Ainsworth (2006), namely complementary roles, constraining interpretation, and constructing deeper understanding, can be applied gradually along the transition from concrete to abstract. At the concrete stage, representations such as real objects or realistic simulations provide meaningful contexts and are easily associated by students, following the principle of complementarity. At the semi-abstract stage, representations such as schematic drawings or graphs help limit possible misinterpretations of concrete experiences, in line with the function of constraining interpretation. Furthermore, at the abstract stage, formal symbols and formulas are used to build deeper conceptual understanding through the integration of various representations that have been previously learned. Thus, integrating multiple representations of concreteness fading allows for smoother cognitive transitions and strengthens understanding, making learning more effective and meaningful.

By applying these two approaches in learning geometric optics, it is expected that students will not only be able to understand the passage of light rays visually but also be able to explain and analyze them mathematically and verbally. The sequence of representations in concreteness fading, from concrete representations towards abstract representations, has the potential to encourage a more generalized and translatable understanding (Fyfe, McNeil, & Borjas, 2015; Jaakkola & Veermans, 2018; Kokkonen & Schalk, 2021). Kokkonen & Schalk (2021) revealed that concreteness fading through multi-representation has potential in physics learning. However, its application requires adaptation and special consideration to fit the nature and specific needs of the discipline. Specifically, this research requires a transition from concrete phenomena (shadows, mirrors, lenses) to an abstract understanding (ray diagrams, laws of optics, and mathematical equations). In addition, the topic of geometric optics has the potential to be one of the physics topics that is closer to a linear transition than other physics concepts. McNeil & Fyfe's (2012) research confirms that the transition from concrete to abstract is more effective if the abstract representation retains a clear connection to concrete experience. The next consideration is to involve prospective physics teacher students who have dual learning needs: conceptual and pedagogical, meaning that they not only learn to understand concepts, but also to teach them later (Nousiainen, 2013). Prospective physics teacher students are at the formal operational stage, meaning they can think abstractly, logically, and systematically (Sorge et al., 2019), which should support the transition from concrete to abstract representation.

The uniqueness of this research lies in the integration of concreteness fading and multi-representation in college-level physics learning. Although these two approaches have been widely researched separately, studies that specifically combine them, especially in physics learning, are still very limited (Kokkonen & Schalk, 2021; Lichtenberger et al., 2024). Most of the research on concreteness fading is mostly applied in mathematics learning, such as in understanding numbers or algebraic equations, and has been proven successful in gradually improving understanding of abstract concepts, from concrete to symbolic representations (Donovan & Fyfe, 2022; Fyfe et al., 2015; Ottmar & Landy, 2017; Zhao, 2024). This difference in results can occur because the characteristics of mathematics and physics lessons are different (Kokkonen et al., 2022). Mathematics tends to be logical and structured, so students find it easier to move from concrete things to symbols or formulas (Kim, 2020; Kollosche, 2021). For example, students learn fractions by dividing a cake into four parts. From there, they easily understand that 1/4 means one of four parts, then continue to fraction operations with symbols and arithmetic rules. However, in physics, the concept is often directly related to complex real-world events that are difficult to translate (Kokkonen et al., 2022). For example, students observe the shadow of a candle projected onto a mirror. They know that the shadow is formed, but are often confused when asked to explain the position of the shadow using the law of reflection or when translating it into an equation that relates the focal length, object distance, and shadow distance. Therefore, in physics, moving from the concrete to the abstract is not always easy (Kokkonen & Schalk, 2021). Thus, this research is expected to contribute to the improvement of physics learning, especially in the topic of geometric optics at the university level.

Based on the description above, this study aims to analyze the effect of applying the concreteness fading and multi-representation approach on students' understanding of geometric optics concepts. The primary issues addressed in this study are: Can concreteness fading and multi-representation learning enhance students' conceptual understanding of the topic of geometric optics?

▪ METHOD

Research Design and Procedures

This study used a quasi-experimental design involving a control group and an experimental group (Creswell & Creswell, 2017). Both groups came from the same study program. For the control group, a lesson on geometric optics was taught using conventional methods. The same lesson topic was taught to the experimental group using concreteness fading and multiple representations. Table 1 shows a summary of the design in this study. The study was conducted over six weeks, with time allocated to cover all subtopics of geometric optics, ranging from reflection to refraction of light.

Table 1. Research design

Group	*Pre-test*	Treatment	*Post-test*
Experiment	O_1	*Concreteness fading* and multi-representation learning	O_2
Control	O_1	Conventional learning	O_2

Participants

In this study, a total sampling technique was employed, where all members of the population served as research samples. According to Sugiyono (2011), total sampling is a sampling technique in which the number of samples is equal to the population size. This technique is often used when the population is relatively small. In the context of this study, the population refers to all prospective teacher students who take Optics courses in the Physics Education Department of two classes with a total of 20 students each (40 students in total). Of the two available classes, the researcher designated one as the experimental class and the other as the control class through simple randomization.

Procedures and Treatment

This study was conducted over six weeks, with two meetings per week, each lasting 100 minutes. Learning in the experimental class was designed using a combination of concreteness fading and multi-representation learning, which was applied in several stages. The stages include concrete representation, semi-concrete representation, and symbolic representation, as referenced in several studies (Fyfe et al., 2014; Fyfe & Nathan, 2019; Kokkonen et al., 2022). In more detail, the learning stages in the experimental class, exemplified by the subtopic of refraction of light in a thin convex lens, are presented in Table 2. Each stage of learning is designed with customized scaffolding, allowing students to receive gradual support in understanding concepts in depth. Various research results show that scaffolding can help students overcome difficulties in understanding lesson content (Alanazi et al., 2024; Donovan & Fyfe, 2022; Rokhmat & Putrie, 2019).

