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Development of Functional Thinking-Based Student Worksheets to Improve Mathematical Generalization Abilities

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Abstract: This study aims to develop valid, practical, and efficient functional thinking-based student worksheets to improve students' mathematical generalization abilities. This study used a 4D development research approach, divided into four stages: Defining, Designing, Developing, and Disseminating. Thirty-eight-grade students of class VIII H MTsN 9 Jombang became the subjects of this study. The research instruments were validation sheets, teachers' response questionnaires, students' response questionnaires, interview guidelines, and test questions. This functional thinking-based worksheet was designed based on functional thinking indicators. Functional thinking in this research consists of recursive patterns, covariational thinking, and correspondence relationships. The data analysis results showed that the functional thinking-based student worksheets developed have a validity score of 87,5% from teachers and 90% from lecturers, categorized as very good (valid). The results of the teacher response questionnaire were 80%, and the student response questionnaire was 82,25%, both classified as good. The average pre-test score was 53, and the average post-test score was 75. N-gain scores for each indicator of mathematical generalization ability in the medium and high categories. It is found that there was an increase in students' generalization ability. Generalization ability studied includes perceptions about generalization, expressions of generalization, formulating generalities symbolically, and solving problems using the results of generalization. The most prominent finding in this study is the improvement in the ability to formulate generalities symbolically. This indicates that the use of functional thinking-based worksheets is sufficiently effective in enhancing students' generalization ability. The results of this study provide theoretical implications for how to improve students' generalization ability.

Keywords: functional thinking, student worksheets, mathematical generalization ability.

▪ INTRODUCTION

Generalization is the extrapolation of observed similarities about relationships or qualities from the sample context to a broader context (Oz & Ciftci, 2024). Ellis argues that generalization can be used to build new knowledge and is a fundamental part of mathematical activity (Jackson & Stenger, 2024). In addition, generalization is the process of identifying similarities across all cases, extended reasoning, and extending results based on specific cases (Jackson & Stenger, 2024). Mathematical generalization is closely related to functional thinking. This is because generalization is the core of functional thinking and has been learned in exploration by students (Kieran et al., 2016).

Generalization ability is an important component of mathematics learning (Chua & Hoyles, 2014). Generalization is needed in learning mathematics to understand various mathematical concepts (Callejo & Zapatera, 2017; Karabulut & Ozmen, 2018). The curriculum standards also show the importance of generalization and research in Mathematics Education (Tillema & Gatza, 2017). Complex and deep generalizations can be formed through patterns (Mata-Pereira & Da Ponte, 2017). Tasks for generalization ability tests often use numerical and geometric patterns (Joanna, 2017; Jureczko, 2017; Mulenga & Marban, 2020). Mathematical generalization can be measured by students'

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ability to work on questions about patterns. The indicators of generalization ability in this study include perception of generality, expression of generality, symbolic expression of generality, and manipulation of generality (Mason, 1996). Perception of generality is the process of identifying patterns. Expression of generality is the process of determining the following data based on pattern identification, symbolic expression of generality is formulating the results of generalization symbolically (formulating general rules), while manipulation of generality is the process of solving problems using the results of generalization.

There are various strategies that students use in generalizing. The strategy of finding differences is a strategy that pays attention to the differences between terms in number patterns (Montenegro et al., 2018). The strategy of finding patterns is a strategy that pays attention to the patterns that make up the terms of a number pattern (Ellis, 2007). Quantity relationship strategies are strategies that involve the relationship between input values and output values (Ellis, 2011). Trial and error strategy, which is trying everything to solve the problem (Becker & Rivera, 2005). Linear pattern strategy is a strategy using a linear pattern formula (Stacey, 1989). Visual strategies are visual grouping strategies and visual growth strategies (Becker & Rivera, 2005). However, students often use recursive strategies, which is a strategy for determining the next n-th term by using the previous n-th term (Hourigan & Leavy, 2015).

Based on previous research, it shows that out of 35 students in class VIII, eight students can generalize from the problems given (Suryowati & Tristanti, 2022). In addition, TIMSS results in 2019 have shown that Indonesian students are low in mathematical generalization ability (Ambussaidi & Yang, 2019). Based on these results, it shows that students' generalization ability needs to be improved. Some studies on efforts to improve generalization ability, among others, show that there is an increase in mathematical analogy skills (including generalization) by applying discovery learning methods rather than expository methods (Maarif, 2016). While the results of other studies show that cooperative learning developed based on real needs and problems can improve generalization skills through three stages of learning, namely the active stage, the collaborative stage, and the inductive stage (Nirfayanti et al., 2023).

One other innovation to improve students' mathematical generalization ability is through the use of student worksheets. The student worksheets used are functional thinking-based. This worksheet is designed according to functional thinking indicators. Student worksheets are based on functional thinking, which is important for algebraic thinking (Lichti & Roth, 2019; Stephens et al., 2017). Functional thinking is part of algebraic reasoning and mathematical reasoning (Martins et al., 2023; Pittalis et al., 2020). In addition, functional thinking involves a generalization about how quantities are related (Tanıslı, 2011). The characteristics of functional thinking are being able to recognize and analyze the relationship between quantities (Frey et al., 2022; Martins et al., 2023; Stephens et al., 2017). The essence of functional thinking is how students express relationships between quantities, represent related generalizations, and reason with various representations such as words, tables, diagrams, graphs, or symbols (Kieran et al., 2016).

