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# Misconceptions on Interpreting and Modeling the Concept of Division Among Elementary School Teachers

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**Abstract:** This study investigates the nature and extent of conceptual misconceptions held by elementary school teachers in interpreting and modeling the mathematical concept of division. Motivated by the recognition that students' misunderstandings often originate from teachers' inadequate conceptual grasp, particularly regarding the use of partitive and quotative models, this research addresses a critical gap in the literature on teachers' mathematical representations. Although division is a foundational concept in mathematics instruction, limited empirical research has explored how teachers misconstrue its meanings in classroom contexts. Employing a descriptive qualitative design within a multiple-case study framework, the study involved 80 fifth-grade teachers from four major Indonesian cities: Jakarta, Bandung, Yogyakarta, and Surabaya. Participants were selected through purposive sampling. Data were collected through classroom observations, semi-structured interviews, and lesson plan analysis. Thematic analysis was used to identify recurring patterns of misconception across instructional practices. Findings in this study revealed misconceptions among many teachers in distinguishing between the partitive (repeated subtraction) and quotative (equal sharing) interpretations of division. This confusion results in the use of inappropriate, rigid, or overly simplified concrete models. The misconception distorts mathematical representations and contributes directly to the propagation of student misconceptions. The most prominent patterns occurred during story problem interpretation, where teachers struggled to match the semantic structure with the appropriate division model. These conceptual misconceptions not only distort instructional representations but also contribute to students' way of thinking. These findings highlight the urgent need for targeted professional development programs. Those would emphasize semantic analysis of word problems and the flexible use of multiple representations. Such interventions are essential to help teachers deliver instruction that fosters conceptual understanding beyond procedural fluency. Aligning teacher training with findings in this study may prevent the transfer of fundamental misconceptions to students and promote deeper mathematical thinking in early education contexts.

**Keywords:** misconceptions, interpretations, division, elementary school, teacher.

## ▪ INTRODUCTION

Division, at its conceptual foundation, can be interpreted as a process of repeated subtraction, where a given quantity is decreased incrementally by equal parts until it reaches zero (Van de Walle et al., 2019). This representation offers a cognitively accessible entry point for learners, bridging intuitive strategies and formal operations. Embedding this structure into students' understanding is critical for shifting their perspective from mere computational routines toward meaningful mathematical reasoning. Instead of treating division merely as a set of steps to follow, this representation strategy helps students understand the logic behind it, because it focuses on why division makes sense in real situations.

Beyond repeated subtraction, division is commonly taught through two structurally distinct models: partitive division (repeated subtraction) and quotative division (equal

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sharing). In partitive contexts, partitive division involves identifying how many groups of a fixed size can be formed from a total. In contrast, a total quantity is divided into a known number of groups to determine the size of each group (Van de Walle, Karp, & Bay-Williams, 2019; Greer, 1992). Although both models share similar symbolic forms (e.g., a ÷ b), their cognitive demands and contextual applications diverge significantly. Understanding these models more deeply is important because each one reflects a different way of thinking about division. Also, each of them helps highlight different ideas about how division works and what it means.

Additionally, division is often introduced as the inverse operation of multiplication, where understanding the relationship among factors and products reinforces multiplicative reasoning (Usiskin, 2007). Division also functions as a vehicle for ratio interpretation (Lamon, 2005) and as a basis for fraction construction, especially in contexts where whole-number division yields non-whole-number results (Charles et al., 2015). For example, interpreting ¾ as "three parts of a whole partitioned into four" illustrates how division connects to proportional reasoning and part-whole relationships. The variety of ways in which students divide underscores its importance in shaping their mathematical literacy. It also points to the importance of teachers having strong pedagogical content knowledge, so they can choose representations that fit the lesson's goals. (Canogullari & Isiksal, 2024).

However, this study found that many teachers experienced cognitive difficulties in distinguishing between the partitive (repeated subtraction) and quotative division (equal sharing) models of division, which significantly narrowed their instructional perspective. These challenges often came from relying too heavily on text-based materials. The mathematical meanings in narrative problems were rarely interpreted reflectively. As Dixon & Tobias (2022) noted, when teachers fail to evaluate the structural representations in problems, they may unintentionally pass errors from textbooks into their teaching. Over time, these mistakes can become built-in misconceptions across the classroom. Moreover, Sungur et al. (2021) emphasized that effective mathematics teaching requires the ability to translate abstract ideas into contextual and representational forms. In this case, teachers' struggles to create or adapt narrative problems show that their understanding of division is both rigid and limited in depth. This weakness leads to teaching that focuses only on procedures. It also limits students' ability to see division as the basis for understanding fractions and ratios in later grades, which keeps the conceptual gaps going.

In the documentation study, the researcher identified various issues in elementary school learning resources, particularly in how they convey the meaning of division. Most of these resources still frame division narrowly as "sharing equally," and have not yet extended toward a broader conceptual understanding, such as interpreting division as "how many units are contained within another" (Tim Gakko Tosho, 2021). Furthermore, the researcher observed that the illustrations and narratives in many word problems often fail to align with the appropriate mathematical models. For instance, problems that should represent the model "a ÷ ? = b" are frequently structured instead as "a ÷ b = ?." This misrepresentation leads to incorrect problem-solving procedures, as students are guided to follow strategies that do not correspond with the underlying concept. This phenomenon points to a widespread misconception built into how division is presented in textbooks. Classroom practices then reinforce it even further. Teachers tend to rely on a single model of division, equal sharing, without introducing other conceptual meanings, such as division as repeated subtraction or finding the number of groups. As a result, students end up with a limited understanding of division. Many important conceptual elements are either oversimplified or absent from the learning materials.

