

## Bridging Indigenous Knowledge and Cognition: A Metacognitive-Metaphor Learning Model to Enhance Mathematical Reasoning and Motivation

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**Abstract:** Students' obstacles in learning mathematics are due to weak reasoning and low motivation. This study aims to develop a metacognitive learning model using the metaphor of a rare Kalimantan tree to improve mathematics learning outcomes and motivation. The research method comprised the Recursive, Reflective, Design, and Development stages, with the learning model developed as a single-path prototype and validated through expert, small-group, and field validation. The instruments include expert assessments, student response questionnaires, mathematics achievement tests, and motivation scales. Data analysis used qualitative and quantitative descriptions, bar charts, line charts, and analysis of variance. The research findings are presented through the gradual development of the learning model from a single-path prototype to an alpha and beta version. The learning model, in the form of a single-path prototype, is a rough draft for organizing three components: metacognitive questions, metaphors, and hands-on activities. The alpha version of the learning model was developed through expert and small-group validation. Experts assess the learning model as very effective for mathematical reasoning and learning motivation, and easy to apply. The beta version of the learning model has been proven effective in improving mathematics learning outcomes and in the interaction effect between motivation and the learning model during the mathematics learning process. Teachers use it by creating worksheets with metacognitive questions and assigning hands-on activities. The results of this study contribute theoretically by highlighting elements of local wisdom as components of the model.

**Keywords:** hands-on activity, mathematical reasoning, metaphor, metacognitive questioning strategy, motivation.

### ▪ INTRODUCTION

Mathematical reasoning is an essential ability in the mathematics learning process. Success in understanding mathematics depends on the reasoning process in the learner's cognition. Mathematical reasoning and problem-solving contribute to the mastery of critical thinking (Xu et al., 2023), creativity (Hansen & Naalsund, 2025; Olsson & Granberg, 2024), and communication and collaboration skills (Wathne & Carlsen, 2024; Arnesen & Rø, 2024; Hendriana, Rohaeti, & Hidayat, 2017). Mathematical reasoning is the basis for thinking about and understanding patterns and relationships in mathematics, including algebra, trigonometry, comparison and linearity, exponential functions, and proportion (Guinungco & Roman, 2020; Kop et al., 2020; Pitta-Pantazi, Chimonis & Christou, 2020). On the other hand, mathematical reasoning promoted students' motivation and mediated the problem-solving process (Supriadi, Jamaluddin, & Suherman, 2024).

However, some research shows that the students lack mathematical reasoning. Weak Mathematical reasoning was found in the following research results. Students often make conceptual, procedural, and calculation errors in understanding mathematics, and their ability to solve mathematical problems remains weak (Lestari & Jailani, 2018). Jupri

et al. (2014) reported that students had difficulty understanding the meanings and uses of arithmetic operations in algebraic expressions. Students encountered an obstacle in analogical reasoning when they engaged in mathematical modelling, such as transferring knowledge from the real world to another context (Saeki et al., 2025). Students also faced obstacles in representing the multiplication distributive property (Kim, 2025) and in analogical reasoning in understanding trigonometry (Kristayulita et al., 2018).

(Metacognition helps students think about solving mathematical problems. Metacognitive pedagogy has affected students' abilities in arithmetical or algebraic word problem-solving (Verschaffel et al., 2019). Smith & Mancy (2018) and Vorhölter (2025) found that metacognitive questioning strategies help students understand the problem and choose the right formula to solve it. Yıldız & Öztürk (2023) have used metacognition to mediate students' problem posing serially during the mathematics course. Students have a thinking strategy to understand their solution and their mistakes. Tak, Zulnaidi, & Eu (2025) find that metacognition awareness has a causal relation with mathematical reasoning. Metacognitive awareness mediates students' attitudes and mathematical reasoning. Metacognition refers to thinking about thinking, which means students' awareness of their own thinking (Thi-Nga et al., 2024). The results of this study indicate that metacognitive questioning can foster mathematical reasoning.