Blanton et al. (2015) stated that functional thinking is a generalization of a covariation relationship that can be represented by words, algebraic notation, tables, and graphs. So generalization is part of functional thinking. The tasks given to examine functional thinking can be number patterns and pictures, as well as tables. Interpretation of a variable and generalization is a covariation relationship and correspondence, and connecting various representations (Donevska-Todorova et al., 2022; Oliveira et al., 2021). Recursion patterns, covariation relationships, and correspondence can form functional thinking (Blanton et al., 2015; Pang et al., 2022; Stephens et al., 2017). There are three ways to analyze functional thinking, namely recursive patterning, covariational thinking, and correspondence relationship (Blanton & Kaput, 2011).

The functional thinking indicators in this study include determining recursive patterns, determining covariational relationships, and determining correspondence relationships. Determining recursive patterns is shown by observing objects in a given pattern and determining the next object in the pattern. Determining covariational relationships is shown by being able to determine how changes in one quantity relate to changes in another quantity, while determining the correspondence relationship is shown by generalizing two quantities in a given problem.

The ability to generalize patterns determines functional thinking. There are three types of pattern generalization, including direct generalization tasks, which involve calculating values based on previous steps, near generalization, which involves identifying values that are close to pre-existing values, and far generalization, which involves determining values that deviate from values in previous steps (El Mouhayar & Jurdak, 2016). Based on the explanation above, the research problem of this study is: how to develop a student worksheet based on functional thinking that is valid, practical, and effective in improving the generalization ability of junior high school students?

▪ METHOD

Participants

This study used a development method with 4D stages, which was implemented in class VIII H MTsN 9 Jombang in the even semester of 2024/2025. The population in this study was all students of class VIII MTsN 9 Jombang. The sample of this study was 30 students of class VIII H. This sample was selected through purposive sampling technique.

Research Design and Procedures

According to Thiagarajan (Judijanto et.al, 2024), the 4D stages consist of Define, Design, Develop, and Disseminate. The explanation of the 4D stages is as follows:

Define

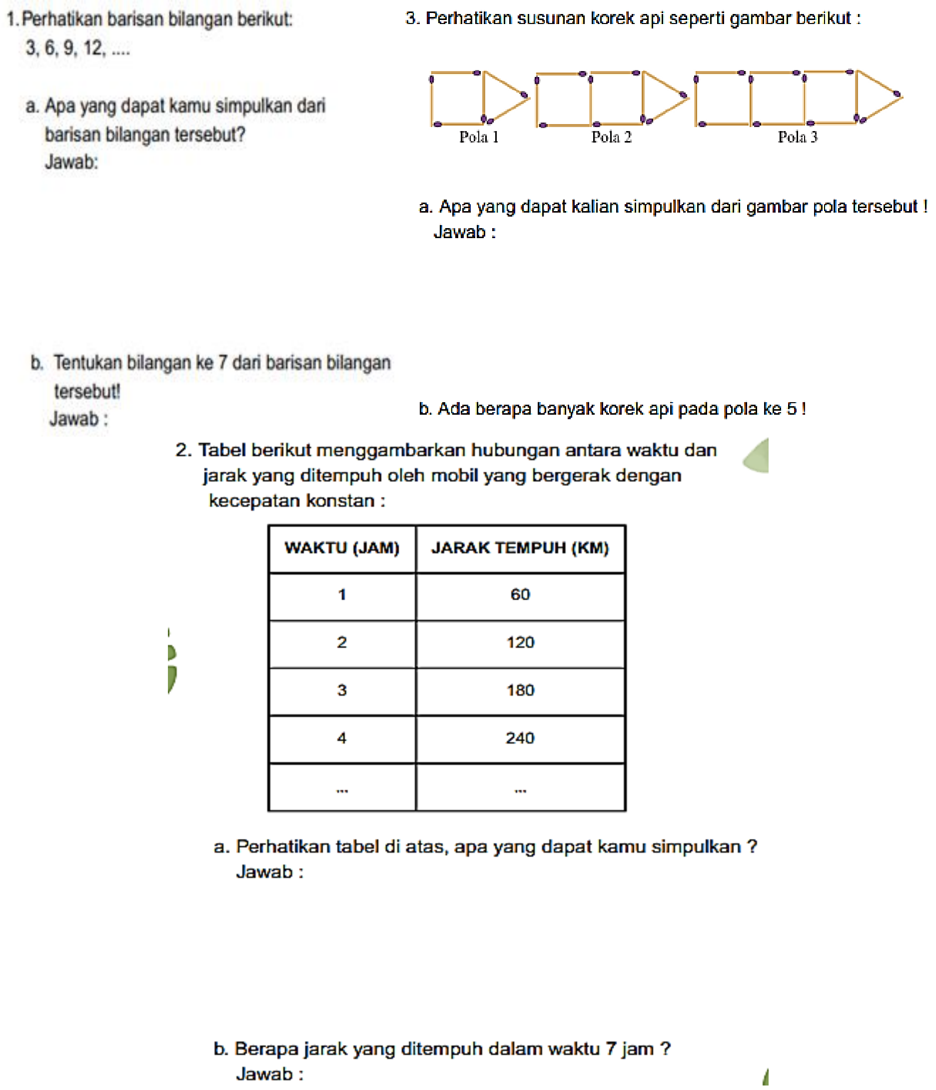
In this stage, the need for development was defined, and a needs analysis was carried out. Information was collected related to the learning by the teacher and how the students' generalization abilities were developed to find out to what extent of development needs. Five steps in this stage include conducting an initial analysis, analyzing students, analyzing tasks, analyzing concepts, and formulating learning objectives.