Although many different ways to teach division exist, research consistently shows that both students and teachers often have trouble with the core conceptual differences. One well-known misconception is mixing up partitive and quotative division. All division is treated as equal sharing, without recognizing the different logical structures and representations each one involves. (Spitzer et al., 2025; Kinboon, 2019). Spitzer et al. (2025), through a large-scale assessment study in Germany, identified that even in-service teachers often defaulted to partitive models regardless of problem structure, suggesting deep-rooted misconceptions. Similarly, Kiymaz (2023) investigated Turkish elementary teachers and found that most could not distinguish semantic differences between "how many groups" and "how many in each group," leading to systematic errors in interpreting contextual problems. Ölmez and Izsák (2023), using eye-tracking and interview data, showed that teachers' visual attention during problem solving disproportionately focused on surface features like keywords rather than on quantitative relationships, indicating weak structural reasoning. However, these studies primarily focused on teachers' reasoning at the problem-solving level, without examining how such misconceptions manifest in actual classroom practices, task construction, or instructional narratives. Moreover, little is known about how teachers' use of visual representations and textbook-based language may further reinforce these misunderstandings. The present study addresses this gap by analyzing not only teachers' conceptual interpretations but also how those interpretations are translated into representational models, classroom demonstrations, and instructional storytelling, offering a comprehensive view of systemic misconceptions in teaching division. Such oversimplifications can make it harder for students to see fractions as quantities that represent parts of a whole (Charles et al., 2015).

These trends reflect deeper issues in teachers' mathematical knowledge for teaching (MKT), particularly in their ability to interpret and model foundational mathematical structures (Ball, Thames, & Phelps, 2008). In newer extensions of the MKT framework, researchers stress that Specialized Content Knowledge (SCK) is crucial for teachers. It helps them choose, interpret, and create mathematical representations that match students' needs and the demands of the task (Depaepe et al., 2015; Blömeke & Kaiser, 2017). The inability to distinguish between partitive and quotative division models reflects a deficiency in SCK. It is not just about knowing how to carry out operations. It also means unpacking mathematical structures, spotting semantic cues in problems, and connecting those cues to the right conceptual model during teaching. Using division models in teaching without fully understanding their conceptual foundations can limit deep learning. When a teacher's choice of model does not align with the problem's semantic structure, it can unintentionally promote or strengthen misconceptions instead of fixing them.

Data collected during the initial implementation of the research instrument revealed that many elementary teachers still struggled to identify and differentiate among the three main models of division. These difficulties contributed to the emergence of conceptual errors, mainly when translating word problems into mathematical representations. Consistent with findings by Li & Schoenfeld (2019), Shih et al. (2023), and Anggiana et al. (2022), this suggests that many teachers face persistent challenges in teaching division conceptually, particularly when dealing with narrative or contextual problems.

Further evidence from Kusmaryono, Basir, and Maharani (2020) indicates that misconceptions about fundamental mathematical concepts are not limited to students but are also prevalent among in-service elementary school teachers. Their findings highlight the systemic nature of conceptual misunderstandings in mathematics instruction, especially regarding operations such as division. Against this backdrop, the research question of this study is as below:

**RQ-1.** What types of conceptual misconceptions elementary school teachers demonstrate when interpreting and modeling the partitive and quotative meanings of division?
**RQ-2.** How do these misconceptions affect their selection and use of mathematical representations in classroom instruction, particularly when designing and delivering story problems?

## ▪ METHOD

### Research Design

This study employed a descriptive qualitative approach using a case study design, selected to enable an in-depth exploration of elementary teachers' conceptual misconceptions in interpreting and modeling the division operation. The case study methodology was chosen for its strength in capturing complex, context-bound phenomena, particularly teachers' understanding of foundational mathematical ideas within authentic instructional settings (Merriam & Tisdell, 2016; Yin, 2018). Rather than testing hypotheses, the study focused on revealing the cognitive processes and representational frameworks teachers utilize when teaching division to their students.

### Participants and Setting

Participants consisted of 80 fifth-grade classroom teachers from public elementary schools located in four urban regions of Indonesia: Jakarta, Bandung, Yogyakarta, and Surabaya. A purposive sampling strategy was used to ensure participants met the following criteria: (1) active classroom teachers responsible for mathematics instruction in grade five; (2) a minimum of three years of teaching experience; and (3) willingness to participate in classroom observations, in-depth interviews, and instructional document analysis.

The selection of these regions considered variation in school profiles and logistical accessibility to support intensive data collection. While not all participants held degrees in mathematics education, the majority (43 out of 80) graduated from elementary teacher education programs, and all had met national subject-area alignment requirements through recognized course equivalency or retraining programs. However, the variety of academic backgrounds was treated as a relevant variable potentially contributing to variation in teachers' conceptual understanding of division, particularly its interpretation as repeated subtraction. Table 1 presents the distribution of participants by region.

**Table 1.** Regional distribution of study participants

| Participants' Area of Origin | Total Subject |
|---|---|
| Jakarta | 15 |

| | |
|---|---|
| Bandung | 30 |
| Yogyakarta | 18 |
| Surabaya | 17 |
| **Total** | **80** |

**Instrument**

To obtain a comprehensive understanding of teachers' misconceptions in teaching division, the study employed three primary data collection techniques: (1) a division concept diagnostic test, (2) classroom observations, and (3) in-depth, semi-structured interviews. This triangulated strategy enhanced data validity and allowed for multi-perspective insights into teachers' instructional practices (Creswell & Poth, 2018). The diagnostic test was designed to assess five key areas:

1. Understanding of division as repeated subtraction,
2. Ability to abstract the concept of division,
3. Construction of mathematical models for division of whole numbers,
4. Representation of contextual division problems, and
5. Use of manipulatives or visual tools to support division concepts.