However, the metacognition process influenced the cognitive and decreased learning motivation. High-level mathematics is not easy for students. According to Arnesen & Rø (2024), learning to strengthen mathematical reasoning has its own complexities. Azevedo (2020) reflects on metacognition, which requires exerting the initial effort in thinking. On the other hand, motivation has a positive correlation with mathematics achievement (Tran & Nguyen, 2021). The best learning environment for supporting metacognition is one in which someone learns in a motivated state. According to Goleman (2000), someone in a highly motivated environment helps students overcome difficulties without realizing it. Research on Mathematics learning shows that motivation to learn Mathematics influences success in Mathematics learning (Schukajlow et al., 2023). Supriadi et al. (2024) find that motivation mediates students' reasoning in successful problem-solving.

The strong motivation to learn Mathematics requires effective strategies. Lim and Chapman (2015) found that motivational strategies are an important key influence on Mathematics learning outcomes. Furthermore, Schukajlow et al. (2023) recommended that the instructional method in mathematics teaching and learning influences motivation. The instructional strategy, such as learning design based on the basic needs of emotion and motivation. The basic need for motivation, fulfilled through the learning model, addresses students' interests. Herpratiwi & Tohir (2022); Perche, Yennek & Léger (2025) find that students' interests influence motivation. Wong & Wong (2019) suggested igniting students' interest to strengthen motivation.

Perche et al. (2025) explained that students' interests can be divided into two types: individual and situational interests. Individual interest refers to an individual's predisposition to re-engage in a learning activity. Situational interest refers to an individual's affective reaction to a learning environment that captures students' attention. Based on these findings, the model of learning design heightens situational interest, as evidenced by the activities favored. The struggle of low motivation could be used to maintain situational interest and emerging individual interest. In practice, motivation

often decreases when students fail to complete the questions. The problem is, what are the effective strategies for increasing learning motivation?

Previous research results suggest that hands-on activities foster motivation to learn Mathematics. Ellah, Achor, & Enemarie (2019) suggest hands-on activities for foster span attention and working memory. Riley et al. (2017) explain an instructional strategy that is more hands-on and provides all students with opportunities to grasp mathematical concepts. Hughes (2019) and Tachie (2019) recommendations for hands-on activities to promote metacognitive awareness. Through hands-on activities, used to maintain situational interest and to foster emerging individual interest. Thibodi (2017) found that metaphors formed a mental picture of one's efforts and perseverance in learning mathematics. The research results of Demitra & Dewi (2021) show that the rare tree metaphor and hands-on activity can foster students' interest and enthusiasm for learning when studying analytical trigonometry.

Based on previous research, it is noted that the metacognition questioning strategy and hands-on mathematics activities can be effectively integrated into a learning model design. These three components of the learning strategy are synergistic and inseparable, strengthening mathematical reasoning and motivation to learn. If learning relies solely on metacognitive questioning strategies, it creates a significant cognitive load. Students easily become tired, bored, and even frustrated while learning mathematics. As a result, the motivation to learn mathematics decreases. Observations in classroom action research suggest that, in an effort to increase learning motivation, teachers employ a hands-on activity strategy by writing, cutting, and pasting trigonometric formulas in notebooks before students work on problems. This effort does not increase motivation; students still appear lethargic and lazy when solving problems.

However, what kind of hands-on activities can attract and maintain situational interest? Teenagers generally enjoy creating by gluing and cutting. Moreover, these students live in a peat swamp environment with forests of rare trees. A rare tree from Kalimantan has the potential to serve as a metaphorical source for mottos in mathematics learning. On the Kalimantan island, there are many types of endemic trees, including ironwood (*eusideoxylon zwageri*), agathis (*agathis borneensis*), belangiran (*shorea belangeran*), and lime trees (*dipterocarpaceae*) (Prasetyo, 2013). These are rare but essential tree types. Scarcity is driven by large-scale logging, high-value deforestation, and forest degradation (Prasetyo, 2013; Herianto et al., 2018).