Table 2. Concreteness fading and multi-representation learning stages

Stages	Objectives	Learning activities	Scaffolding
Concrete stage (direct experience with physical objects)	Students experience directly how convex lenses form shadows before understanding more abstract concepts.	*Experiment with convex lenses:* Students are given a convex lens and a screen to observe directly how shadows are formed when objects are brought closer or further away. *Observation of changes in size and properties of shadows:* Students observe the position of objects, shadows formed upside down or upright, enlarged or reduced, real or virtual, and determine the position of shadows. *Discussion based on experience:* Students are asked to explain in their own words what happens to the shadow when the distance of the object changes and determine the focal distance of a convex lens.	Lead questions, such as “Can the image be captured on the screen?” or “Where is the image positioned if the object is moved further away from the convex lens?” Explicit directions for observing the properties of the image, such as size, position, and whether the image is real or virtual. Explanation of the experimental equipment configuration: what equipment is used, how the equipment is positioned or arranged, what the students do, and how the experimental activity flows.
Semi-concrete stage (using visual representations and diagrams)	Students connect real observations with visual representations in the form of diagrams and tables	*Diagram of the movement of light through a lens: Students draw ray diagrams to illustrate* shadow formation on convex lenses. *Interactive Simulation: Students use computer-based simulations to explore various positions of objects and visually observe how shadows are formed*. *Tabulation of shadow properties:* Students record the properties of shadows based on the object's position relative to the focus and center of curvature.	Step-by-step procedural guides illustrate the shadow formation diagram. A complete example of a shadow formation diagram is used as a reference for comparison and correction. Visual aids, such as interactive computer simulations.
Symbolic stage (using symbolic and mathematical representations)	Students understand the quantitative relationship in the formation of shadows and can predict	*Use of lens equation:* Students use the thin lens equation to calculate the position of the shadow when the distance of the object and the focus of the lens.	Explicit explanations of sign conventions in equations. Worksheets that guide students from variable identification and substitution into

Stages	Objectives	Learning activities	Scaffolding
	shadows mathematically and determine the focal distance of a lens.	*Mathematically analyze* shadow magnification to determine whether the shadow is enlarged or reduced.	formulas to interpretation of numerical results. Explicit explanations of the relationship between calculated results and visually observed experimental observations.
Integration of representations	Connecting various representations (concrete, visual, symbolic) that have been learned so that students understand concepts more deeply.	Students are given problem-solving-based tasks that combine concrete representations, diagrams, and mathematical analysis.	Reflective questions, such as “Are your ray drawing and calculated results consistent?” or “Do the shadow properties from the experiment match the calculated results and diagram?” Re-use of computer simulations to visually test the correspondence between the results of the diagram and the formula.
Evaluation and reflection	Evaluate students' understanding of the concepts that have been learned through various representations.	Students work on evaluation questions and discuss their concept understanding through various representations.	Formative feedback from lecturers, both in the form of assignments and class discussions, that focuses on improving conceptual and representational consistency.

Especially in the control class, geometric optics learning is carried out conventionally by still referring to the standard curriculum content with a teacher-centered learning approach but still provides opportunities for students to actively participate. de Jong et al. (2023) and Hughes et al. (2017) emphasized the importance of a combination of teacher-centered and still-involving-students-in-learning approaches. Each meeting begins with an explanation of basic concepts by the lecturer, accompanied by the use of visual presentations in the form of PowerPoint slides and diagram illustrations on the board. On several occasions, the lecturer also displays simple visual demonstrations using props to show the direction of light rays and the formation of shadows, either on mirrors or lenses. Students discuss in small groups to discuss the results of the demonstration, then present the results of their discussions. To improve understanding, the lecturer presents examples of conceptual and numerical problems. Students then work on practice questions individually. After that, students are allowed to discuss and ask questions about things that are not yet understood. In this phase, the lecturer provides clarification or encouragement to think further.

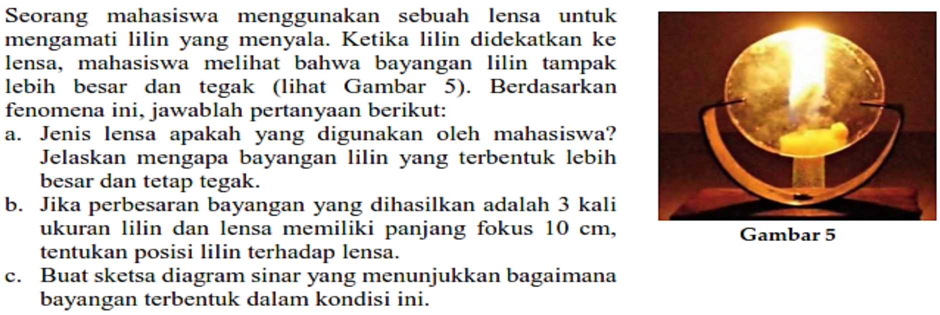
Instrument

The conceptual understanding test instrument used in this study was designed to measure the level of student understanding of geometric optics material. The test used in this study is in the form of objective descriptions, which require students to provide answers in writing based on their conceptual understanding. Each question is designed to measure conceptual understanding, application of theory, and students' ability to represent information in various forms, such as verbal, mathematical, and graphical. This test was developed by the researcher and adapted from various reliable sources relevant to the topic of geometric optics, such as the reference books of Giancoli, (2022), Tipler & Mosca (2007), and Halliday et al. (2013), as well as several studies on the development of comprehension test instruments on geometric optics (Tural, 2015; Uwamahoro et al., 2021). This test instrument consists of five questions covering light reflection and light refraction. These questions were designed to explore students' understanding through explanations with various representations. Table 3 below presents the conceptual understanding test grid used in the data collection.

Table 3. Description of the test for understanding the concept of geometric optics

No.	Question Indicator	Question Number
1.	Analyze the types and properties of shadows formed by spherical curved mirrors using the principles of geometric optics	1
2.	Analyze the position and magnification of the image of an object formed through a double mirror system using the principles of geometric optics.	2
3.	Analyze problems about refraction using Snell's law	3
4.	Analyze the types and properties of shadows formed by thin lenses using the principles of geometric optics.	4
5.	Analyze the position and magnification of an object's shadow formed through a double lens system by applying the thin lens equation.	5

Before use, this test instrument has gone through a peer review process, namely one lecturer who has expertise in physics and one lecturer who has a background in physics learning expertise, both of whom are experienced in teaching geometric optics topics at the higher education level. This review process is carried out to ensure the validity of the content, the suitability of the level of difficulty of the questions, and the acceptability of the questions for students, as well as ensuring the suitability of the items with the learning objectives (Artino Jr et al., 2014). Feedback obtained from the review process is used to improve the test instrument. In addition, to measure its reliability, this test uses the inter-rater reliability (IRR) method or inter-rater agreement. The inter-rater reliability (IRR) test in this study was conducted by the same two lecturers who previously conducted a peer review of the test instrument. IRR measures the extent to which two or more raters give consistent scores to the same set of responses (Eagan et al., 2020). In assessing IRR, Cohen's Kappa is used, which is one of the frequently used coefficients, especially when there are two raters (Eagan et al., 2020). The measurement results show that the Cohen's Kappa coefficient is 0.68, meaning that the instrument has a fairly good reliability value as a conceptual understanding assessment tool (McHugh, 2012). One example of an existing question contained in the instrument is presented as follows.