Design

Activities in this stage include constructing test criteria, determining media, format selection, and initial design. In this research, the design stage selected and designed functional thinking-based student worksheet media to be applied in learning mathematical patterns. The following is a student's worksheet design based on functional thinking and generalization indicators:

Functional thinking indicators: determining recursive patterns, determining covariation relationships. Determining recursive patterns is demonstrated by observing objects in a particular pattern and determining the next object in that pattern. Determining covariation relationships is demonstrated by the ability to determine how changes in one quantity relate to changes in another quantity.

Generalization indicators: recognizing generalizations, expressing generalizations. Recognizing generalizations is the process of identifying patterns. Expressing generalizations is the process of determining the next data based on pattern identification. The tasks on the student worksheet related to these indicators are shown in the following question (Figure 1).



1. Perhatikan barisan bilangan berikut:
3, 6, 9, 12, ...

a. Apa yang dapat kamu simpulkan dari barisan bilangan tersebut?
Jawab:

b. Tentukan bilangan ke 7 dari barisan bilangan tersebut!
Jawab :

3. Perhatikan susunan korek api seperti gambar berikut :

Pola 1 Pola 2 Pola 3

a. Apa yang dapat kalian simpulkan dari gambar pola tersebut !
Jawab :

b. Ada berapa banyak korek api pada pola ke 5 !

2. Tabel berikut menggambarkan hubungan antara waktu dan jarak yang ditempuh oleh mobil yang bergerak dengan kecepatan konstan :

WAKTU (JAM)	JARAK TEMPUH (KM)
1	60
2	120
3	180
4	240
...	...

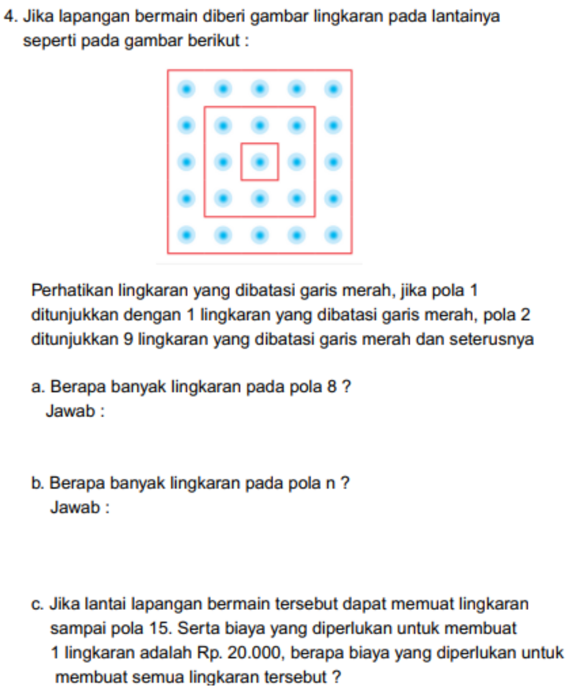
a. Perhatikan tabel di atas, apa yang dapat kamu simpulkan ?
Jawab :

b. Berapa jarak yang ditempuh dalam waktu 7 jam ?
Jawab :

Figure 1. Examples of tasks on the student worksheet

Functional thinking indicators: determining correspondence relationships. Determining correspondence relationships is demonstrated by generalizing two quantities in a given problem.

Generalization indicators: symbolic expression of generality and manipulation of generality. Symbolic expression of generality is formulating the results of generalization symbolically (formulating general rules), while manipulation of generality is the process of solving problems using the results of generalization. The tasks on the student worksheet related to these indicators can be seen in the following question (Figure 2).


4. Jika lapangan bermain diberi gambar lingkaran pada lantainya seperti pada gambar berikut :

Perhatikan lingkaran yang dibatasi garis merah, jika pola 1 ditunjukkan dengan 1 lingkaran yang dibatasi garis merah, pola 2 ditunjukkan 9 lingkaran yang dibatasi garis merah dan seterusnya

a. Berapa banyak lingkaran pada pola 8 ?
Jawab :

b. Berapa banyak lingkaran pada pola n ?
Jawab :

c. Jika lantai lapangan bermain tersebut dapat memuat lingkaran sampai pola 15. Serta biaya yang diperlukan untuk membuat 1 lingkaran adalah Rp. 20.000, berapa biaya yang diperlukan untuk membuat semua lingkaran tersebut ?