Classroom observations took place during regular math lessons in fifth-grade classes. The focus was on observing how teachers delivered division concepts, what hands-on materials they used, and how they interacted with their students. The observations were non-participant in nature, just observing and taking notes, plus recording videos. This follows the usual way researchers study classrooms (Miles, Huberman, & Saldaña, 2019).

To make sure the findings were solid, three different ways to collect information were used: tests for teachers, watching classes, and interviews. This helped verify if what was found was accurate. Like, if a teacher picked the wrong division model on the test (maybe using sharing instead of grouping), classroom observations would show if they made the same mistake during actual teaching, too. Then interviews would explore why they made those choices - was it because they misunderstood the wording, or maybe they just copied what was in textbooks? Every problem that got identified had to show up in at least two of the data sources, not just one. This way, there could be more confidence about the findings (Fusch, Fusch, & Ness, 2018). The whole process helped avoid jumping to conclusions and gave a better understanding of how these misunderstandings showed up both when teachers planned lessons and when they taught them.

After teachers took the diagnostic test, some of them got picked for interviews, choosing ones who had given different types of answers. These interviews helped dig deeper into their thinking and understand why they chose specific models or teaching strategies. There were some prepared questions, but things stayed flexible so follow-up questions could be asked when something interesting came up (Merriam & Tisdell, 2016).

**Data Analysis**

The data were analyzed using thematic analysis, a method well-suited for identifying, organizing, and interpreting patterns in qualitative educational data (Braun & Clarke, 2019). The process followed the six-phase framework suggested by Braun and Clarke (2019), beginning with familiarization, generating initial codes, searching for themes, reviewing themes, defining and naming themes, and finally producing the report.

In this study, we adopted a hybrid coding strategy, combining inductive and deductive coding. Deductive codes were informed by prior literature on division models (e.g., partitive vs. quotative), conceptual understanding (e.g., repeated subtraction, inverse operations), and representational forms (e.g., symbolic, visual, contextual). Simultaneously, inductive coding allowed emergent patterns from raw data, particularly in classroom observations and open-ended interviews, to refine the framework and accommodate contextual nuances specific to Indonesian teaching practices.

The initial coding was conducted by two researchers independently, using a shared codebook developed after initial pilot coding of 10% of the data. Codes included items such as “equal sharing visual,” “division as subtraction,” “symbol use without explanation,” and “reliance on linguistic keywords.” After this first cycle, the researchers met to compare coding consistency. An inter-rater reliability coefficient (Cohen’s Kappa) of 0.82 was achieved, indicating strong agreement. Discrepancies were resolved through consensus discussions involving a third senior researcher, which also helped refine ambiguous code definitions. In the second cycle, these initial codes were organized into broader thematic categories.

For example, the raw classroom observation: "The teacher draws a pie and divides it into four equal parts to explain 12 ÷ 4.” was first coded as “equal sharing visual.” This was then grouped under “partitive representation,” and ultimately subsumed under the overarching theme: “Overreliance on the partitive model.”

Another example is from a diagnostic test item where a teacher responded: “I teach students to subtract four multiple times to find how many groups fit in 20 ÷ 4.” This was coded as “repeated subtraction,” placed under the theme “Division as iterative reduction”, reflecting a procedural but narrow conceptual grasp of division.

Triangulation across three data sources, diagnostic tests, classroom observations, and interviews, ensured analytical depth and trustworthiness. For example, repeated instances of “visual equal sharing” found in lesson plans were validated against classroom video data and teacher statements during interviews. This method allowed each theme to be confirmed from multiple perspectives, which strengthened both its credibility and its transferability (Miles, Huberman, & Saldaña, 2019).

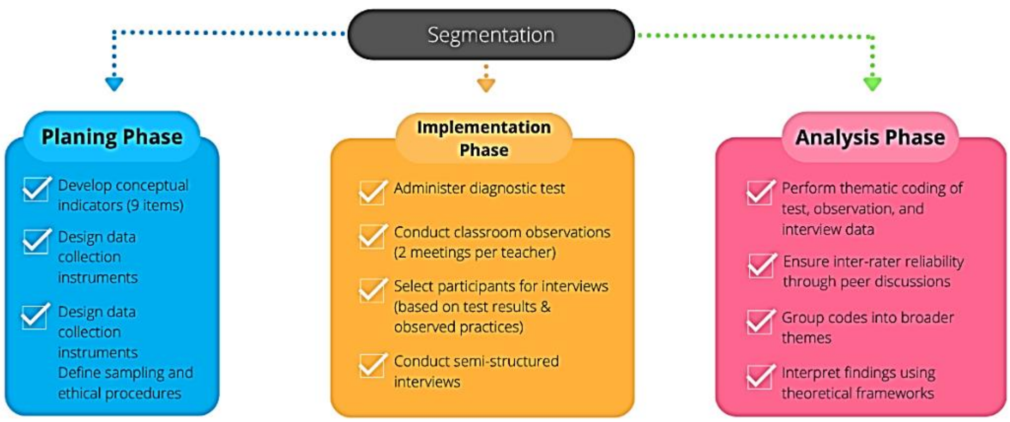



**Figure 1.** Data analysis flowchart

This study followed the methodological structure outlined by Prediger et al. (2015), emphasizing a descriptive approach to identify teachers’ misconceptions related to division. The research procedure included three phases: planning, implementation, and analysis, as can be seen in Figure 1. During the planning phase, nine conceptual indicators were developed to assess teachers' understanding of division based on the repeated subtraction perspective, as detailed in Table 2.

During the implementation phase, participants were asked to complete the diagnostic items and were instructed to document all written work, including incorrect attempts or revisions. These artifacts were considered valuable for capturing teachers' thought processes.