Seorang mahasiswa menggunakan sebuah lensa untuk mengamati lilin yang menyala. Ketika lilin didekatkan ke lensa, mahasiswa melihat bahwa bayangan lilin tampak lebih besar dan tegak (lihat Gambar 5). Berdasarkan fenomena ini, jawablah pertanyaan berikut:

a. Jenis lensa apakah yang digunakan oleh mahasiswa? Jelaskan mengapa bayangan lilin yang terbentuk lebih besar dan tetap tegak.
b. Jika perbesaran bayangan yang dihasilkan adalah 3 kali ukuran lilin dan lensa memiliki panjang fokus 10 cm, tentukan posisi lilin terhadap lensa.
c. Buat sketsa diagram sinar yang menunjukkan bagaimana bayangan terbentuk dalam kondisi ini.

Gambar 5

Figure 1. Screenshot for one of the conceptual understanding items

Figure 1 shows one of the items in the conceptual understanding test. All items are designed to test deep understanding, not just calculation ability, but require students to connect one representation to another. The items shown in Figure 1 help students understand the formation of shadows on a convex lens in various representations: conceptual (verbal), mathematical, and graphical. This strengthens students' understanding by connecting various representations to understand the concept of a convex lens more fully.

Data Analysis

Data analysis techniques in this research used independent samples t-test, analysis of covariance (ANCOVA), and N-gain to measure the effectiveness of the applied learning. Before conducting the main analysis, prerequisite tests were carried out in the form of a normality test and a homogeneity test. The normality test aims to ensure that the data on student learning outcomes are normally distributed, while the homogeneity test is used to determine the similarity of variance between groups (Bulut et al., 2016; Pallant, 2020). If these two prerequisites are met, then the analysis continues using the independent samples t-test to compare differences in concept understanding between experimental and control classes. The decision-making criteria in the t-test is if the p-value < 0.05, then there is a significant difference between the two groups.

ANCOVA was used to analyze differences in learning outcomes by controlling covariate variables in the form of pretest scores so that the analysis results are more accurate (Senocak et al., 2007). Controlling for pre-test scores while satisfying these assumptions increases the precision of learning effect estimates (Vázquez-Bernal et al., 2012). This approach ensures that any observed differences in post-test conceptual understanding are primarily due to the learning intervention and not to pre-existing differences in students' prior knowledge.

In addition, the N-gain test was used as an analytical tool to measure the improvement of concept understanding before and after learning. The N-Gain test was calculated based on students' pretest and posttest scores (Hake, 1998). This calculation was done for each student isn both classes, and then the average was analyzed to illustrate the effectiveness of the learning treatment as a whole. The criteria listed in Table 4 were used to determine the interpretation of students' concept understanding improvement (Hartanto et al., 2023).

Table 4. N-gain learning effectivity interpretation

***Mean* N-gain**	**Students understanding interpretation**
<g> < 0.3	Low
0.3 ≤ <g> ≤ 0.7	Moderate
<g> > 0.7	High

▪ RESULT AND DISSCUSSION

Based on the results of the research, descriptive statistical analysis in Table 5 shows that the average pre-test score in the experimental class was 11.30 and increased to 21.55 in the post-test. Meanwhile, in the control class, the average pre-test score was 11.10 and increased to 19.10 in the post-test. Descriptively, these results show that both learning can improve students' concept understanding, but learning with concreteness fading and multi-representation provides a greater improvement.

Table 5. Descriptive statistical analysis results in the experimental class and the control class

Results	**Mean (*M*)**	**Std. Error**	**Std. Deviation (*SD*)**	**Min.**	**Max.**
Pre-test experiment	11.30	0.514	2.296	7	15
Post-test experiment	21.55	0.667	2.982	16	25
Pre-test control	11.10	0.486	2.174	8	15
Post-test control	19.10	0.556	2.532	14	25

Based on the normality test results in Table 6 using the Shapiro-Wilk test, it shows that the significance value of the pretest data for the control class is 0.287, the significance value of the control class posttest data is 0.109, the significance value of the pretest data for the experimental class is 0.125, and the significance value of the posttest data for the experimental class is 0.051. That is, based on the normality test with Shapiro-Wilk, it shows that the pre-test and post-test data in both classes are normally distributed ($p > 0.05$). The results of the homogeneity test of pretest data in the control class and experimental class using Levene's Test are shown in Table 6; the significance value obtained is 0.651. Since this value is greater than 0.05 ($p = 0.651 > 0.05$), it can be concluded that the variance of pretest data between experimental and control classes is homogeneous. The results of the homogeneity test of posttest data in the control class and experimental class obtained a significance value of 0.291, which is greater than 0.05 ($p = 0.291 > 0.05$). It can be concluded that the variance of post-test data between the experimental class and control class is homogeneous.

Table 6. The results of the analysis of the normality and homogeneity

Source	**Group**	**Shapiro-Wilk**			**Variant homogeneity**		
		Statistic	**df**	**Sig.**	**Levene's Test**	**Sig.**	**Description**
Pre-test	Experiment	.925	20	.125	.208	.651	Homogeneous
	Control	.944	20	.287			
Post-test	Experiment	.905	20	.051	.147	.291	Homogeneous
	Control	.922	20	.109			

The results of the independent samples test (Table 7) show that there is no difference between the pretest scores of the experimental and control classes with a Sig.

(2-tailed) of 0.779 ($p > 0.05$). That is, the statistical comparison revealed that there was no significant difference in concept understanding between the two groups before the treatment was given. Meanwhile, the results of the independent samples test showed that there was a significant difference between the post-test scores of the experimental class and the control class with a Sig. (2-tailed) of 0.008, which is smaller than 0.05 (Sig. (2-tailed) = 0.008 < 0.05), which means that learning with concreteness fading-multi-representation is proven to be more effective in improving students' concept understanding than conventional learning.