Figure 2. Examples of tasks on the student worksheet

Develop

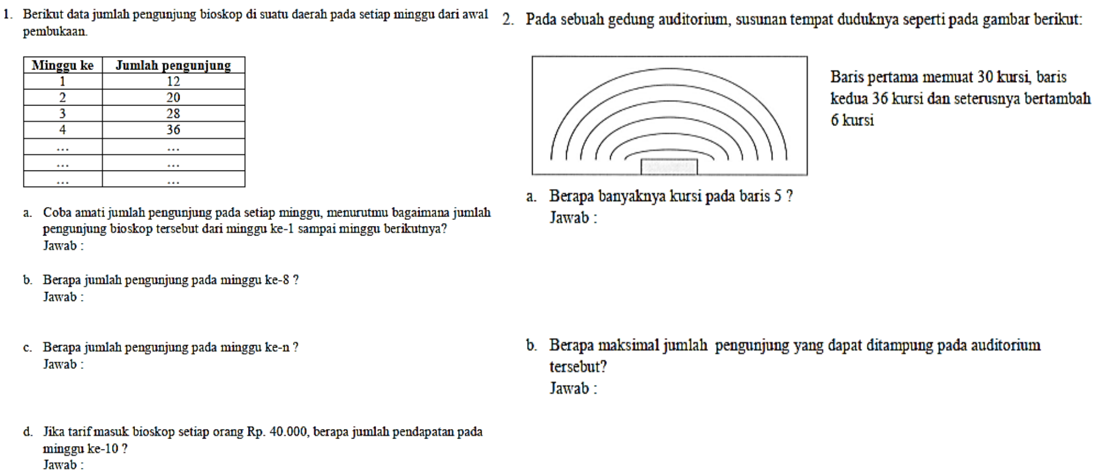
This development stage is to produce student worksheets based on functional thinking. In this stage, there are two activities, namely expert assessment accompanied by revision and trial. Developmental trials were carried out to obtain direct input in the form of responses or comments from students, experts/and observers on the functional thinking-based student worksheets that had been prepared. Trials and revisions were carried out to obtain effective and consistent student worksheets.

Dissemination

This dissemination stage was carried out through the socialization of functional thinking-based student worksheets to mathematics teachers.

Instruments

Data collection in this study used student worksheet validation sheets, questionnaires for students and teachers, interview guidelines, and tests, which included pre- and post-tests on the generalization of image and number patterns. All instruments were validated by the lecturer and the teacher. Instrument validation process related to the content and language used. The student worksheet validation sheet consisted of 3 aspects, namely language, content, and design. The response questionnaire for teachers consisted of 10 questions, and the response questionnaire for students consisted of 7 questions. For the student and teacher questionnaires, this is to provide an assessment of the student worksheets. Three aspects were assessed, namely the content of the student worksheet, the language used, and the appearance of the student worksheet. The interview guideline was created as a reference for conducting the interview process with students to confirm their answers and determine how to proceed. Interviews were conducted with several students to find out the generalization process carried out. The post-test questions used are shown in Figure 3.



1. Berikut data jumlah pengunjung bioskop di suatu daerah pada setiap minggu dari awal pembukaan.

Minggu ke	Jumlah pengunjung
1	12
2	20
3	28
4	36
...	...
...	...
...	...

a. Coba amati jumlah pengunjung pada setiap minggu, menurutmu bagaimana jumlah pengunjung bioskop tersebut dari minggu ke-1 sampai minggu berikutnya?
Jawab :

b. Berapa jumlah pengunjung pada minggu ke-8 ?
Jawab :

c. Berapa jumlah pengunjung pada minggu ke-n ?
Jawab :

d. Jika tarif masuk bioskop setiap orang Rp. 40.000, berapa jumlah pendapatan pada minggu ke-10 ?
Jawab :

2. Pada sebuah gedung auditorium, susunan tempat duduknya seperti pada gambar berikut:

Baris pertama memuat 30 kursi, baris kedua 36 kursi dan seterusnya bertambah 6 kursi

a. Berapa banyaknya kursi pada baris 5 ?
Jawab :

b. Berapa maksimal jumlah pengunjung yang dapat ditampung pada auditorium tersebut?
Jawab :

Figure 3. Post-test questions

The indicators of generalization ability in this study include perception of generality, expression of generality, symbolic expression of generality, and manipulation of generality (Mason, 1996). Perception of generality is the process of identifying patterns. For example, in the number pattern 3, 6, 9, 12, 15, ... students realize that these numbers always increase by three.

This ability in this study was assessed in post-test question number 1a. Expression of generality is the process of determining the following data based on pattern identification. For example, in the number pattern 3, 6, 9, 12, 15, ..., students can determine the following number from the number pattern, or each number is obtained by multiplying 3 by its positional order. This ability in this study was assessed in post-test questions 1b, 2a, and 2b. Symbolic expression of generality is formulating the results of generalization symbolically (formulating general rules). For example, in the number pattern 3, 6, 9, 12, 15, ..., students can determine the n-th number by writing the general formula 3n. This ability in this study was assessed in post-test question number 1c. In contrast, manipulation of generality is the process of solving problems using the results of generalization. For example, in the number pattern 3, 6, 9, 12, 15,..., students determine the sum of the numbers 1 to 10 in the pattern. This ability in this study can be seen from post-test question number 1d. This test question was adapted from Pang et al. (2022).

