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Mapping Critical Thinking Skills through Newman's Error Analysis in Secondary Students' Problem-Solving

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Abstract: Mathematics plays a crucial role in fostering students' logical, critical, and analytical thinking skills. However, many students still face challenges in understanding and applying mathematical concepts. This study aims to map students' critical thinking skills through an analysis of student errors in solving problems involving systems of linear equations in three variables using Newman Error Analysis (NEA). The research was conducted using a qualitative descriptive approach, supported by written tests and interviews. The research sample was 33 eleventh-grade high school students in Majalengka Regency, West Java. The findings revealed that while no errors occurred at the reading stage, 48% of students made comprehension errors, 57% made transformation errors, 66% experienced process skill errors, and 82% committed encoding errors. The most dominant errors were encoding errors, reflecting weaknesses in the aspects of evaluating evidence and drawing conclusions. Most encoding errors were influenced by students' lack of confidence and learning motivation. These results suggest that difficulties in earlier stages of problem-solving significantly affect students' ability to arrive at correct final answers. The study emphasizes the importance of strengthening students' procedural fluency and supporting their cognitive processes to improve mathematical problem-solving performance.

Keywords: contextual, critical thinking skills, Newman's error analysis, three-variable linear equation.

▪ INTRODUCTION

Mathematics plays a crucial role in the development of students' logical thinking (Hariri et al., 2025; Johar et al., 2023), critical reasoning (Arisoy & Aybek, 2021), and analytical abilities (Huincahue et al., 2021). Moreover, it significantly contributes to cognitive growth and effective problem-solving (Coolen et al., 2023). However, many students continue to struggle with understanding fundamental mathematical concepts (Uegatani et al., 2023). These difficulties are influenced by various factors, including an unconducive learning environment (Juan & Chen, 2022), limited access to effective instructional media (Silva et al., 2024), and insufficient parental engagement in academic activities (Acharya, 2017). Consequently, students face challenges in grasping mathematical ideas (Hia & Harefa, 2023), modeling real-world problems mathematically (Velez et al., 2023), and performing correct computations (Rusek, 2025). These persistent challenges highlight the need for a detailed investigation into students' errors and misconceptions in mathematics learning.

In response to these challenges, error analysis emerges as an effective diagnostic tool for identifying specific learning difficulties in mathematics (Makgakga, 2023; Xu, 2023). By recognizing common error patterns, teachers can develop targeted interventions to address students' misconceptions (Takaendengan et al., 2022; Valdez & Taganap, 2024). The presence of systematic errors often reflects deeper cognitive or conceptual issues (Chiphambo & Mtsi, 2021; Hindi & Muthahharah, 2021), and thus, understanding the nature of these errors is vital for improving instructional strategies.

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Newman's Error Analysis (NEA) is a framework to analyze students' problem-solving processes. NEA facilitates the identification of specific cognitive stages where errors occur during problem-solving activities (Lamadoken & Dinulloh, 2022). Typical student errors include misreading problems, failing to interpret the problem context mathematically, incorrect calculations, misapplication of formulas, inaccurate final answers, or the absence of conclusions (Díaz et al., 2020; Lee & Byun, 2022). NEA classifies these into five key categories: reading, comprehension, transformation, processing, and encoding errors (Surya & Edriati, 2024). Each category reveals insights into students' reasoning and the depth of their critical thinking.

Several previous studies have explored the relationship between student error analysis and higher-order thinking skills (HOTS). Tanujaya et al. (2021) identified the implementation of active learning and textbook quality as key factors influencing HOTS development in mathematics education in Indonesia. Liu et al. (2024) proposed a framework for evaluating K-12 students' HOTS, emphasizing critical thinking and innovation as crucial components. However, the approaches used in these studies tended to be general and insufficiently detailed in systematically mapping essential processes of thinking. In this context, this study employed the NEA framework, which provides a more systematic analytical structure for identifying weaknesses in students' critical thinking. However, there is limited research that comprehensively examines the application of NEA to context-based mathematics problems that are defined in this study as problems in real-life or meaningful situations. These problems require students to interpret, model, and solve them through mathematical reasoning, particularly in complex topics such as systems of linear equations in three variables.