Table 7. Independent Samples t-test results in the control and the experimental group

Source	Group	t-test for equality of means				
		t	**df**	**Sig.**	**Mean difference**	**Std. error difference**
Pre-test	Experiment	-.283	38	.779	-.200	.707
	Control					
Post-test	Experiment	2.801	38	.008	2.450	.875
	Control					

Furthermore, from the ANCOVA analysis (Table 8), the pretest variable had a significance value of 0.416 ($p > 0.05$), indicating that the pretest score did not have a significant influence on the posttest in this model. Meanwhile, from the analysis, the F value was 7.553 with a significance of 0.009 on the group variable, which means that the difference between the experimental and control groups was statistically significant ($p < 0.05$). This means that learning with concreteness fading and multi-representation has a greater impact on students' conceptual understanding than conventional learning.

Table 8. ANCOVA test results

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	65.247[a]	2	32.624	4.228	.022	.186
Intercept	497.764	1	497.764	64.503	.000	.635
Pretest	5.222	1	5.222	.677	.416	.018
Group	58.289	1	58.289	7.553	.009	.170
Error	285.528	37	7.717			
Total	16875.000	40				
Corrected Total	350.775	39				

a. R Squared = .186 (Adjusted R Squared = .142)

In addition to statistical tests, the results of the N-Gain analysis showed that the experimental class had an average N-Gain of 0.75, which was categorized as high, while the control class had an average N-Gain of 0.56, which was categorized as medium (Figure 2). In the experimental class, 13 students had high N-Gain and 7 students had medium N-Gain. Meanwhile, in the control class, 5 students had high N-Gain, 13 students had medium N-Gain, and 2 students had low N-Gain. These results strengthen the conclusion that learning with concreteness fading-multi-representation is more effective in improving students' concept understanding than conventional learning methods.

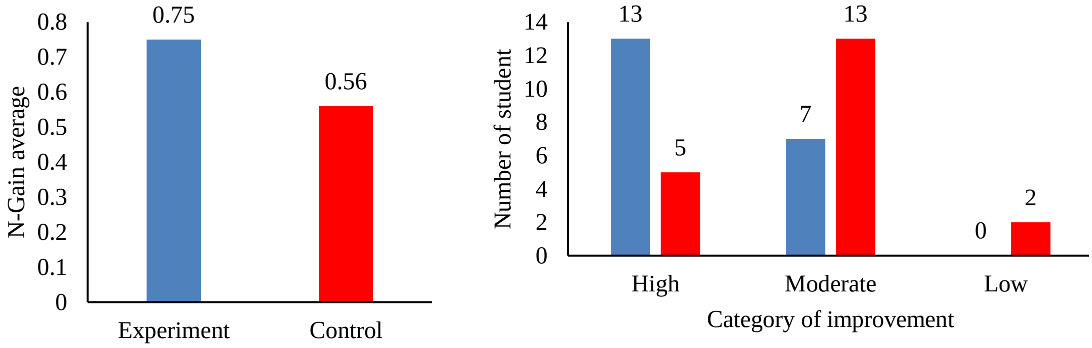


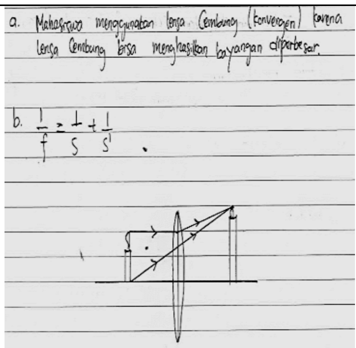
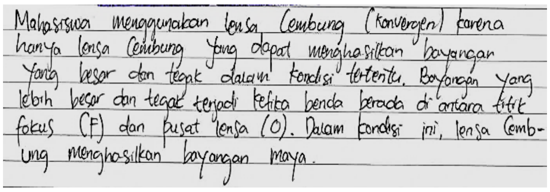
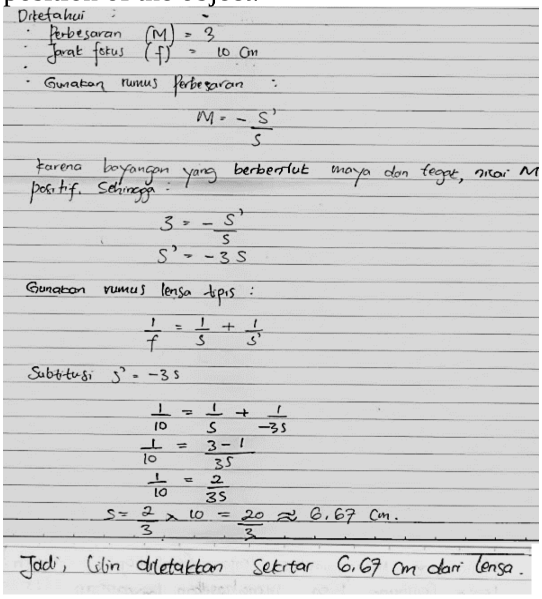
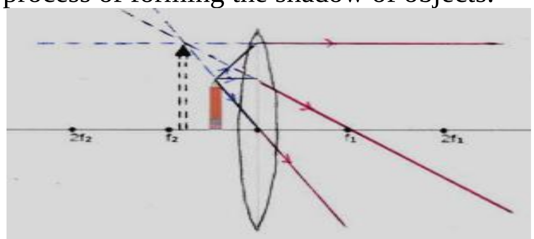
Figure 2. Average *N*-Gain in the control and experiment class and the number of students based on the acquisition of N-Gain

This study reveals that the application of learning approaches that incorporate concreteness fading and multiple representations has a positive impact on student understanding in geometric optics. Students in the experimental class demonstrated a greater increase in concept understanding compared to the control class, which used conventional learning methods. The results of the analysis showed that students in the experimental class were better able to connect various representations (concrete, pictorial, mathematical, and verbal) in solving physics problems, which reflected a deeper and more flexible understanding of the concepts learned. These results align with several studies that demonstrate the effectiveness of multiple representations in enabling students to connect verbal, mathematical, and visual representations with real experiments to gain a deeper understanding of concepts (Hubber et al., 2010; Sutopo & Waldrip, 2014). Combining multiple representations will encourage deeper understanding when students integrate information from different modes of representation (Munfaridah et al., 2021). This is based on cognitive theory, which states that learning is more effective when information is presented through multiple modalities, as each representation offers a unique viewpoint that complements the others (Prain & Tytler, 2012).