In the final analysis phase, participants' responses were categorized based on emerging reasoning patterns. Misconceptions were classified according to the predefined indicators drawn from the literature and are presented in Table 2. Responses were broadly categorized into four levels, that is:

1. Demonstrates clear conceptual understanding,
2. Shows partial understanding of uncertainty,
3. Displays misconceptions, or
4. It provides no relevant or coherent responses (Trivena et al., 2017).

Each category was anchored by both conceptual indicators and concrete response patterns. The distinction between "displays some understanding but shows no confidence" and "displays misconception" lies in the correctness of the underlying reasoning: the former shows partial accuracy or hesitation without conceptual error, while the latter includes confident yet incorrect mental models. The rubric criteria are detailed in Table 2.

**Table 2.** Rubric for categorizing teachers' conceptual understanding of division

| Category | Criteria for Classification |
|---|---|
| Understands the concept well | Accurately explains the operation and correctly matches it with the appropriate real-world context; able to distinguish quotative from partitive division. |
| Displays some understanding but shows no confidence | Provides a partially correct explanation or answer with hesitant or unsure phrasing, often lacking in complete reasoning. |
| Displays misconception | Offers confident yet incorrect reasoning, such as applying partitive reasoning to a quotative context or misinterpreting inverse structure. |
| Shows no understanding | Leaves the item blank, writes unrelated responses, or provides no coherent explanation of the division model. |

In this study, the test items and interview prompts were carefully crafted to reflect diverse aspects of division, each corresponding to a specific conceptual indicator as outlined in Table 3. Prior to implementation, all instruments were reviewed by experts in mathematics education to ensure conceptual coherence and pedagogical relevance. Rather than focusing on the validation of a new instrument, the emphasis was placed on using these items diagnostically to uncover patterns of reasoning, hesitation, and misinterpretation among teachers. Consequently, while a structured rubric adapted from Trivena et al. (2017) was used to categorize participants' responses into levels of understanding, the detailed mapping of items, scoring weights, and classification criteria is not included in full here due to space considerations and the study's focus on thematic insights rather than instrument development.

## ▪ RESULT AND DISSCUSSION

### Quantitative Patterns of Misconception

The findings from this study, organized across three major thematic domains, highlight a pervasive and deeply rooted misconception among elementary teachers: the inability to accurately identify and apply appropriate mathematical models for division. A more detailed breakdown of teachers' responses based on the conceptual indicators of division is presented in Table 3.

**Table 3.** Distribution of teachers' conceptual understanding across indicators

| Conceptual Indicators of Division | Understands the Concepts Well | Displays Some Understanding but Shows no Confidence | Displays Misconception | Shows No Understanding of the Concept |
|---|---|---|---|---|
| Division as Repeated Subtraction | 60 (75%) | 20 (25%) | 0 (0%) | 0 (0%) |
| Abstraction of the Division Concept | 1 (0.125%) | 23 (28.75%) | 40 (50%) | 16 (20%) |
| Modeling Division within Word Problem Contexts | 50 (62.5%) | 30 (37.5%) | 0 (0%) | 0 (0%) |
| Teachers' Interpretation of Division Representations | 20 (25%) | 60 (75%) | 0 (0%) | 0 (0%) |
| Interpretation of Mathematical Models in Quotative Word Problems | 10 (12.5%) | 0 (0%) | 70 (87.5%) | 0 (0%) |
| Development of Word Problems Involving Division | 10 (12.5%) | 40 (50%) | 0 (0%) | 30 (37.5%) |
| Illustrating Division Models Using Concrete Objects | 30 (37.5%) | 20 (25%) | 10 (12.5%) | 20 (25%) |
| Analysis of Division Representations | 10 (12.5%) | 0 (0%) | 40 (50%) | 30 (37.5%) |
| Understanding Division as a Ratio Concept | 40 (50%) | 10 (12.5%) | 0 (0%) | 30 (37.5%) |

Most notably mentioned from the results are the majority of participants interpreted division exclusively through the lens of partitive (equal sharing) models, without critically considering the contextual structure or narrative logic embedded in word problems. This tendency suggests more than a surface-level lack of knowledge; it reflects entrenched cognitive habits shaped by limited exposure to diverse mathematical representations during their own education and professional training. As noted by Hill & Chin (2018) and Pincheira & Alsina (2021), such conceptual rigidity may stem from insufficient Specialized Content Knowledge (SCK), a subdomain of Mathematical

Knowledge for Teaching (MKT), which involves the ability to select, interpret, and adapt representations to specific instructional goals. Moreover, these misconceptions appear to be sustained by unreflective teaching practices and the uncritical adoption of textbook-based examples, leading to what Ball, Thames, & Phelps (2008) describe as a procedural orientation rather than a conceptual one. Thus, the prevalence of partitive bias is not merely an instructional flaw but a cognitive default shaped by systemic limitations in both teacher preparation and ongoing professional development. This pattern also reinforces prior claims by Wu (2020), who argues that teachers lacking representational fluency often struggle to translate abstract mathematical ideas into instructional forms that foster student understanding. Overgeneralizing division as simply “equal distribution” creates a systemic instructional misconception. It affects both visual representations and abstract forms, such as repeated subtraction. This pattern of misapplication aligns with what Greer (1992) called the reliance on “primitive models,” intuitive understandings of arithmetic formed early in life and rarely examined in later years. For many teachers in this study, the partitive model serves as such a primitive model, easy to visualize and supported by everyday language. While this familiarity makes it accessible, it also encourages its use in situations where it does not fit. As a result, teachers often apply the partitive schema even in contexts better represented by quotative (equal sharing) division. This overextension keeps a narrow view of division across different types of problems. It misrepresents the underlying mathematical relationships. It also limits the ability to use multiple representations effectively. In turn, this hinders the development of flexible representational competence. In line with Greer’s perspective, the dominance of partitive reasoning in the participants’ responses suggests that their internalized concept of division remains anchored in early-learned structures that were never expanded through targeted conceptual enrichment during teacher preparation or practice. Therefore, the issue is not merely pedagogical but deeply cognitive, rooted in entrenched representational habits that fail to evolve in response to instructional complexity.