Hands-on activities through gluing and cutting of the coloured papers, creating the rare three fantasies. Moreover, the students exploring the learning motto based on the rare three growth and benefits of the Bourneo forest is an effort to situate interest and strengthen motivation, the dimension of cultural influence on teacher and student teaching and learning, specifically on mathematics learning. Previous research finds that cultural factors influence differences in the lesson design of Japanese and Swiss teachers (Clivaz & Miyakawa, 2020). Parra & Trinick (2018) found that teachers can explore sociolinguistic and epistemological issues when the use of the indigenous language is elaborated over a short period. Aikenhead (2017) said teaching school mathematics in a culturally sensitive manner in Canada by integrating Indigenous people into the mathematics lesson curriculum. The rare three of Bourneo is a part of the cultural aspect that influenced the students' mathematics learning.

So, three strategies can be used to facilitate the mathematical reasoning process and maintain stable motivation. These three strategies are metacognitive questioning, hands-on activities, and incorporating elements of local wisdom about rare trees. These three elements support one another to create a reasoned, motivated learning process. If one is omitted, the learning process will not proceed as expected. The problem is how those three strategies orchestrate a learning model that effectively enhances the quality of mathematics learning. The following problems relate to the development of a metacognition-metaphor-based mathematics learning model.

1. What does the learning model look like in the single path prototype?
2. What is the feasibility of the learning model based on the results of expert review, small group validation?
3. What does the learning model look like after expert review and small group validation to get the alpha version?
4. How effectiveness is of learning model on the learning outcome and motivation to get the beta version?

The research aims to develop a mathematics learning model by organizing metacognitive questioning, metaphor, and hands-on activities into mathematics instruction. These objectives are described as follows.

1. Develop a draft learning model in the form of a single path prototype;
2. Expert and small group validation to refine and obtain an alpha version of the learning model;
3. Field validation of the effectiveness of the learning model on learning outcomes and motivation to learn Mathematics at school, to obtain the beta version of the model.

## ▪ **METHOD**

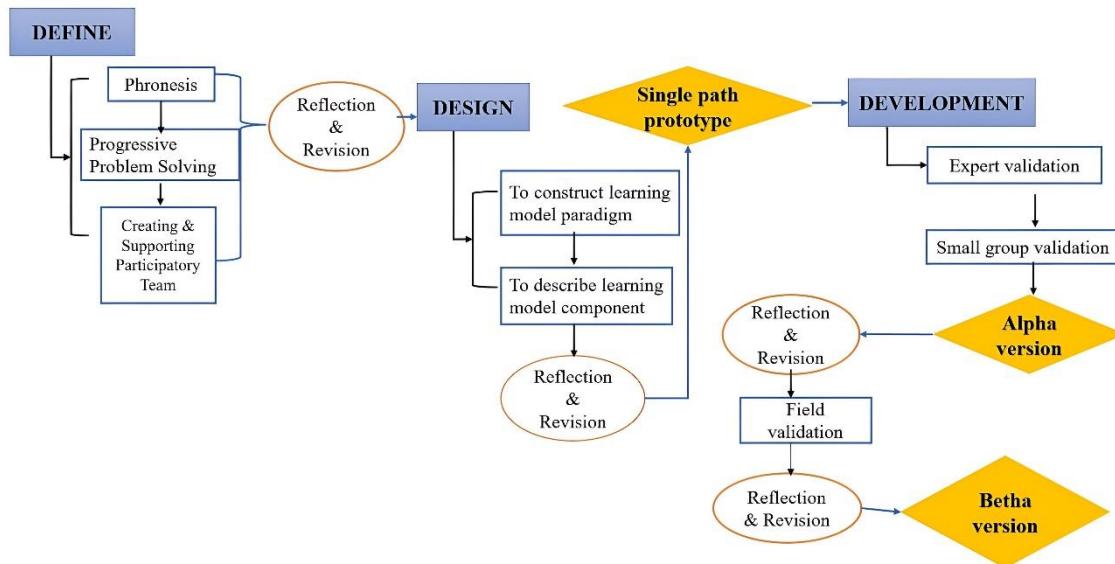
### **Participants**

Participants were selected based on the validation stages. The expert validation stage involved two experts in learning technology and mathematics learning. The first expert has expertise in mathematics learning. The second expert has expertise in instructional technology. The small-group validation stage involved a lecture and ten prospective mathematics teachers. Ten students from the Mathematics Education Study Program were involved in this validation. The goal was to assess the implementation of the learning steps.