As an illustration, the following examples of student answers from the experimental class are provided in solving questions related to the concept of shadow formation on thin lenses (Table 9). Here are some examples of student answers from the pretest and posttest for questions related to thin convex lenses (item number 4). Based on the analysis of student answers on the pretest and posttest, there is a change in understanding of the concept.

Table 9. An example of student answers in pretest and posttest

Student code number	Pre-test answer	Post-test answer
Pertanyaan 4: Seorang mahasiswa menggunakan sebuah lensa untuk mengamati lilin yang menyala. Ketika lilin didekatkan ke lensa, mahasiswa melihat bahwa lilin tampak lebih besar dan tegak (lihat gambar). Berdasarkan fenomena ini, jawablah pertanyaan-pertanyaan berikut: (a) Jenis lensa apa yang digunakan oleh mahasiswa? Mengapa bayangan lilin yang terbentuk tampak lebih besar dan tegak? (b) Jika perbesaran bayangan yang dihasilkan adalah 3 kali ukuran lilin dan lensa memiliki panjang fokus 10 cm, dimana posisi lilin terhadap lensa? (c) Buat sketsa diagram sinar yang menunjukkan bagaimana bayangan terbentuk dalam kondisi ini.		

Student code number	Pre-test answer	Post-test answer
M-08	 a. Mahasiswa menggunakan lensa Cembung (Konvergen) karena lensa Cembung bisa menghasilkan bayangan diperbesar. b. $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$	Text representation to explain the characteristics of a thin convex lens:  Mahasiswa menggunakan lensa Cembung (Konvergen) karena hanya lensa Cembung yang dapat menghasilkan bayangan yang besar dan tegak dalam kondisi tertentu. Bayangan yang lebih besar dan tegak terjadi ketika benda berada di antara titik fokus (F) dan pusat lensa (O). Dalam kondisi ini, lensa Cembung menghasilkan bayangan maya. Mathematical representation to determine the position of the object:  Diketahui : · Perbesaran (M) = 3 · Jarak fokus (f) = 10 cm · Gunakan rumus perbesaran : $M = -\frac{s'}{s}$ Karena bayangan yang berbentuk maya dan tegak, nilai M positif. Sehingga : $3 = -\frac{s'}{s}$ $s' = -3s$ Gunakan rumus lensa tipis : $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$ Substitusi $s' = -3s$ $\frac{1}{10} = \frac{1}{s} + \frac{1}{-3s}$ $\frac{1}{10} = \frac{3-1}{3s}$ $\frac{1}{10} = \frac{2}{3s}$ $s = \frac{2}{3} \times 10 = \frac{20}{3} \approx 6.67$ cm. Jadi, lilin diletakkan sekitar 6,67 cm dari lensa. A diagrammatic representation to describe the process of forming the shadow of objects: 

The students' answers above are a typical pattern found in the initial test. In the answers above, in the pretest, many students used only a single representation in answering the questions, relying solely on formulas, verbal descriptions, or simple diagrams to explain optical phenomena, or even providing no explanation at all. Such limited representations suggest an understanding that remains quite basic or is confined to formal knowledge that has not been integrated with other concepts (Mainali, 2021; Rexigel et al., 2024). It can be seen that they still struggle to explain the concept of a lens and draw the path of rays correctly. The drawings made by students demonstrate their efforts to understand the concept of shadow formation on a convex lens. Students seem to have difficulty determining the position of objects and shadows formed, especially in describing the course of special rays, so that the resulting shadows do not follow the principles of shadow formation on a convex lens. This result is consistent with findings from previous studies (Grusche, 2017; Mitrović et al., 2020). However, from these answers, students have a basic understanding. This can be seen from several indicators; for example, they can sketch a convex lens, although there are still errors in the position or direction of the rays. In addition, they also realize that lenses can form shadows but still make mistakes in identifying the type of lens or the nature of the shadow produced. These errors do not mean that they do not understand the concept at all, but rather indicate that their initial understanding is not entirely accurate (Marcelina & Hartanto, 2021). The theory of knowledge from DiSessa (Kali, Goodyear, & Markauskaite, 2011) suggests that students' understanding is not formed from a single, unified concept, but rather from a collection of smaller pieces of knowledge. Students enter the class with initial knowledge, either acquired through daily experiences or previous formal learning; however, it is not uncommon for this initial knowledge to be partial or erroneous (misconceptions), so it needs to be identified and addressed carefully in the learning process.

In the post-test, with the application of concreteness fading and multi-representation learning, there was a significant change in the way students organized their answers. Many students were able to provide multi-representations, such as combining diagrams of shadow formation with mathematical equations and more in-depth verbal explanations of the phenomena that occurred. This increased use of multi-representations indicates a deeper and more holistic understanding (Hubber et al., 2010; Sutopo & Waldrip, 2014) of geometric optics. Fredlund et al. (2012) stated in their study that if students can use various representations and understand the meaning behind them, they will more easily connect or translate between representations to deepen their concept understanding.

The pre-test results showed that most students used only a single representation, such as a formula without a conceptual explanation or a simple diagram without considering the rules of optics, to explain the formation of images by convex lenses. Specifically, as shown in Table 9, at the beginning of the learning process, students were predominantly wrong in drawing the virtual image formation diagram when the object was very close to the convex lens and failed to explain the nature of the image formed from this position. At the concrete stage, students directly observe the phenomenon of image formation using a convex lens and a real object, allowing them to visually experience that the virtual image does not appear on the screen, even though it can be seen through the lens. At the semi-concrete stage, students are asked to draw a ray diagram based on the experimental situation, with the help of scaffolding from the lecturer and the use of interactive simulations that allow them to dynamically re-test the relationship between the position of the object and its image. Furthermore, the symbolic stage strengthens conceptual understanding through the application of the lens formula mathematically. The results of mathematical calculations are analyzed and compared with those of previous experiments and diagrams. This stage strengthens the connection between mathematical symbols and the physical reality that students have experienced. The representation integration stage requires students to combine observations, diagram visualizations, and mathematical calculations into one unified answer. Finally, in the evaluation and reflection stage, students are asked to compare their initial answers with the new understanding they have gained so that they can explicitly recognize and correct their initial errors, as illustrated by the answers on the posttest in Table 9. Thus, all stages of this intervention directly target students' representational weaknesses and build deeper and more integrated conceptual understanding.