Table 3 summarizes the number and percentage of participants who demonstrated a conceptual understanding of division as repeated subtraction, as assessed through the diagnostic indicators. The results revealed that a substantial proportion of participants exhibited significant misconceptions across several indicators. For instance, 87.5% of participants misunderstood the indicator on interpreting mathematical models within quotative word problems, while only 12.5% showed an accurate understanding. Similarly, on the indicator related to developing contextual division problems, only 12.5% of respondents demonstrated adequate conceptual understanding.

Another critical indicator, “Analysis of Representations of Division Concepts”, also revealed notable gaps, with 50% of participants displaying misconceptions and 37.5% failing to provide any relevant response. These findings point to widespread conceptual challenges among teachers, particularly when working with contextualized or word-problem-based division tasks. The data suggest that many respondents struggled not only with mathematical modeling but also with selecting appropriate problem-solving strategies.

## Thematic Analysis of Teachers' Conceptual Understanding

### *Theme 1: Division as Repeated Subtraction*

In the first thematic domain, although most respondents were able to articulate the concept of division as repeated subtraction, their instructional practices gravitated toward visual representations consistent with partitive models. This disconnect reveals that their conceptual knowledge may remain superficial, verbally acknowledged but not deeply internalized or integrated into instructional reasoning (Van de Walle et al., 2019). The findings suggest a critical gap between teachers' theoretical understanding and their ability to implement mathematically sound representations in classroom settings.

Most respondents stated that they understood division as a process of repeated subtraction. Although some failed to perform well on related tasks, overall, no critical misconceptions were identified under this indicator. This finding aligns with literature in elementary mathematics education, which defines division as the iterative removal of equal quantities until zero is reached (Van de Walle et al., 2019).

Nonetheless, two respondents described division primarily as the inverse of multiplication. This suggests the presence of divergent mental models among teachers regarding the foundational meaning of division. As Gibim et al. (2023) noted, such variations in conceptual understanding often influence the instructional approaches teachers employ in the classroom. Similarly, Suryadi (2019) highlighted that conceptual knowledge has a direct impact on teachers' pedagogical choices, even when such decisions are not made explicitly. We analyze some respondents' answers who were asked to solve the following division task: *"Ms. Tuti has eight softball balls. The balls will be evenly distributed among two children. How many balls will each child receive?"*

All respondents interpreted the story problem as a case of partitive division and modeled it mathematically as 8÷2. Their justifications were supported by visual demonstrations, as illustrated in Figure 2 below:

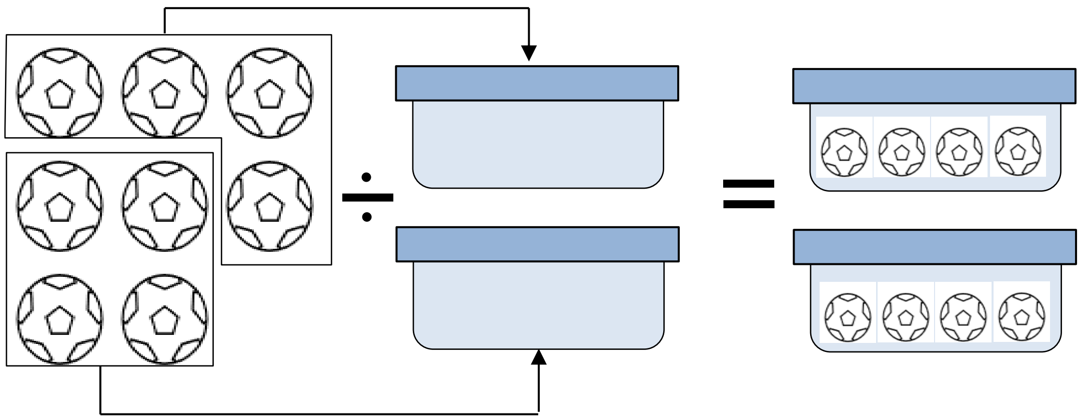


**Figure 2.** The process of visualizing mathematical models in division instruction

The visual in Figure 2 reflects the respondents' understanding of division as "equal sharing", creating two groups with four objects each. While mathematically correct, this interpretation fails to align with the intended conceptual framing of division as repeated subtraction. This finding suggests that, although teachers may be familiar with the terminology of repeated subtraction, their instructional representations predominantly reflect equal partitioning models.

There are several reasons why teachers might understand division as repeated subtraction but tend to use a partitive model in practice. First, the conceptual understanding of division as repeated subtraction is theoretically simple and aligns with foundational arithmetic operations. However, in practice, the partitive model is often employed because it provides a more practical and intuitive way of distributing objects into equal parts, which is a common scenario in real-world applications and educational settings (Matitaputty et al., 2024).

One hypothesis for this discrepancy is that while repeated subtraction is easy to grasp, it can become inefficient and cumbersome for larger numbers or more complex problems. The partitive model offers a simpler way to visualize and carry out the process of dividing quantities into equal parts. This can be an advantage in classroom settings. It is especially useful when the goal is to build students' intuitive understanding of division (Riera et al., 2023). Additionally, the way math is taught and structured in the curriculum often focuses on practical use. For division, this tends to favor the partitive approach because it fits easily into many everyday situations (Riera et al., 2023).

Moreover, teacher training often overlooks the different models of division. This leads many educators to default to the traditionally emphasized partitive model (Yoon et al., 2017). This lack of comprehensive teacher preparation may limit the use of the repeated subtraction model beyond theory. It reinforces the preference for the partitive method in classroom practice. Model choice is often shaped by practicality, ease of understanding for students, and limited exposure during training to alternatives like repeated subtraction (Matitaputty et al., 2024; Riera et al., 2023).