The field validation stage involved 54 junior high school students from Madrasah Tsanawiyah Negeri 2 Palangka Raya, selected from a total of 288 students. The unit of analysis for the field test was 288 students from 10 eighth-grade classes at Madrasah Tsanawiyah Negeri 2, Palangka Raya. The sample was selected via cluster random sampling from the 10 eighth-grade classes, with clusters defined by parallel classes. The mathematics teacher involved in the learning process justified the homogeneity of mathematical ability, stating that each class comprised students with high, medium, and low abilities. These student characteristics underpinned the selection of cluster random sampling, which requires that the population characteristics within each cluster be homogeneous.

## Research Design and Procedures

This research was carried out using the Recursive, Reflective, Design, and Development-Dissemination (R2D2) by Willis (2009), with the stages of defined, design and development, and dissemination. The development stages and activities with the R2D2 model are presented in Figure 1.



**Figure 1.** The flow of development activities through R2D2

The research and development process for the learning model of metacognition-metaphor of rare Kalimantan trees, based on the R2D2 stages, is shown in Table 1.

**Table 1.** The phase of development of the learning model through R2D2

Stages	Activities
<i>Define</i>	Study the components of learning strategies, such as the metacognition questioning strategy, the rare trees of Kalimantan metaphors, and hands-on activities of rare trees fantasy.
a. Phronesis	Collecting and selecting relevant learning strategies fostering mathematical learning.
b. Progressive problem solving	Review the variances of Kalimantan's rare trees as a source of metaphor.
c. Creating and supporting a participatory team	Formation of the core team and participatory team
<i>Design and development</i>	
a. Design	To construct the paradigm of the learning model as a theoretical frame.
b. Development:	
a) Develop the learning model in the form of a single path prototype	To develop a learning model in a single-path prototype through the activities: ✓ Collected the literature and mathematics learning materials

	<ul style="list-style-type: none"> <li>✓ Finalizing the learning model framework through focus group discussions</li> <li>✓ Developed the learning model in the form of a book reference to realize the single path prototype version model.</li> </ul>
b) Experts' validation and reflection	Invited an expert in mathematics learning and an expert in instructional technology.
c) Small-group validation and reflection	<ul style="list-style-type: none"> <li>✓ Implemented the syntax of the learning model involving 10 undergraduate students of the Mathematics Education Department at the University of Palangka Raya.</li> <li>✓ Reflect and revise the learning model to obtain an alpha version form.</li> </ul>
<i>Dissemination: Field validation and reflection</i>	<ul style="list-style-type: none"> <li>✓ To validate the effect of the learning model on mathematics learning outcomes and learning motivation.</li> <li>✓ Implementation of the learning model involved 54 students of the junior high school through experimental research.</li> <li>✓ Reflect and revise the learning model to realize the beta version form.</li> </ul>

Development at the focus of the define stage includes three activities, namely phronesis, progressive problem-solving, and creating and supporting participatory teams. Phronesis activities have been carried out by collecting and selecting relevant learning strategies fostering mathematical reasoning and learning motivation, as components of the learning model. Progressive problem-solving activities focus on analysis and reflection on the components of metacognition questions, the metaphor of rare trees in Kalimantan, and hands-on activities. Creating and supporting participatory teams has formed a core team and a participant team. The core team is responsible for creating the learning model. Meanwhile, the participants team is (a) a team of mathematics learning and learning technology experts, (b) a small group of students and lecturers, and (c) a group of teachers and high school students.

The design focus is achieved by constructing a framework and a description of the components of the learning model, which are realized in book form. This team conducted a focus group discussion to reflect on the learning model and revise it to obtain a single-path prototype. The learning model in the form of a single-path prototype is still a rough draft and needs to be refined through gradual validation.