This study aims to examine the relationship between students' errors in solving context-based mathematics problems and their critical thinking skills using NEA, specifically through contextual three-variable linear equation system problems designed to assess critical thinking. Specifically, this study aims to answer the question: What types of errors occur, and how do these errors indicate weaknesses in students' critical thinking abilities? By triangulating data from written tests and interviews, this study is expected to offer a comprehensive understanding of the most frequent error types and their underlying causes.

▪ METHOD

Participants

The participants in this study were eleventh-grade students from a public senior high school in Majalengka Regency, West Java, Indonesia. The population comprised students who had completed instruction on the topic of three-variable linear equation systems (TVLES). Using a purposive sampling technique, 33 students were selected to participate in the study by completing a written problem-solving test. Based on their performance, the students were categorized into three levels of mathematical ability (high, medium, and low) using classification criteria adapted from Indrawati et al. (2019). One representative student from each category was selected for in-depth, semi-structured interviews, resulting in a total of three interview participants.

Research Design and Procedures

This study adopted a qualitative descriptive research design to systematically investigate students' mathematical problem-solving errors through the lens of NEA. Conducted over two months from February to March 2025, the research followed a structured and theoretically grounded sequence of procedures. The initial phase involved the development of research instruments, particularly open-ended essay questions constructed to reflect students' HOTS and informed by established critical thinking frameworks. A comprehensive validation process was subsequently undertaken, encompassing content validity, construct validity, and pilot testing to ensure the clarity, relevance, and theoretical alignment of the instrument. The validated test was then administered to 33 eleventh-grade students under standardized conditions. Their written responses were evaluated and categorized into three levels of mathematical ability based on predefined performance criteria. To obtain deeper cognitive insights, one representative student from each ability group was selected for semi-structured interviews, which were designed to elicit error patterns and reasoning processes and were guided by both NEA and critical thinking dimensions. All interviews were audio-recorded, transcribed verbatim, and analyzed using thematic analysis techniques to identify recurring themes and cognitive tendencies. Finally, the study employed a triangulated integration of both quantitative data from the written test and qualitative insights from the interviews, facilitating a nuanced and holistic understanding of students' error types, underlying misconceptions, and the critical thinking processes involved in solving complex mathematical problems.

Instruments

The primary instrument employed in this study consisted of six contextual, open-ended essay items specifically developed to assess students' critical thinking skills in solving problems related to TVLES. The construction of these items was theoretically anchored in two complementary frameworks. First, the critical thinking indicators were adapted from Kania et al. (2024), emphasizing components such as logical reasoning, evaluative judgment, and informed decision-making. Second, the design of the items was aligned with Bloom's revised taxonomy (Anderson & Krathwohl, 2001), with a particular focus on the higher-order cognitive domains of analysis, evaluation, and creation. These theoretical underpinnings ensured that the items not only captured students' procedural proficiency but also elicited their capacity for deep, reflective, and structured thinking. Each item was explicitly mapped to distinct critical thinking indicators, which are summarized in Table 1, to ensure transparency, coherence, and alignment between the intended constructs and the actual cognitive demands imposed by the test items.

Table 1. Indicators of critical thinking skills assessed in the test

No	Indicator of Critical Thinking Skills	Item Number
1	Identifying problems involving TVLES using logical principles in contextual settings	1. 2
2	Analyzing mathematical problems by integrating various sources of contextual information	3
3	Determining the solution set of a TVLES by logically evaluating data and arguments	4

4	Applying the solution set based on appropriate mathematical standards and context relevance	5
5	Critically evaluating the validity of obtained solutions using mathematical reasoning	6

To ensure the validity and reliability of the written test instrument, a three-phase validation process was conducted, encompassing content validity, construct validity, and pilot testing. These steps were carried out rigorously to align the instrument with the study's theoretical foundations and to ensure its efficacy in capturing students' critical thinking abilities in solving TVLES problems.