***Theme 2: Teachers' Abstraction of the Division Concept***

The gap between teachers' theoretical understanding and their ability to implement mathematically became more perceptible in the second theme, which focused on abstraction, the ability to translate contextual word problems into formal mathematical expressions. Alarmingly, only one out of 80 teachers accurately constructed a mathematical model that aligned with the story context. The vast majority either misrepresented the problem structure or failed to provide a relevant model at all. A primary factor appears to be unfamiliarity with the full range of division sentence structures, such as a $a \div b = ?$, $a \div ? = b$, and $? \div a = b$. Many participants defaulted to the first form due to its dominance in textbooks and standardized assessments, reflecting what Greer (1992) and Charles et al. (2015) describe as representational rigidity driven by curricular exposure rather than conceptual reasoning.

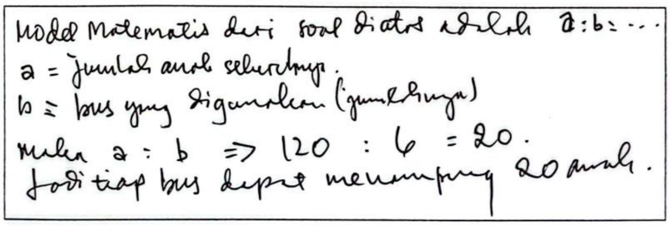
Analysis revealed that only one teacher demonstrated a comprehensive understanding of abstraction in teaching division as repeated subtraction. This individual effectively applied abstraction principles to scaffold students' conceptual development. In contrast, the remaining 79 participants, including 76 classroom teachers and 3 school leaders, either misapplied the abstraction process or failed to grasp its core essence.

A clear example of this misunderstanding emerged when participants were asked to construct mathematical models for a division word problem. As is well established, the division operation can be expressed through three fundamental mathematical sentence structures:

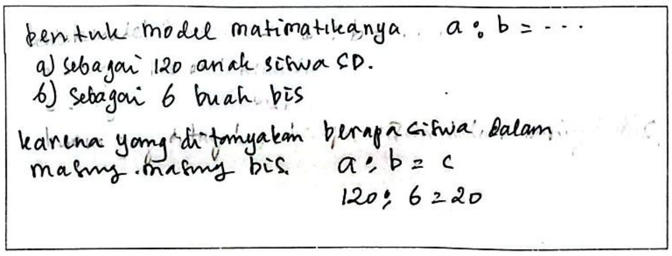
1. $a \div b = ?$; (partitive division)
2. $a \div ? = b$; (quotative division) and
3. $? \div a = b$ (unknown dividend division)

Participants were asked to determine the appropriate mathematical model for the following word problem: *"120 elementary students from SD Taruna will go on a school trip to a certain place using six buses. How many students should be placed on each bus so that every bus carries the same number of students?"*

The given problem falls under Model (2): Quotative division, which is also referred to as equal sharing. In this context, the number of holders (6 buses) is known. The task of the problem is to determine how many students should be placed in each holder (bus) in order to achieve equal distribution. This type of division cannot be approached using repeated subtraction, because it would require subtracting "6 buses" from "120 students," which is conceptually invalid. While partitive division, which relies on a repeated subtraction paradigm, only applies when the size of the holder (number of students in the bus) is known, not when the number of holders is fixed, as is the case in this problem. However, most respondents selected the model a÷ ? = b, citing its frequent appearance in standard textbooks. In contrast, alternative forms, such as ? ÷a = b and a÷ b = ? were rarely recognized or applied. This suggests that limited textbook exposure narrows teachers' representational flexibility and contributes to cognitive difficulties in distinguishing among different division structures.

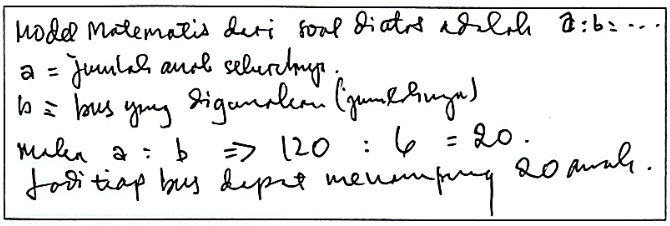

Model Matematis dari soal diatas adalah a : b = ...
a = jumlah anak seluruhnya.
b = bus yang digunakan (jumlahnya)
maka a : b => 120 : 6 = 20.
Jadi tiap bus dapat menampung 20 anak.

**The mathematical model is a : b = ...**
a = total students
b = used bus (total)
then a : b ⇒ 120 : 6 = 20
So, every bus may carry 20 students.

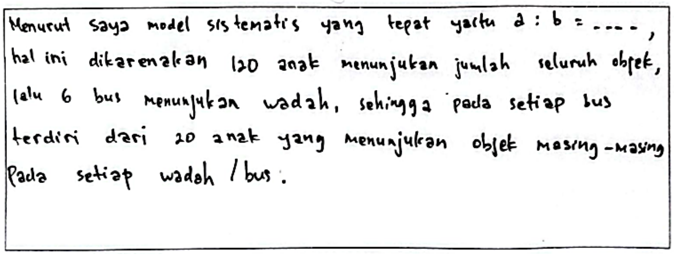

bentuk model matimatikanya a : b = ...
a) sebagai 120 anak siswa SD.
b) sebagai 6 buah bis
karena yang ditanyakan berapa siswa dalam masing masing bis. a : b = c
120 : 6 = 20

**The mathematical model form is**
**a : b = ...**
a) represents 120 elementary school students
b) represents six buses

Since the question asks how many students per bus,
**a : b = c**
**120 : 6 = 20**


Model Matematis dari soal diatas adalah a : b = ...
a = jumlah anak seluruhnya.
b = bus yang digunakan (jumlahnya)
maka a : b => 120 : 6 = 20.
Jadi tiap bus dapat menampung 20 anak.