Data Analysis

This research design used a one-group pretest-posttest pre-experiment, and this is also a limitation of this research. Quantitative data was analyzed by percentage and N-gain value. The student worksheets validity was examined by the lecturer. This validation was used to assess the feasibility of student worksheets based on functional thinking. The results of the validation and response questionnaire are used to determine the validity and practicality of the functional thinking-based student worksheet. The percentage of validity and practicality of all aspects was calculated by dividing the score obtained by the total score, then multiplying by 100%. Furthermore, the percentage results were grouped into several categories. The category is very good for the interval 84% < Score ≤ 100%. Good category for the interval 68% < Score ≤ 84%. Fair category for the interval 52% < Score ≤ 68%. Less category for the interval 36% < Score ≤ 52%. Very poor category for the interval 20% < score ≤ 36%.

To evaluate the generalization ability using a pre-test and a post-test. The pre-test and post-test scores were analyzed using the N-gain score to determine the effectiveness of functional thinking-based student worksheets in improving students' mathematical generalization abilities. Effectiveness is analyzed using the N-gain score. N-gain (g) scores were grouped into categories. High category for g higher than 0.7, medium category for $0.3 \leq g < 0.7$ and a low category for g less than 0.3 (Nissen et al., 2018). In addition, a t-test was also conducted on the results of the pre-test and post-test for each indicator of mathematical generalization ability.

▪ RESULT AND DISSCUSSION

This research developed functional thinking-based student worksheets to improve students' generalization abilities. This student worksheet is a valid, practical, and effective learning medium. The following are the stages of developing student worksheets based on functional thinking.

Define

In this study, the initial stage was determining the subject and location of the research, analyzing learning materials, and communicating with teachers. The results of the needs analysis showed that class VIII H needed learning media for mathematics learning resources, especially on the topic of patterns. Students of class VIII H have varied family backgrounds and academic abilities. The mathematics learning in this school is good, but students do not use any math reading books. So students record the material learned. Learning objectives are also relevant and need to be added for the pattern topic, namely, students can generalize from number patterns and image patterns through the media used.

Design

The selected media were functional thinking-based student worksheets. This worksheet refers to functional thinking indicators, namely recursive patterns, covariational relationships, and correspondence. So the recursive pattern in this functional thinking-based student worksheet, students determine the number pattern by examining the changes in the order of values. In the covariational relationship in this student worksheet, students observe the change in value that occurs in two quantities. In the correspondence relationship in this student worksheet, students generalize the given problem by expressing it in algebraic form. Some problems in the functional thinking-based student worksheets can be seen in Figures 4-6.