Content validity was established through expert judgment involving one senior lecturer in mathematics education and one experienced high school mathematics teacher. The experts assessed the test items against several key criteria: (1) the relevance to the high school TVLES curriculum, (2) clarity and appropriateness of language and instructions, (3) alignment with critical thinking indicators adapted from Kania et al. (2024), and (4) coherence with Bloom's revised taxonomy, particularly the cognitive levels of analysis, evaluation, and creation. Based on the panel's feedback, the research team revised ambiguous wording, refined the alignment between items and indicators, and improved instructional clarity to strengthen the representativeness of the instrument.

Construct validity was addressed by ensuring that each item explicitly reflected observable and measurable aspects of critical mathematical thinking. Each question was systematically mapped to one or more critical thinking indicators derived from Kania et al. (2024) and was designed to activate multiple cognitive operations as outlined in Bloom's revised taxonomy. Expert reviewers examined the consistency between the theoretical intention of each item and its surface-level representation. This process ensured that the test did not merely assess procedural knowledge but meaningfully elicited reasoning, logical inference, and evaluative judgment, thus reinforcing the construct validity of the instrument.

In the final phase, a pilot test was conducted with a comparable group of Grade 11 students who were not involved in the main study. The objective was to evaluate the practical effectiveness of the test in real classroom settings, specifically assessing (1) the clarity of instructions, (2) the appropriateness of item difficulty to differentiate among varying ability levels, and (3) the capacity of items to elicit genuine critical thinking processes. The pilot results informed a final round of revisions, leading to the removal of non-discriminatory items, the refinement of item formulations, and the enhancement of internal coherence across the test. Through this iterative and theory-driven validation approach, the instrument achieved a high degree of credibility and robustness, rendering it suitable for in-depth qualitative analysis in high-impact educational research.

In addition to the written test, the study employed a semi-structured interview protocol to gain deeper insight into students' cognitive, metacognitive, and epistemic processes during problem-solving. The development of this protocol was grounded in the NEA model, which categorizes mathematical errors into five stages: reading, comprehension, transformation, process skills, and encoding. This framework enabled a systematic diagnosis of error types and helped trace the origins of students' problem-solving difficulties across different cognitive domains.

To further enrich the qualitative dimension, interview questions were structured based on the critical thinking framework according to Kania et al. (2024). These prompts were designed to elicit how students interpreted problem statements, justified their solution strategies, applied relevant mathematical concepts, and reflected on the accuracy and validity of their responses. The protocol followed an open-ended format, allowing for adaptive, in-depth conversations while maintaining a strong alignment with the study's theoretical constructs. All interviews were conducted with informed consent, audio-recorded, and transcribed verbatim to ensure the authenticity and completeness of the data. The rich narrative data derived from these interviews played a crucial role in triangulating and contextualizing the findings from the written test, ultimately contributing to a more nuanced and comprehensive understanding of students' mathematical reasoning and critical thinking trajectories.

Semi-Structured Interview Protocol

To complement the written test data and gain deeper insights into students' mathematical thinking, a semi-structured interview protocol was developed to examine the cognitive and metacognitive processes underlying students' problem-solving behaviors, particularly with regard to error identification and critical reasoning. The construction of the protocol was theoretically grounded in the NEA model, which categorizes student errors across five hierarchical stages, including reading, comprehension, transformation, process skills, and encoding (Abdullah et al., 2015; Darmayanti et al., 2024; Hadi et al., 2018). This framework allowed systematically tracing of the origin and nature of students' problem-solving difficulties. In parallel, the protocol also incorporated elements of critical thinking frameworks (Asari et al., 2019; Göran & Britt-Marie, 2014; Kania et al., 2024), enabling the exploration of students' logical interpretation of problem contexts, justification of strategies, and self-evaluation of outcomes. The interview prompts were intentionally designed to be open-ended and flexible, thereby allowing for an adaptive yet theory-driven dialogue that captured the depth and variability of students' responses. All interviews were conducted with prior informed consent, audio-recorded, and transcribed verbatim to ensure accuracy, transparency, and fidelity in data representation. The rich qualitative data obtained through this process served not only to triangulate findings from the written test but also to reveal nuanced dimensions of students' reasoning patterns and critical thinking dispositions that would otherwise remain inaccessible through quantitative measures alone.