**The mathematical model from the given problem is**
**a : b = ...**
a = total number of students
b = buses used (their number)
So,
a : b ⇒ 120 : 6 = 20
Therefore, each bus can carry 20 students.


Menurut saya model sistematis yang tepat yaitu a : b = ...., hal ini dikarenakan 120 anak menunjukan jumlah seluruh objek, lalu 6 bus menunjukan wadah, sehingga pada setiap bus terdiri dari 20 anak yang menunjukan objek masing-masing pada setiap wadah / bus.

**In my opinion, the correct systematic model is a : b = ...**
This is because the 120 students represent the total number of objects, while the six buses represent containers. Therefore, each bus contains 20 students, which represents the number of objects in each container/bus.

**Figure 3.** Participant response samples illustrating the concept of division

***Theme 3: Teachers' Interpretation of Division Representations***

The third theme, which examined teachers' interpretation of division representations, confirmed the persistence of these misconceptions. An overwhelming 98.75% of respondents treated all division problems as partitive, regardless of structural cues. This suggests a significant lack of awareness regarding the representational flexibility of division models, and an overreliance on linguistic markers such as "equally" or "evenly." Such reliance indicates a shallow processing strategy based on narrative keywords rather than a structural analysis of the quantitative relationships involved. This kind of surface-level reasoning can limit teachers' ability to formulate deeper classroom learning (Ball, Thames, & Phelps, 2008; Li & Schoenfeld, 2019)

One of the limitations observed during classroom observation is a limitation in reasoning, interpret, and representing the mathematical model. Without a good understanding of the model, teachers will not be able to provide a cognitive thinking bridge to students. Such a limitation results in students' lack of mathematical abstraction skills. As evidenced by the test we conducted, 98.75% of respondents interpreted all of the division problems we test concluded the problem as a quotative model. Those indicate the lack of flexibility when dealing with contextual problems (Downtown & Maffia, 2025).

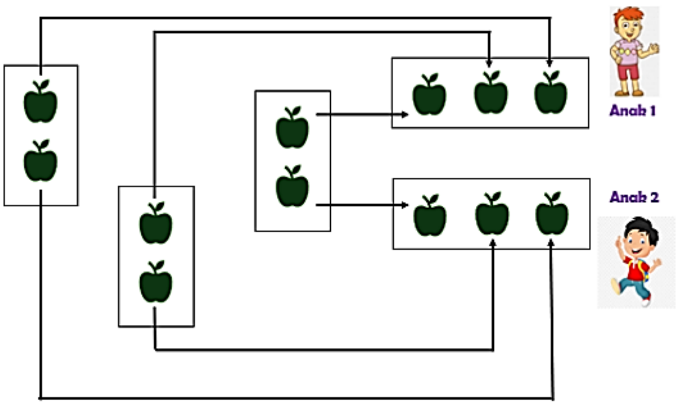
Teachers often relied on linguistic cues such as "equally" or "evenly" to determine the mathematical structure of the problem, rather than analyzing the quantitative relationships embedded in the scenario. While intuitive, this approach has been consistently criticized for encouraging surface-level processing, leading to incorrect model selection when keywords are absent or misleading (Booth et al., 2017; Durkin, Star, & Rittle-Johnson, 2021). In this study, for example, teachers consistently failed to recognize quotative division structures in problems that omitted these linguistic cues. Consider the word problem: "A rope is 24 meters long and is cut into pieces that are 6 meters each. How many pieces are there?", a classic quotative case. Even though the problem did not use typical keywords like "equal sharing" or "divided among," most teachers still thought about it in terms of sharing things out evenly. They used equal-grouping ideas instead of thinking about it as a measurement problem.

This shows a real problem with how teachers break down word problems. Instead of looking at what the numbers mean in the story - like how you keep subtracting 6 meters over and over to figure out how many pieces you get - teachers just fell back on the same old patterns they always use. Bråten et al. (2020) found something similar - that solving problems well is not just about knowing math stuff, but also about being able to build mental pictures that make sense. When teachers skip this step of really understanding what the problem is asking, they might confuse students and make them think math is just about finding certain words instead of understanding what is going on (Verschaffel et al., 2019). If this keeps happening, students might not learn how to think flexibly about math problems, which could hurt them when they hit more complicated stuff later on.

The whole pattern shows teachers rely way too much on spotting certain words in story problems, and they are missing the bigger picture of how the problem works. From a teaching perspective, this creates issues: when teachers only show students one way to think about division (a÷b=?), students miss out on other ways to approach problems where different parts are unknown or where they need to think backwards (a÷?=b). To probe further, the study examined teachers' responses to three specific division representations:

*Case 1: Quotative Partition (a÷? = b)*

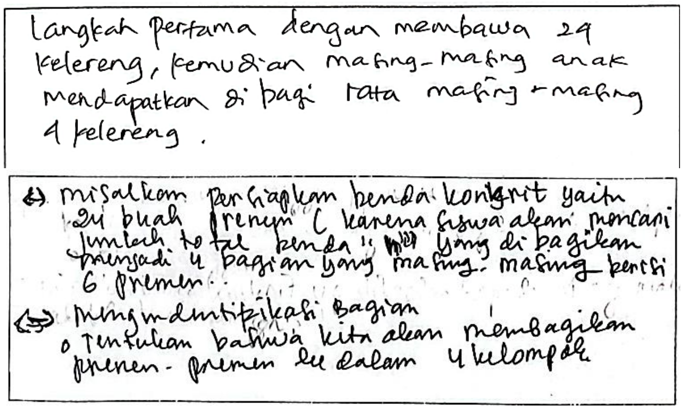
Participants were asked to solve the following problem:
*"Ms. Fitri has six apples. The apples will be equally distributed among two children. How many apples will each child get?"*




This problem is categorized as Model (2): Quotative Division, because the number of holders (2 children) is known. The task is to determine how many apples should be given to each child; this concept means "equal sharing". This structure is similar to the bus problem, where 120 students are distributed into six buses, and we are asked to find how many students per bus for equal distribution. In both cases, the solution involves distributing the total into a fixed number of groups, which is not repeated subtraction. The common solution process was described as "take two apples, distribute; take two more, distribute," and so on, visually, every child needs to have an equal distribution. However, the appropriate mathematical model in this case is $six \div ? = 2$ Despite this, 79 respondents incorrectly identified the problem structure as $a \div b \; = ?$ Interpreting it as partitive division.