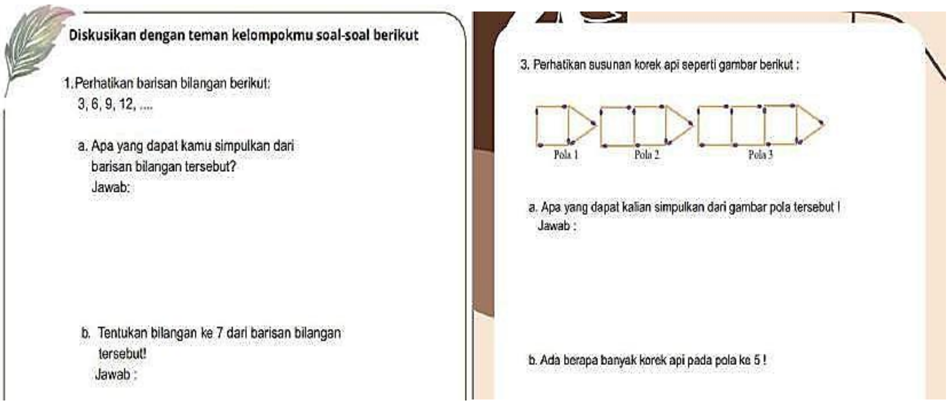


Figure 4. Recursive pattern

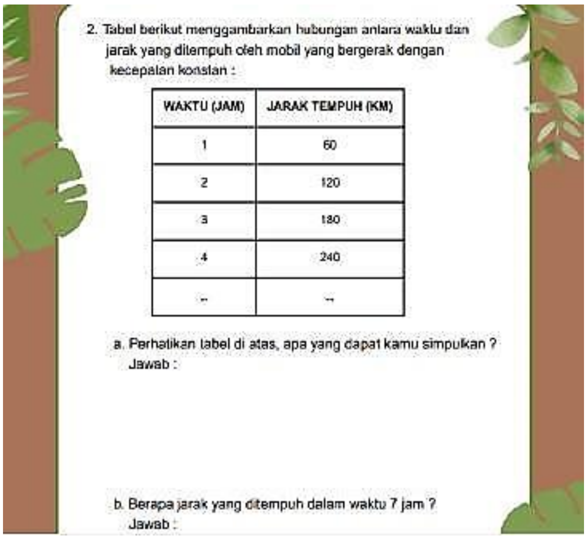


Figure 5. Covariational relationship

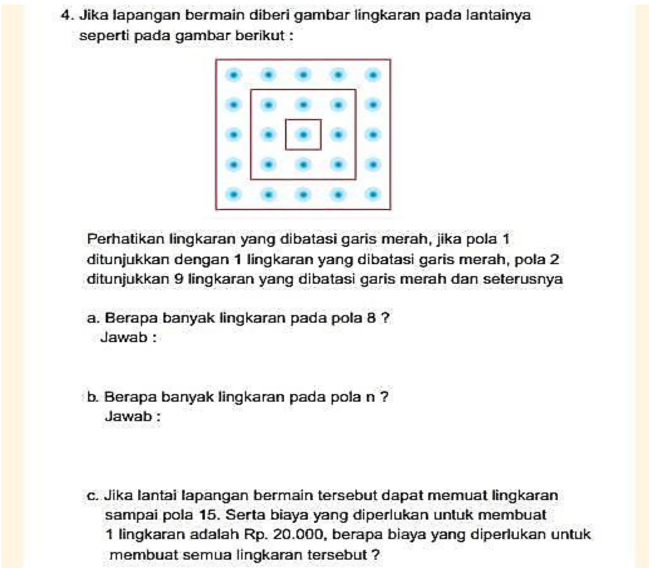


Figure 6. Correspondence relationship

In this developed student worksheet, students generalized number patterns and image patterns. Generalization ability includes the process of perceiving generalizations, expressing generalizations, formulating generalities symbolically, and solving problems using the results of generalizations.

Develop

This stage consists of two activities, namely expert assessment/validation accompanied by revisions and developmental testing. An expert appraisal was conducted by a mathematics education lecturer and a mathematics teacher. The results of validation of pre-test and post-test questions by lecturers obtained a score of 85.71%. The results of the validation of the pre-test and post-test questions by the math teacher obtained a score of 89.29%. The results of validation by lecturers for student worksheets obtained a score of 90%. The results of validation by the math teacher obtained a score of 87.5%. There were minor revisions to the writing on the student worksheets. The trial of student worksheets based on functional thinking was carried out four times on students of class VIII H MTsN Jombang in the even semester of 2024/2025. Before the trial, a pre-test was given first, and after the student worksheet trial, a post-test was conducted. The purpose of this test is to examine the effectiveness of student worksheets in improving the generalization ability of students in class VIII H MTsN 9 Jombang. Based on the pre-test results, the average score was 53, and the post-test score was 75. At this stage, a response questionnaire was also given to two mathematics teachers and students of class VIII H. The average teacher response questionnaire was 80%. At the same time, the average student response questionnaire was 82.25%. These results show that the results of the teacher and student response questionnaires are in the good category. Based on these results, it shows that the student worksheet based on functional thinking is valid and practical.

Based on the N-gain value of the pre-test and post-test for each indicator of mathematical generalization ability. The perception of generality indicator obtained an N-gain value of 0.49; meanwhile, the expression of generality indicator obtained 0.35. The symbolic expression of generality indicator obtained an N-gain value of 0.40, while the manipulation of generality obtained an N-gain value of 0.72. All of the N-gain values obtained are in the medium and high categories. This is shown in Figure 7 below.

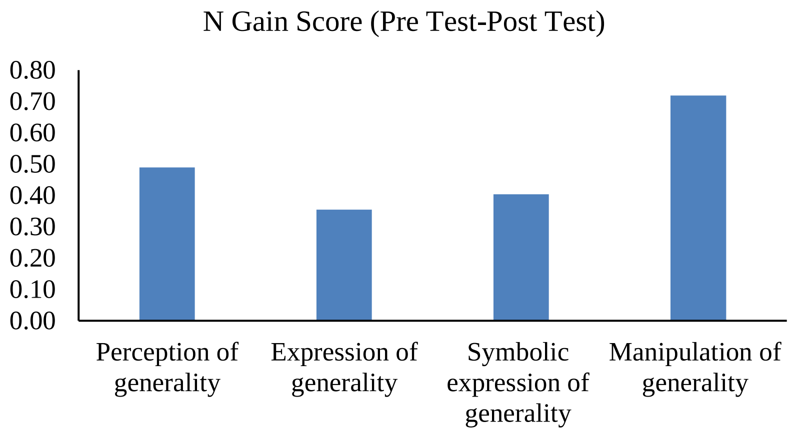


Figure 7. N-Gain value for mathematical generalization ability

The effectiveness of functional thinking-based student worksheets in improving students' mathematical generalization ability is indicated by the N-gain value and the results of the pre-test and post-test t-test according to the indicators of mathematical generalization ability. The test used was the paired sample t-test, and this t-test uses SPSS. Based on the SPSS results, a sig (2-tailed) value of 0,001 was obtained for the generality perception indicator. This result indicates less than 0,05, thus concluding that there is a difference in generality perception before and after the implementation of functional thinking-based student worksheets. The sig value (2-tailed) for the expression of generality indicator is 0,037; this value is less than 0,05, so it can be concluded that there is a difference in students' expression of generality before and after using functional thinking-based worksheets. The sig value result (2-tailed) for the symbolic expression of generality indicator is less than 0,001. This value also shows less than 0,05, so it can be concluded that there is a difference in the symbolic expression of generality before and after the use of functional thinking-based student worksheets. The sig value result (2-tailed) for the manipulation of the generality indicator is 0,002. This value also shows less than 0,05, so it can be concluded that there is a difference in the manipulation of generality before and after the use of functional thinking-based student worksheets.