The findings of this study align with the theoretical concept proposed by Fyfe & Nathan (2019), who hypothesized that a gradual transition from concrete representation to abstract representation is more effective than simultaneous presentation. The findings in this study support the hypothesis, as students taught with concreteness fading and multi-representation demonstrated a better understanding compared to the control class. This suggests that concept understanding in geometric optics develops more optimally when students are given real experiences before moving to symbolic representations. Practically, in learning geometric optics on the topic of lenses, phased learning starts with experiments using real lenses to observe shadow formation (concrete representation), followed by drawing diagrams of light rays to understand the principles of reflection and refraction (iconic representation), and finally applying the mathematical equations of thin lenses (abstract representation). This stepwise presentation ensures that students have sufficient time to understand each stage before moving to a higher level of abstraction (Fyfe et al., 2015).

Fyfe & Nathan (2019) and Kokkonen et al. (2022) emphasize that learning with concreteness fading requires not only clear stages but also “additional interventions” that support the linkages between stages. Researchers use the term “scaffolding” for these “additional interventions.”. In the context of concreteness fading and multi-representation learning, scaffolding plays a crucial role in guiding students as they transition from one form of representation to another. This means that the transition from concrete to abstract concepts is not sudden but must be gradual, with support tailored to each student's readiness. If students have difficulty in a stage, the teacher can return to the previous representation and provide additional forms of scaffolding before proceeding to the next stage. This means that scaffolding is provided to address the difficulties faced by students at each stage (details of the scaffolding form are in Table 2). For example, when students have difficulty observing the shadow of an object in front of a convex lens, the lecturer does not immediately provide an answer but asks a provocative question, such as "Can the shadow be captured on the screen?" or "Where is the position of the shadow if the object is moved further from the convex lens?" The lecturer provides explicit directions to observe the properties of the shadow, such as size, position, and whether the shadow is real or virtual. Scaffolding is designed to connect various types of representations, such as linking experimental results to ray diagrams or simulation results to mathematical calculations. For example, the lecturer can ask students to explain what they see in the simulation using a ray diagram or calculate the distance of the shadow based on the configuration in the simulation. The lecturer provides digital scaffolding by integrating technology as visual and interactive support in the form of computer simulations.

The provision of scaffolding is believed to be one of the components that positively impacts students' conceptual understanding in the experimental class. This is relevant to the research by Simon & Klein (2007), which demonstrated that scaffolding interventions led to significant improvements in students' abilities. The study by Rokhmat & Putrie (2019) found that learning with scaffolding in learning activities proved effective in improving understanding of physics concepts. Likewise, the study by Alanazi et al. (2024)

concluded that scaffolding strategies have a positive impact on science learning, particularly in teaching physics at the college level.

Jaakkola & Veermans (2020) and Ainsworth (2006) emphasize the importance of identifying components in concreteness fading that can impact the outcome, such as the timing of transitioning from one representation to another. Based on this research, timing and scaffolding have a complementary relationship in transitioning from one representation to another in learning, especially in concreteness fading or multi-representation-based approaches. Proper timing ensures that students move from one representation stage to the next at the optimal moment, which is when they have achieved sufficient understanding of the previous representation but are still in a cognitive state that allows them to accept new challenges. Meanwhile, scaffolding acts as support provided by teachers or instructors to help students in the process of transitioning from one representation to another. Specifically for scaffolding, it helps students when facing difficulties in the transition by providing support that is gradually reduced as their understanding increases. This statement is also supported by experimental studies that reveal that learners who are given explicit guidance to make connections between different representations show better results (Fyfe & Nathan, 2019). This is relevant to the research conducted by Donovan & Fyfe (2022), whose study highlights that without adequate support (scaffolding), learners have difficulty connecting one representation with another, making learning less effective. The study by Fyfe et al. (2014) demonstrates that without sufficient guidance, students may struggle to connect concrete and abstract representations, leading to misconceptions in their understanding of concepts. Kokkonen & Schalk (2021) stated that through this “additional intervention,” explicit references between stages will help students see stronger connections between concrete representations, diagrammatic visualizations, and mathematical formulations, thereby strengthening their understanding as a whole and as a coherent whole.

In the context of geometric optics, the transition from concrete to abstract representations has unique characteristics that require in-depth analysis. According to Kokkonen & Schalk (2021), transitions in physics are not always linear and necessitate a more robust approach. However, the topic of geometric optics has the potential to be one of the physics topics that is closer to a linear transition than other physics concepts. This is because the fundamental laws in geometric optics (reflection and refraction) can be directly modeled using ray diagrams, which, although abstract, still have a strong visual connection to concrete phenomena. For example, when students learn about shadow formation using lenses and mirrors, they can gradually transition from direct observation to ray path mapping with the help of simple mathematical rules. McNeil & Fyfe's (2012) research confirms that the transition from concrete to abstract is more effective if the abstract representation retains a clear connection to concrete experience. However, although the transition in geometric optics tends to be more linear than in other physics topics, again, appropriate pedagogical interventions are needed so that students not only understand the concepts procedurally but can also generalize the principles to broader situations (Fyfe & Nathan, 2019; Kokkonen et al., 2022).

The success of the study is also attributed to the fact that it was conducted among pre-service teachers, who possess higher levels of abstract thinking skills. Therefore, university students may be better equipped to connect various representations, which may explain the difference in results between this study and Kokkonen & Schalk (2021), where the research subjects were secondary school students. Research by Fyfe et al. (2014) demonstrated that the effectiveness of concreteness fading is higher in individuals with strong prior knowledge. Students, especially those in physics education, may already possess a basic understanding of physics and math concepts from their first year of college and previous educational levels. This allows them to adapt to concrete representations before moving to abstract forms, in stark contrast to high school students. Donovan & Fyfe (2022) found that the transition from concrete to abstract representations is more effective when learners have sufficient cognitive capacity to connect the two forms of representation. College students have generally acquired better analytical and problem-solving skills than high school students, so they are better prepared to follow the fading stage optimally.

▪ CONCLUSION

The independent sample t-test results showed a significant difference between the experimental group and the control group ($p = 0.008 < 0.05$), with the experimental group scoring higher on average ($M = 21.55$) than the control group ($M = 19.10$). This finding is supported by the results of the N-Gain analysis, which showed that the increase in conceptual understanding in the experimental group reached 0.75, categorized as high. In contrast, the control group experienced an increase of 0.56, categorized as moderate. Based on the results of statistical tests and N-Gain analysis, learning with concreteness fading and multi-representation proved to be more effective in improving students' concept understanding compared to conventional learning in the control class, especially on the topic of geometric optics.