Data Analysis

To provide a comprehensive understanding of students' mathematical problem-solving behaviour, this study employed a qualitative analysis. This methodological triangulation was instrumental in uncovering not only the frequency and distribution of students' mathematical errors but also the underlying cognitive and metacognitive processes that contributed to those errors. The dual-mode analysis ensured that the findings were both empirically grounded and theoretically enriched, in line with the interpretive depth required in qualitative educational research.

Students' written responses from the test were systematically analyzed using the NEA framework to identify and categorize the types of errors made at each stage of the problem-solving process. This model categorizes errors into five progressive stages: reading, comprehension, transformation, process skills, and encoding. Each student's answer was independently coded based on these categories, allowing for precise identification of error types. The frequency of each error type was calculated (Elugbadebo & Johnson, 2020; Omoyemiju & Omotosho, 2023). This quantitative procedure facilitated the mapping of dominant error patterns across ability levels and enabled the researcher to isolate critical bottlenecks in students' problem-solving trajectories.

The analysis focused on an in-depth thematic examination of interview data to uncover the cognitive dimensions of student reasoning. All interviews were transcribed verbatim to preserve the authenticity of participants' expressions. Thematic analysis was conducted through a combination of inductive and deductive coding approaches: initial codes were derived directly from the data (based on participants' language). In contrast, deductive codes were developed based on the theoretical frameworks of the NEA and the critical thinking model. The coded data were then organized into broader thematic categories, including: (1) misconceptions and reasoning gaps, (2) cognitive strategies employed in navigating problem contexts, and (3) metacognitive reflections on accuracy, confidence, and justification.

In the final phase, thematic synthesis was carried out to link participants' verbal responses to specific stages of error development and dimensions of critical thinking. This process enabled the researcher to make interpretive inferences regarding students' conceptual understanding, procedural fluency, and self-regulatory awareness in problem solving. To enhance analytical rigor and credibility, the coding process involved researcher triangulation and peer debriefing to ensure consistency and reduce subjective bias. The integration of qualitative insights with quantitative patterns of error provided a more holistic portrayal of students' mathematical difficulties, revealing not only what types of errors were made but also why they occurred and how students rationalized their problem-solving decisions.

▪ RESULT AND DISSCUSSION

The analysis results, crucially facilitated by the NEA framework, reveal that each type of error is linked to specific critical thinking indicators. For instance, errors at the reading and comprehension stages point to deficiencies in the ability to clarify and interpret information, indicating a lack of understanding of the problem's contextual requirements. Similarly, transformation errors often signal weak inference and decision-making skills, as students struggle to convert verbal information into appropriate mathematical representations. Errors at the process skills stage signify shortcomings in logical evaluation and the selection of solution strategies, while encoding errors are frequently linked to inaccuracies in communicating results logically, indicating a poor ability to construct coherent arguments or conclusions.

A total of six problem-solving questions related to three-variable linear equations were administered to 33 students. Each item varied in difficulty, ranging from basic to complex tasks. These questions were analyzed using NEA, which classifies students' problem-solving stages into five categories: reading, comprehension, transformation, process skills, and encoding errors.

Analysis of Student Errors

The distribution of errors across the five Newman stages is shown in Table 2.

Table 2. Percentage of student errors based on NEA

Newman Stages	Incorrect (%)	Correct (%)
Reading	0%	100%
Comprehension	48%	52%
Transformation	57%	43%
Process Skills	66%	34%
Encoding	82%	18%

All students demonstrated success at the reading stage (100%), indicating the ability to read questions and identify relevant mathematical symbols. However, at the comprehension stage, nearly half (48%) failed to determine the known and unknown elements of the problem. At the transformation stage, 57% failed to correctly formulate mathematical models, often as a consequence of miscomprehension. Difficulties significantly increased at the process skills stage, with 66% of students making procedural errors. This ultimately impacted their performance in the encoding stage, where 82% failed to write correct final answers or appropriate conclusions.