Figure 8. Student worksheet trial

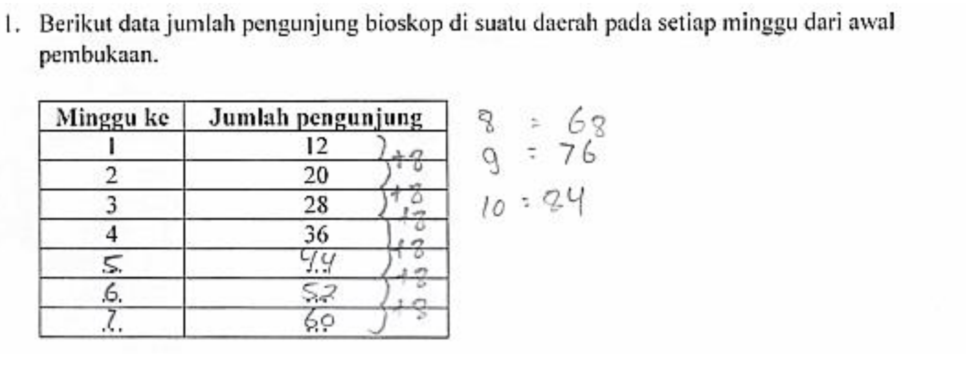
Dissemination

At this stage, student worksheets based on functional thinking are developed and implemented more widely, and outreach is conducted to mathematics teachers. The assignments given on the worksheets are in the form of number patterns and geometric patterns. This is in accordance with research that the most common method of teaching generalization is using pattern assignments (Jackson & Stenger, 2024). The gain score for the expression of the generality indicator is the lowest compared to the other indicators. This is because there is not too much improvement, as almost all students determine the n-th pattern determined by the pre-test and post-test questions. Students can continue the pattern until the n-th pattern requested by the question. Through the use of functional thinking-based worksheets, students practice problems about number patterns and image/geometric patterns. The teacher also plays a significant role in utilizing this worksheet. The teacher directed students in understanding the worksheet so that students could generalize. However, not all students can make a general rule or make a general rule symbolically. In this student worksheet, problems related to everyday life are given so that students can understand more easily.

Based on the post-test results, one of the students' generalization abilities is shown in Figure 9 below. The student can identify patterns (perceiving generalizations) by noticing that the number of visitors each week increases by 8. In the Expression of a Generalization, students determine the number of visitors in week 8 by adding the number of visitors in week 4 by 8, and so on until week 8. This strategy is recursive (Hourigan & Leavy, 2015). Students can express generalization and formulate generality symbolically. Students can formulate the number of visitors in week n. Based on interviews conducted with students, the formulation process starts from the previous problem to observe the number of visitors each week. Then, from observing and connecting with many visitors each week, the same difference is obtained each week, which is eight visitors. Then students try to get the same way to determine the number of visitors in each week (as students write, 8.1 + 4 = 12, 8.2 + 4 = 20, and so on). This strategy is called a linear pattern strategy (Stacey, 1989). This method applies to determine the number of visitors in week 1, 2, and so on until the n-th week, students write 8n+ 4. This formulation is to express the number of visitors in the n-th week.

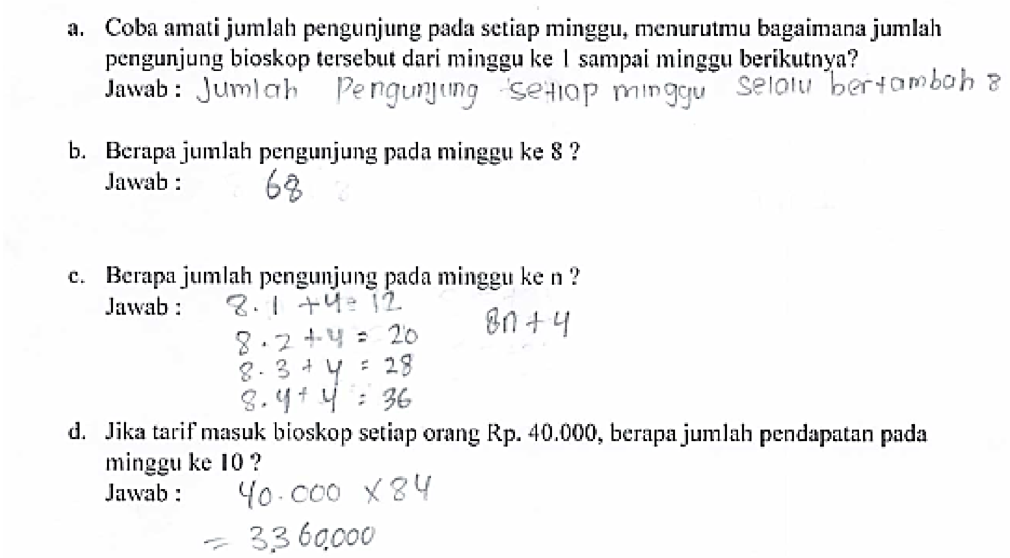
The students' generalization process is in accordance with Ellis (Park & Kim, 2017), which states three student activities in the generalization process, namely identifying similarities between cases, extending reasoning from its original range, and obtaining broader results from a particular case. After students formulate the number of visitors in the n-th week, students use the generalization results to solve the problem of the amount of income in a particular week (manipulation of generality).