This study was conducted on college students who have had sufficient academic experience in understanding abstract concepts. Therefore, the results of this study may not be generalizable to high school students with varying levels of cognitive readiness. To improve the generalizability of the results, further research can be conducted on secondary school students. This study only examined the effectiveness of concreteness fading on geometric optics material. Concepts in physics have various levels of abstraction, so this approach may yield different results when applied to other topics, such as electromagnetism. Additionally, students' understanding is evaluated through conceptual test-based pre- and post-tests. This method has not fully explored qualitative aspects, such as students' thought processes in transferring concepts from concrete to abstract, so future research can utilize case study interview methods to gain a deeper understanding of how students construct their understanding.

▪ REFERENCES

Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction*, *16*(3), 183–198.

Alanazi, A. A., Osman, K., & Halim, L. (2024). Effect of scaffolding strategies and guided discovery on higher-order thinking skills in physics education. *Eurasia Journal of Mathematics, Science and Technology Education*, *20*(9), em2496.

Aregehagn, E., Lykknes, A., Getahun, D. A., & Febri, M. I. M. (2023). Representation of image formation observation in optics in ethiopian textbooks: student learning difficulties as an analytical tool. *Education Sciences*, *13*(5), 445.

Artino Jr, A. R., La Rochelle, J. S., Dezee, K. J., & Gehlbach, H. (2014). Developing

questionnaires for educational research: AMEE Guide No. 87. *Medical Teacher*, *36*(6), 463–474.
Bulut, M., Akçakın, H. Ü., Kaya, G., & Akçakın, V. (2016). The effects of geogebra on third grade primary students' academic achievement in fractions. *International Electronic Journal of Mathematics Education*, *11*(2), 347–355.
Ciobanu, A., Miron, C., Berlic, C., & Barna, V. (2023). Teaching geometrical optics by means of geogebra software. *Romanian Reports in Physics*, *75*(1), 1–14.
Corradi, D., Elen, J., & Clarebout, G. (2012). Understanding and enhancing the use of multiple external representations in chemistry education. *Journal of Science Education and Technology*, *21*, 780–795.
Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches*. Sage publications.
de Jong, T., Lazonder, A. W., Chinn, C. A., Fischer, F., Gobert, J., Hmelo-Silver, C. E., … Linn, M. C. (2023). Let's talk evidence–The case for combining inquiry-based and direct instruction. *Educational Research Review*, *39*, 100536.
Donovan, A. M., & Fyfe, E. R. (2022). Connecting concrete objects and abstract symbols promotes children's place value knowledge. *Educational Psychology*, *42*(8), 1008–1026.
Eagan, B., Brohinsky, J., Wang, J., & Shaffer, D. W. (2020). Testing the reliability of inter-rater reliability. *Proceedings of the Tenth International Conference on Learning Analytics \& Knowledge*, 454–461.
Fredlund, T., Airey, J., & Linder, C. (2012). Exploring the role of physics representations: an illustrative example from students sharing knowledge about refraction. *European Journal of Physics*, *33*(3), 657.
Fyfe, E. R., McNeil, N. M., & Borjas, S. (2015). Benefits of "concreteness fading" for children's mathematics understanding. *Learning and Instruction*, *35*, 104–120.
Fyfe, E. R., McNeil, N. M., Son, J. Y., & Goldstone, R. L. (2014). Concreteness fading in mathematics and science instruction: A systematic review. *Educational Psychology Review*, *26*, 9–25.
Fyfe, E. R., & Nathan, M. J. (2019). Making "concreteness fading" more concrete as a theory of instruction for promoting transfer. *Educational Review*, *71*(4), 403–422.
Gacovska Barandovska, A., Mitrevski, B., & Barandovski, L. (2023). Students' ability to use geometry knowledge in solving problems of geometrical optics. *International Journal of Mathematical Education in Science and Technology*, *54*(10), 2071–2085.
Gao, L., Xu, K., Wang, H., & Peng, Y. (2022). Multi-representation knowledge distillation for audio classification. *Multimedia Tools and Applications*, *81*(4), 5089–5112.
Giancoli, D. C. (2022). *Physics: Principles and applications* (7th ed.). California: Pearson.
Grusche, S. (2017). Developing students' ideas about lens imaging: teaching experiments with an image-based approach. *Physics Education*, *52*(4), 44002.
Hake, R. R. (1998). Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. *American Journal of Physics*, *66*(1), 64–74.
Halliday, D., Resnick, R., & Walker, J. (2013). *Fundamentals of physics*. New York: John

Wiley & Sons.

Hansen, J., & Richland, L. E. (2020). Teaching and learning science through multiple representations: Intuitions and executive functions. *CBE Life Sciences Education*, *19*(4), ar61.

Hartanto, T. J., Dinata, P. A. C., Azizah, N., Qadariah, A., & Pratama, A. (2023). Students' science process skills and understanding on Ohm's law and direct current circuit through virtual laboratory based predict-observe-explain model. *Jurnal Pendidikan Sains Indonesia (Indonesian Journal of Science Education)*, *11*(1), 113–128.

Hubber, P., Tytler, R., & Haslam, F. (2010). Teaching and learning about force with a representational focus: Pedagogy and teacher change. *Research in Science Education*, *40*, 5–28.

Hughes, C. A., Morris, J. R., Therrien, W. J., & Benson, S. K. (2017). Explicit instruction: Historical and contemporary contexts. *Learning Disabilities Research & Practice*, *32*(3), 140–148.

Jaakkola, T., & Veermans, K. (2018). Exploring the effects of concreteness fading across grades in elementary school science education. *Instructional Science*, *46*, 185–207.

Jaakkola, T., & Veermans, K. (2020). Learning electric circuit principles in a simulation environment with a single representation versus "concreteness fading" through multiple representations. *Computers \& Education*, *148*, 103811.

Kali, Y., Goodyear, P., & Markauskaite, L. (2011). Researching design practices and design cognition: contexts, experiences and pedagogical knowledge in pieces. *Learning, Media and Technology*, *36*(2), 129–149.

Kim, H. (2020). Concreteness fading strategy: A promising and sustainable instructional model in mathematics classrooms. *Sustainability*, *12*(6), 2211.