Analysis of the Case Study of Three Subjects

To gain deeper insight into the errors, three students representing high (S1), medium (S2), and low (S3) performance levels were interviewed and analyzed in detail. Table 3 summarizes the achievement of each student across the Newman stages.

Table 3. Student achievement by newman stage

Newman Stages	S1 (high)						S2 (medium)						S3 (low)					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Reading	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Comprehension	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✓	✓	✓	✗	✗	✗
Transformation	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✓	✓	✓	✗	✗	✗
Process Skills	✓	✓	✓	✓	✗	✓	✗	✓	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗
Encoding	✗	✓	✓	✓	✗	✓	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗

S1: High-Performing Student

S1 (high-level student) has reached the reading stage on each question, which means that he can read the questions and recognize the symbols contained. In addition, S1 has also reached the comprehension stage, determining the observable information stated in the question. Likewise, S1 has reached the transformation stage for all questions. In the process skills stage, S1 has been able to get it on most questions except 1 question. Likewise, in the encoding stage, S1 has not yet fully reached that stage. The following are the results of the interview with S1.

Q: In question number 1, why could you not finish it and not get the final answer?
S1: It took me quite a long time to remember the concepts and formulas, so in the last few questions, the time was up, and I could not do them anymore.

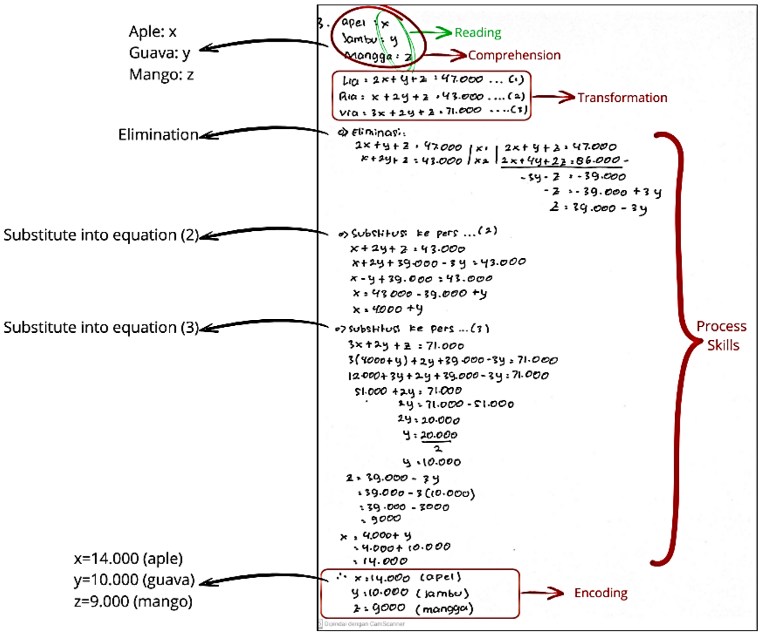

Figure 1. S1 Learning outcomes (1)

From the learning outcomes in Figure 1, students with high learning outcomes have been able to achieve all of Newman's stages, starting from reading and recognizing symbols in questions, determining what is known and asked, being able to change verbal and visual information into mathematical equations, and of course, being able to use process skills well so that they can code correctly. From achieving all stages of question number 3 in this S1, several stages cannot be achieved in other questions.