*Case 2: Unknown Dividend (? ÷a = b)*

Participants were asked to generate or identify a word problem that aligns with this structure. Performance was somewhat improved: 50% responded correctly, 12.5% failed to produce any answer, and 37.5% displayed misconceptions. These errors were largely due to insufficient understanding of alternative division structures. Most respondents reverted to previously used forms, especially quotative partition models. Representative responses are shown in Figure 4: *"Suppose you are asked to explain how to find the result of a division problem in the form ... : 4 = 6 to your students using concrete objects. How would you do it?"*




- The first step is to bring 24 marbles, then each child receives an equal share of 4 marbles.
- For example, use concrete objects, such as 24 candies, because students will be finding the total number of candies that are divided into 4 groups, with each group containing six candies.
- Identify the groups and determine that we are dividing the candies into four groups.

**Figure 4.** Participant response samples illustrating the concept of division with an unknown dividend

These findings collectively reveal a limited conceptual repertoire among teachers in modeling division. The dominance of partitive reasoning across all themes underscores the need to expand teachers' representational and structural understanding of division beyond procedural templates. Pedagogically, these findings point to a critical deficit in teachers' Mathematical Knowledge for Teaching (MKT), specifically within the domains of representation and modeling (Usiskin, 2007). The consistent misapplication of partitive division across diverse tasks reflects a conceptual reductionism in instruction, reducing division to mere equal partitioning. This procedural framing not only misrepresents the mathematical structure but also inhibits the development of flexible reasoning among students, particularly when encountering problems that require relational or inverse reasoning. These concerns echo the findings of Siswono et al. (2019) and Kang & Breiten (2024), who argue that the depth of students' conceptual learning is influenced by teachers' knowledge of representations and mathematical structures.

The limited number of teachers able to accurately interpret all three fundamental division structures underscores an urgent need to reinforce conceptualization processes within professional development programs. Understanding partitive, quotative, and inverse structure is important for helping students develop flexibility in mathematics and solve problems in ways that fit the context (Dalimunthe et al., 2020). Furthermore, the dominance of procedural approaches in teaching keeps long-standing misconceptions in place. This points to the need for instructional changes that focus more on meaning and teaching mathematics in context (Zulkardi & Putri, 2022).

The misconceptions in foundational mathematical concepts, such as division, represent a deep-rooted issue in teacher knowledge that extends beyond isolated instructional errors. This study reveals how such misconceptions manifest not only in the dominance of procedural reasoning but also in teachers' limited use of structural and representational strategies. Similar patterns are documented by Kusmaryono, Basir, and Maharani (2020), highlighting the systemic nature of these misunderstandings among in-service teachers. These findings suggest an underlying weakness in how teachers understand mathematics. They often struggle to connect abstract ideas with real-world contexts in a coherent way. Without addressing these foundational weaknesses, instructional practices risk remaining superficial, limiting students' opportunities for genuine mathematical understanding.

**Limitations of the Study**

This study revealed an overview of elementary school teachers' misconceptions regarding division in four significant Indonesian cities, despite its limitations. One of which is that the sample size makes it harder to get deep qualitative insight from each case. Even with a quite lot sample size, however, the depth analysis may be somewhat considered lacking depth caused the instruments and data collection techniques which are still limited. As a result, the cognitive process within each teacher cannot be clearly reflected and represented in this study. As well as with the cognitive transfer in classroom learning cannot be fully captured due to the limited observation.

This study was also conducted in a one-shot manner. This study did not conduct follow-up interviews or observations on subsequent emergent findings. This resulted in, although it is already in line with the methodology, the discussion provided remained at the surface level. Despite the data collected considering respondents' geographical location, all respondents come from urban locations. We decided with consideration that if those in the city experienced many misconceptions, then those in rural areas are likely to face even more. Future research can be carried out continuously, thus providing more in-depth analysis.

## ▪ CONCLUSION

The findings of this study reveal a persistent disconnect between teachers' conceptual understanding of division as repeated subtraction and their actual instructional practices. Although many participants were able to articulate this framework, their classroom implementations remained procedural and centered almost exclusively on the partitive model, without distinguishing it from other important structures such as quotative or unknown dividend forms. This lack of representational flexibility emerged in the ways teachers constructed story problems, selected visual aids, and interpreted contextual division tasks. These results directly address a gap in the existing literature by providing empirical evidence that misconceptions about division models are not only widespread among students but may originate from the instructional limitations of their teachers. Unlike previous studies that focus on student errors, this research highlights how such errors may be reinforced by the teachers' own reliance on simplistic textbook narratives and insufficient pedagogical content knowledge. The findings suggest that the issue is not merely about inadequate content knowledge but reflects deeper weaknesses within the domain of mathematical knowledge for teaching, particularly in representational and interpretive competence. Although this study did not trace the direct effects on student learning, the consistent overuse of the partitive model raises a valid concern about the potential for these instructional patterns to limit students' conceptual development in the long term.

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