The development activity is carried out through expert validation and small-group validation to obtain the alpha version of the learning model, and field validation in the context of Mathematics learning at school to reach the beta version. The learning model in the alpha version is a single-path prototype, revised based on results from expert and small-group validations. Expert validation assesses the suitability of the learning model, while small-group validation evaluates the implementation of its syntax. The validation results become a reference for revising the learning model. The beta version of the learning model has been proven effective through an effectiveness examination in the context of mathematics learning. Two groups of students were designated as the experimental and control groups. These two groups of students came from two parallel

classes. The math teacher justified the two classes' math abilities as equivalent. To ensure statistical equivalence, the experimental design used a pretest on both groups.

The results of the reflection validation become a reference for revising the learning model. Small-group validation was conducted to implement the learning model in Algebra instruction. The results of expert and small-group validation serve as a reference for revising the learning model. At this stage, the revised learning model has reached the alpha version. Field validation through a quasi-experiment method using a non-equivalent control group design of the learning model in the school context of Mathematics learning.

### **Data Collection and Instrument**

Data collected through expert assessment on the learning model and students' responses when they implement it in small-group validation. Field validation data were collected by providing motivation questionnaires and mathematics learning outcome tests.

Instruments of expert validation using analytic rubrics. The rubric includes three indicators: such as the effectiveness of (a) the metacognitive question strategy, four items, hands-on activities, and metaphors separated into five items, hands-on activities, and metaphors integrated into three items, and asking experts to predict learning model success. Small-group validation is achieved by developing the lesson plan, recording the implementation learning, and providing the student response questionnaires.

Instrument of field validation using Mathematics learning outcomes tests and Mathematics learning motivation questionnaires. The motivation questionnaires and the Mathematics learning outcomes test have an Alpha-Cronbach reliability coefficient  $r_{xx}'$  of 0.84 and 0.94, respectively. The motivation scale indicators are perseverance (5 items), tenacity (5 items), motivation to succeed (8 items), and independence (2 items). The 30 motivational statements were constructed on a Likert scale with five response options. The motivation questionnaire has been tested and analyzed using the Pearson product-moment correlation. The twenty items have validity coefficients ( $r_{ij}$ ) ranging from 0.30 to 0.71, including 12 positive and eight negative items. The mathematics learning outcomes were assessed using an essay test consisting of five problems have good difficulty (0.27 to 0.80) and differential power (0.29 to 0.50) indexes. The essay test indicators include of the following abilities (a) to differentiate the linear equation of two variables or not 1 item; (b) to solve a system of linear equations in two variables using the graphical, substitution, and elimination methods three items; (c) to solve the contextual problem using a system of linear equations in two variables 1 item.

### **Data Analysis**

The expert validation and small-group data were analyzed in descriptive quantitative and qualitative forms. The results of the two experts' assessments of the model, on a scale of 1-4, were presented in a bar chart. The expert opinion comments were qualitative data, summarized by analyzing effectiveness keywords, grouping them, and drawing conclusions. The data on small-group validation results, such as students' hands-on projects and the model documentation implementation stage, were presented in a photo description. The data on students' scores for solving the mathematics problem were analyzed using a scatter diagram.

Data for field validation were analyzed using descriptive statistics, including mean and standard deviation. The assumption of homogeneity was analyzed through Levene's

test of equality of error variances. Meanwhile, the normality assumption was analyzed through the One-Sample Kolmogorov-Smirnov Test. The effect of the learning model was analyzed using Analysis of Variance (ANOVA). The learning effect on mathematics learning outcomes and motivation was practically determined based on F coefficients for the main and interaction effects of the learning model in ANOVA at  $\alpha = 0.05$ .

#### ▪ **RESULT AND DISSCUSSION**

The idea for the metacognition-metaphor learning model for the Kalimantan rare tree was developed during the action research conducted by the author and a mathematics teacher. It was reported in Demitra & Dewi (2021). This finding has been strengthened through research and development of mathematical learning models for reasoning and motivation, based on the R2D2 stage by Willis (2009). The research and development results will be presented as follows.

##### **Define** **Phronesis**

The phronesis stage has been done through studying the learning model component. Initially, the teaching and learning in the action research of Demitra & Dewi (2021) have been thoroughly reviewed. Three instructional strategies have been identified and can be used as components in developing a mathematical learning model. The components are the metacognition questioning strategy, the metaphor of Kalimantan trees, and the hands-on activity.