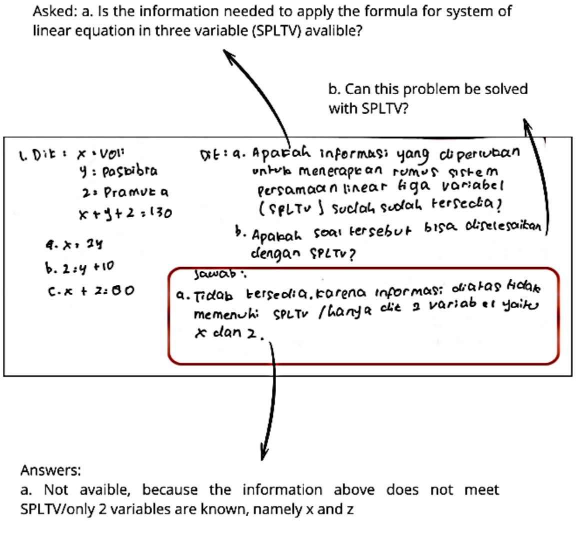

Figure 2. S1 learning outcomes (2)

As seen in Figure 2, the answers of students with a high performance level (S1) at the encoding stage deviated slightly from the expected answers. Further analysis showed that the approach taken reflected a deep understanding of the concept. They tended to interpret problem situations critically and attempted to present solutions more flexibly, even if they did not conform to commonly used formats. This suggests that what appears to be an encoding error may reflect a higher-order problem-solving strategy, while also indicating the presence of interpretive room or wording inaccuracies in the problem that could trigger different interpretations. This finding is interesting because it reveals how high-ability students critically interact with problems, even when they encounter suboptimal problem formats

S2: Moderate-Performing Student

S2 (intermediate level students) have almost reached all stages, where they have reached the encoding stage on some questions. However, on other questions, S2 still made mistakes at the process skills stage, so the encoding stage was also incorrect. In addition, on the last three questions, S2 only reached the reading stage. This is because the last three questions are related to the previous questions, so when students cannot carry out the process to the encoding stage in the earlier questions, they cannot answer the last three questions. This results in students being unable to code correctly, as supported by the results of interviews with S2.

Q: Why didn't you finish it?
S2: Because I am confused about what else to do, I can not continue
Q: What makes you get confused? From the beginning, it seemed you could do it smoothly.
S2: I did not find results x, y, and z, so if I continue working on it, I am afraid of making mistakes.

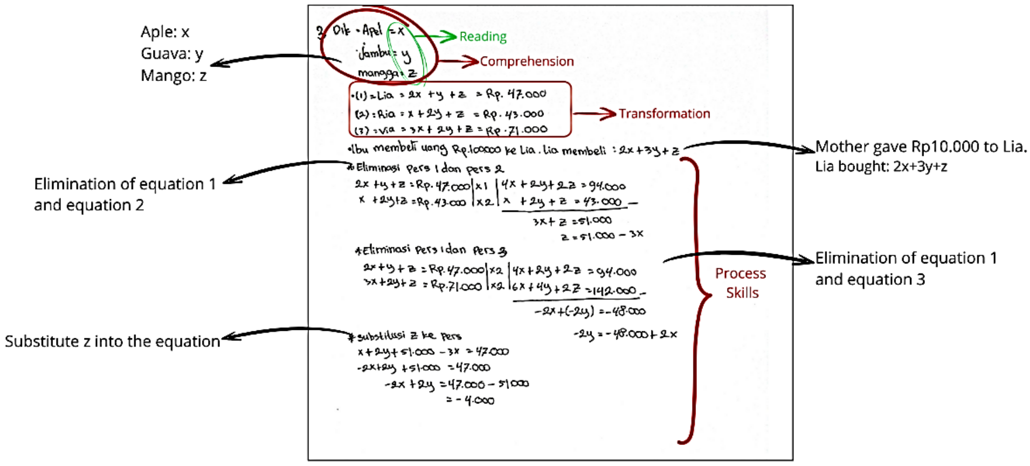

Figure 3. S2 learning outcomes (1)

In general, students can understand the question. This shows that students have good reading skills. He can write the observable information and understand the illustration stated in the questions. He can do transformations, describing students' ability to change verbal or visual information into mathematical equations. However, at the process skills stage, he cannot complete the substitution of z into equation 2.