The generalization made by students when doing the tasks on the student worksheet is the empirical generalization, because students compare and identify the external characteristics of number patterns and geometric patterns (Rubinshtein in Dumitrascu, 2017). Students can determine the next pattern because they identify and compare the difference between the first term and the next term or the first model and the next model. Students find the same difference in the given pattern, so the similarity of this difference is used to continue the next term or model in the given pattern. This is in accordance with the generalization activity of searching (Ellis, 2007). The generalization ability of class VIII H can be seen in Table 1, which shows the average percentage of students who can complete questions on each indicator.



1. Berikut data jumlah pengunjung bioskop di suatu daerah pada setiap minggu dari awal pembukaan.

Minggu ke	Jumlah pengunjung
1	12
2	20
3	28
4	36
5.	44
6.	52
7.	60


a. Coba amati jumlah pengunjung pada setiap minggu, menurutmu bagaimana jumlah pengunjung bioskop tersebut dari minggu ke 1 sampai minggu berikutnya?
Jawab : Jumlah Pengunjung setiap minggu selalu bertambah 8

b. Berapa jumlah pengunjung pada minggu ke 8 ?
Jawab : 68

c. Berapa jumlah pengunjung pada minggu ke n ?
Jawab : 8 . 1 + 4 = 12
8 . 2 + 4 = 20
8 . 3 + 4 = 28
8 . 4 + 4 = 36
8n + 4

d. Jika tarif masuk bioskop setiap orang Rp. 40.000, berapa jumlah pendapatan pada minggu ke 10 ?
Jawab : 40.000 × 84
= 3360.000

Figure 9. Student work results

Table 1. Mathematical generalization ability

Generalization Ability Indicators	Percentage of Pre-Test	Percentage of Post-Test
Perceptions about generalization	76.67%	83.33%
Expression of a generalization	86.67%	90%
Formulating generality symbolically	0%	33.33%
Solving problems using generalization results	50%	86.67%

Based on Table 1, it can be seen that there is an increase in the mathematical generalization ability of class VIII H students. Students' abilities in perception of perception of generalization have increased by 6.66%. The ability to express a generalization has increased by 3.33%. The ability to formulate generalizations symbolically has increased by 33.33% and the ability to solve problems using generalization results has increased by 36.67%. The ability to formulate generalizations symbolically increased significantly because previously, all students could not make general rules symbolically. After the use of functional thinking-based student worksheets, there were 10 students who could make general rules symbolically. These students can determine the number of visitors in the n-th week. In addition, solving problems using generalization results has also increased significantly. Although students cannot make general rules symbolically, they can solve problems by continuing until the pattern is requested in the problem. This indicates that students experience difficulties in making algebraic generalizations, similar to the research by İmre et al. (2017).

Several studies on the development of student worksheets to improve mathematical generalization skills have been conducted. Hayuningrat & Rosnawati's research (2022) on the development of learning tools with a realistic mathematics approach oriented to the mathematical generalization ability of high school students shows that lesson plans and student worksheets are effective in facilitating students' generalization ability. This research was conducted in class X high school on trigonometry material. The research of

Nirfayanti et al. (2025) on the development of worksheets based on an inductive approach to improve students' mathematical generalization ability showed a significant increase in the mathematical generalization ability of grade VIII junior high school students. This research has a novelty from previous research, namely, developing worksheets based on functional thinking. The results of this study are in line with previous research, namely that the development of worksheets based on functional thinking can improve students' generalization skills.

This study is also in line with the results of the study by Çakıroğlu & Muştu (2025) that through scratch, it can improve the generalization skills of 7th-grade students. This study focuses on the generalization process in terms of mathematization and verification. Although the media used in this study are different from previous studies, both can improve the generalization abilities of high school students. The results of this study are expected to be an addition to learning number patterns in schools because there has been no research related to student worksheets based on functional thinking. This research can be developed into ICT-based media.

▪ CONCLUSION

Based on the research results that have been presented previously, it can be concluded that functional thinking-based student worksheets developed are valid and practical. These student worksheets get good responses from students and teachers, and are also quite effective in improving students' generalization abilities. There was an increase in students' generalization abilities, including generalization perception, generalization expression, formulating generalizations symbolically, and solving problems using generalization results. The most important finding in this study is that students can make general rules symbolically (make general formulas for the n-th pattern). This can motivate mathematics teachers to be more creative in making other learning media that can improve.

The impact of the results of this study can theoretically add references in making or developing learning media. No matter what media is used, the teacher's role is also very supportive in reaching learning objectives. Future research can develop this student worksheet in digital form.

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