Kokkonen, T., Lichtenberger, A., & Schalk, L. (2022). Concreteness fading in learning secondary school physics concepts. *Learning and Instruction*, *77*, 101524.

Kokkonen, T., & Schalk, L. (2021). One instructional sequence fits all? A conceptual analysis of the applicability of concreteness fading in mathematics, physics, chemistry, and biology education. *Educational Psychology Review*, *33*(3), 797–821.

Kollosche, D. (2021). Styles of reasoning for mathematics education. *Educational Studies in Mathematics*, *107*(3), 471–486.

Lichtenberger, A., Kokkonen, T., & Schalk, L. (2024). Learning with multiple external representations in physics: Concreteness fading versus simultaneous presentation. *Journal of Research in Science Teaching*, *61*(9), 2258–2290.

Mainali, B. (2021). Representation in teaching and learning mathematics. *International Journal of Education in Mathematics, Science and Technology*, *9*(1), 1–21.

Marcelina, S., & Hartanto, T. J. (2021). Correcting students' understanding about simple direct current (DC) circuits through scientific approach. *Jurnal Penelitian & Pengembangan Pendidikan Fisika*, *7*(2), 153–160.

McHugh, M. L. (2012). Interrater reliability: the kappa statistic. *Biochemia Medica*, *22*(3), 276–282.

McNeil, N. M., & Fyfe, E. R. (2012). "Concreteness fading" promotes transfer of mathematical knowledge. *Learning and Instruction*, *22*(6), 440–448.

Meltzer, D. E. (2005). Relation between students' problem-solving performance and

representational format. *American Journal of Physics*, *73*(5), 463–478.
Mitrović, M. M., Misailović, B. M., Maksimović, B. Z., & Žekić, A. A. (2020). Conceptual difficulties in interpreting the real image of an object. *American Journal of Physics*, *88*(2), 141–147.
Müller, A., Hettmannsperger, R., Scheid, J., & Schnotz, W. (2017). Representational competence, understanding of experiments, phenomena and basic concepts in geometrical optics: A representational approach. In D.F. Treagust, R. Duit, & H. Fischer (Eds), *Multiple Representations in Physics Education* (pp. 209–229). Springer.
Munfaridah, N., Avraamidou, L., & Goedhart, M. (2021). The use of multiple representations in undergraduate physics education: what do we know and where do we go from here? *Eurasia Journal of Mathematics, Science and Technology Education*, *17*(1), em1934.
Nousiainen, M. (2013). Coherence of pre-service physics teachers' views of the relatedness of physics concepts. *Science & Education*, *22*, 505–525.
Opfermann, M., Schmeck, A., & Fischer, H. E. (2017). Multiple representations in physics and science education--why should we use them? In D.F. Treagust, R. Duit & H. Fischer (Eds), *Multiple representations in physics education* (pp. 1–22). Springer.
Ottmar, E., & Landy, D. (2017). Concreteness fading of algebraic instruction: Effects on learning. *Journal of the Learning Sciences*, *26*(1), 51–78.
Pallant, J. (2020). *SPSS survival manual: A step by step guide to data analysis using IBM SPSS*. Routledge.
Prain, V., & Tytler, R. (2012). Learning through constructing representations in science: A framework of representational construction affordances. *International Journal of Science Education*, *34*(17), 2751–2773.
Rexigel, E., Kuhn, J., Becker, S., & Malone, S. (2024). The more the better? A systematic review and meta-analysis of the benefits of more than two external representations in STEM education. *Educational Psychology Review*, *36*(4), 124.
Rokhmat, J., & Putrie, S. D. (2019). A strategy of scaffolding development to ıncrease students' problem-solving abilities: the case of physics learning with causalitic-thinking approach. *Journal of Turkish Science Education*, *16*(4), 569–579.
Roth, K. J. (2006). *Teaching science in five countries: Results from the TIMSS 1999 video study: Statistical analysis report*. Washington, DC: US Department of Education, National Center for Education Statistics.
Senocak, E., Taskesenligil, Y., & Sozbilir, M. (2007). A study on teaching gases to prospective primary science teachers through problem-based learning. *Research in Science Education*, *37*, 279–290.
Simons, K. D., & Klein, J. D. (2007). The impact of scaffolding and student achievement levels in a problem-based learning environment. *Instructional Science*, *35*, 41–72.
Skulmowski, A. (2023). Do concreteness fading and guidance fading aid learning from perceptually rich visualizations? Changes in style lead to more cognitive load and interfere with learning. *Current Research in Behavioral Sciences*, *4*, 100112.
Sorge, S., Kröger, J., Petersen, S., & Neumann, K. (2019). Structure and development of pre-service physics teachers' professional knowledge. *International Journal of Science Education*, *41*(7), 862–889.

Sugiyono. (2011). *Metode penelitian pendidikan*. Bandung: Alfabeta.

Sutarto, S., Indrawati, I., Prihatin, J., & Dwi, P. A. (2018). Geometrical optics process image-based worksheets for enhancing students' higher-order thinking skills and self-regulated learning. *Jurnal Pendidikan IPA Indonesia*, *7*(4), 376–382.

Sutopo & Waldrip, B. (2014). Impact of a representational approach on students'reasoning and conceptual understanding in learning mechanics. *International Journal of Science and Mathematics Education*, *12*, 741–765.

Tipler, P. A., & Mosca, G. (2007). *Physics for scientists and engineers*. New York: W.H. Freeman and Company.

Tippett, C. D. (2016). What recent research on diagrams suggests about learning with rather than learning from visual representations in science. *International Journal of Science Education, 38*(5), 725–746.

Tural, G. (2015). Cross-grade comparison of students' conceptual understanding with lenses in geometric optics. *Science Education International*, *26*(3), 325–343.

Uwamahoro, J., Ndihokubwayo, K., Ralph, M., & Ndayambaje, I. (2021). Physics students' conceptual understanding of geometric optics: Revisited analysis. *Journal of Science Education and Technology*, *30*(5), 706–718.

Vázquez-Bernal, B., Mellado, V., Jiménez-Pérez, R., & Leñero, M. C. T. (2012). The process of change in a science teacher's professional development: A case study based on the types of problems in the classroom. *Science Education*, *96*(2), 337–363.

Zhao, Y. (2024). From tiles to worksheet: Exploring concreteness fading in learning vector addition. *Education Sciences*, *14*(7), 730.