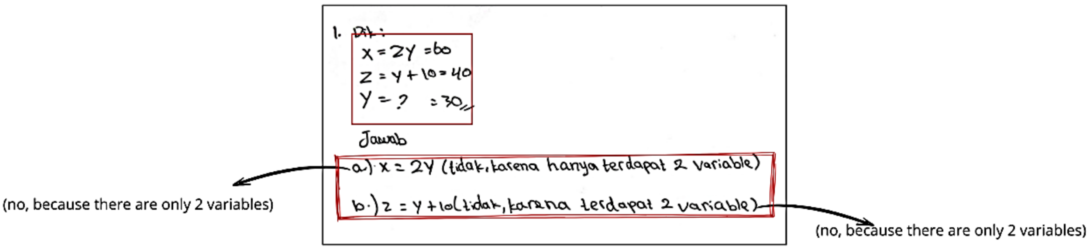

Figure 4. S2 Learning Outcomes (2)

S2 made a transformation error in another question: writing a mathematical model of the question provided. S2 added numbers to each mathematical model not listed in the question. In addition, similar to S1, S2 had a misconception about the equation concepts, also assumed that if the equation had three variables, then all mathematical models had to contain three variables, so he made the same error in encoding.

S3: Low-Performing Student

S3 (low-level students) have not reached all of Newman's stages. S3 made mistakes in the process skills stage on all questions, which caused encoding errors in each question. Like S2, S3 could only reach the reading stage on the last three questions because he made encoding errors in the previous questions. Moreover, S3 also made process skills errors in the earlier questions. The following are the learning outcomes of students from all levels. The following are the results of the interview with S3.

Q: Why didn't you complete the answers to the questions given?
S3: Because I forgot the formula and the way to solve it.
Q: What is the reason you forgot the formula?
S3: This concept I got when I was in grade 10, but now I am in grade 11 and can not remember it anymore

S3 can only reach the transformation stage. Students can read, recognize symbols in questions, write the observable information, and write a mathematical model of the questions presented. However, as seen in Figure 3, he did not perform process skills and could not eliminate two equations, then continued by substituting if one of the variables is known.

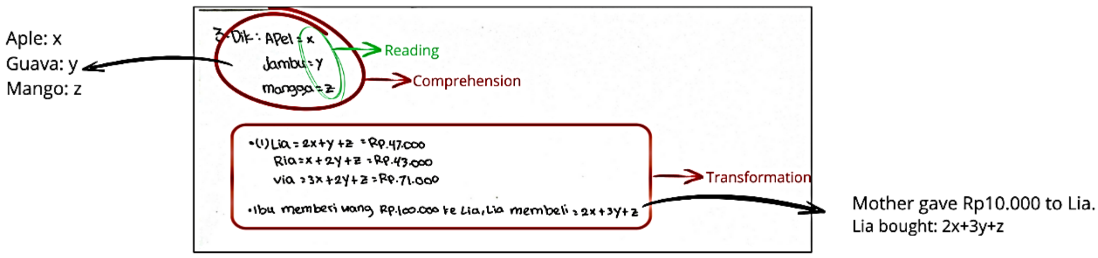

Figure 5. S3 Learning outcomes (1)

This student with low learning outcomes (S3) did not complete his answers to all the questions given. He could only get to the stage of writing the mathematical model in questions 1, 2, and 3. Questions 4, 5, and 6 could not be answered because he had to know the answer to question 3. Just like S2 did not complete the process skill stage and did not find the results, S3 also could not work on questions 4, 5, and 6.

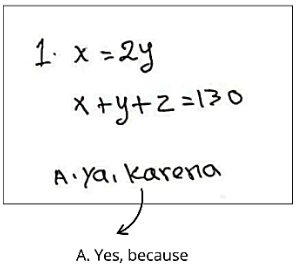

Figure 6. S3 learning outcomes (2)

In another question, S3 made mistakes, from comprehension to encoding. S3 can not write down the observable information and is asked in the question. He just wrote down 2 of the four mathematical models in the question and a short answer at the encoding stage without an apparent reason.

The results of this study indicate that the process skills stage is the most error-prone in the students' problem-solving process. This finding aligns with previous studies that emphasize the significance of process skills in mathematics learning. As noted by Yantoro et al. (2022), process skills refer to students' abilities to execute mathematical procedures correctly and efficiently. These include essential components such as interpreting data, performing calculations, and organizing logical steps toward a solution (Utami et al., 2021).

The current study revealed that 66% of students made errors at the process skills stage, reflecting difficulties in applying previously learned concepts to solve contextual mathematical problems. This proportion is considerably higher than that reported in some prior studies, such as Irianti et al. (2024), who found 26% of students committing process-related errors, and Agustiani (2021), who reported a rate of 42.2%. These differences may stem from the increased complexity of three-variable linear equation systems compared to topics examined in those studies.

The qualitative data from student interviews shed further light on the causes of these errors. Several students stated that they forgot the relevant formulas and lacked confidence in continuing their solutions. This can be attributed to the temporal gap between learning the material in Grade 10 and applying it in Grade 11. As highlighted by Lindquist et al. (2024)students often struggle to recall mathematical formulas when they are not regularly reinforced, particularly when those formulas must be applied in novel or more complex problem contexts.

These findings underscore the importance of continuous reinforcement of previously learned material, as forgetting is a natural cognitive phenomenon that affects students' ability to perform in later tasks. Teachers are advised to implement the spiral review strategy more specifically, for example, by beginning each lesson with a short quiz that reviews previously learned material, or providing weekly cumulative practice that covers older concepts. This strategy helps reinforce understanding gradually, and as students successfully navigate previously difficult material, their confidence naturally grows.

Another major factor identified in this study is the role of self-confidence in students' ability to complete process skills and encoding stages. S2 and S3, for example, expressed doubt and hesitation during problem-solving, often choosing not to proceed rather than risk making mistakes. This observation aligns with previous studies that have found self-confidence to be a significant predictor of students' mathematical performance (Christensen & Knezek, 2020). Students with higher levels of confidence tend to engage more persistently in problem-solving, whereas low-confidence students often disengage, which can further reduce achievement (Zakariya, 2021).

The role of math anxiety also cannot be overlooked. Several studies have found that low levels of anxiety combined with high self-confidence correlate with better performance in mathematical tasks (Hiller et al., 2022; Jameson et al., 2022; Wahyuni et al., 2024). Furthermore, Foster et al. (2022) identified a hypercorrection effect, where students who initially answered questions incorrectly with high confidence were more likely to correct those errors upon receiving feedback. This suggests that a certain degree of self-assurance, even if initially misplaced, may contribute positively to learning in the long term.

Based on the findings of this study, teachers are encouraged to more closely observe the types of student errors as indicators of weaknesses in critical thinking. Implementing strategies such as spiral review, guided discussions, and structured contextual practice exercises can help strengthen students' abilities to understand, process, and logically convey information. Furthermore, teachers can also use the NEA framework in simple classroom reflections to encourage students to recognise their stages of thinking, thereby developing self-confidence and metacognitive awareness.

▪ CONCLUSION

This study was conducted on students who had previously studied the material of three-variable linear equation systems. Participants were categorised into three groups based on their learning achievement: high, medium, and low. The primary objective of this study was to identify the most common types of errors made by students in solving problems and to explore the causal factors behind these errors.

The findings indicate that the encoding stage was the most problematic, with an error rate of 82% among participants. Furthermore, the data also indicated that most student errors began in the process skills stage and then impacted subsequent stages. This suggests that accuracy at each stage of problem solving is cumulative, so errors in early stages, such as transformation or processing, can trigger subsequent errors up to the encoding stage.

The primary contribution of this study lies in the application of the NEA framework to link student error types to identified aspects of critical thinking, thereby providing a more structured and granular analysis of weaknesses in students' thinking processes. The implications of these findings are highly relevant for teachers and educators, who can use this approach to design more targeted learning interventions and strengthen students' conceptual and procedural skills across the board.

More broadly, the results of this study underscore the importance of process-based assessment in mathematics learning, as well as the need for approaches that uncover students' thinking processes, rather than simply assessing final answers. This effort aligns with the vision of 21st-century education, which places critical thinking skills as a core competency in developing a generation of reflective and adaptive learners.

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