The metacognition questioning strategy for mathematical reasoning has been studied in the relevant research by Tachie (2019) and Engelmann, Bannert, & Melzner (2021). Metaphor has been studied extensively in mathematics learning to foster motivation, based on research by Thibodi (2017), Gray & Holyoak (2018), Wang, Zhang & Cai (2019), and Huang, Spector & Yang (2020). Hands-on activities are a strategy for fostering student interest in mathematics, which correlates with motivation (Huang, Spector, & Yang, 2020; Weigand, Trgalova & Tabah, 2024; Tambunan, Sinaga & Widada, 2021).

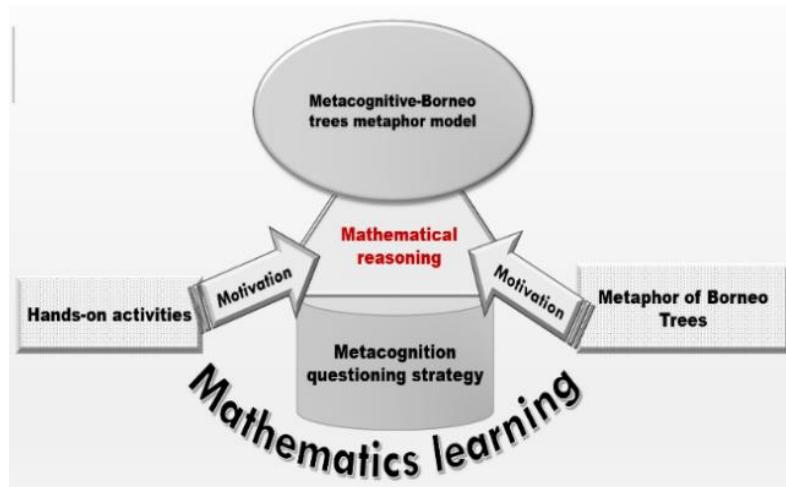
##### **Progressive Problem Solving**

The stage of progressive problem-solving through reviewing the source metaphor, based on the Kalimantan rare tree. The source of the metaphor is explored through the local wisdom of Kalimantan, such as the growth of rare trees in the forest (Herianto et al., 2018). Some research explains the cultural influence on students' learning in mathematics (Clivaz & Miyakawa, 2020; Demitra & Sarjoko, 2018). The result of phronesis and progressive problem-solving is a model of learning components, such as metacognition, questioning, hands-on activities, and metaphor based on rare Kalimantan trees, where each component is separated. The learning model produced a single-path prototype form.

##### **Design and Development** **Design**

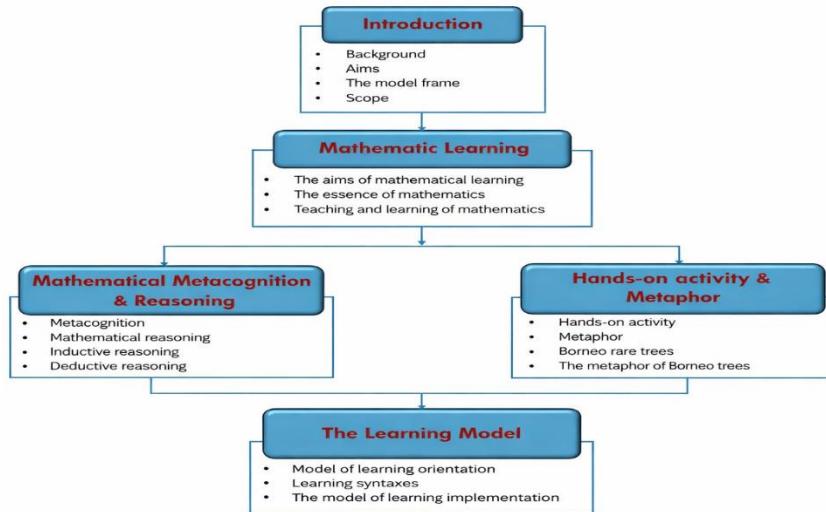
The components of the learning model were integrated to develop the paradigm and framework, and the resulting draft was presented as a book reference. The result is an integrated learning model paradigm, as presented in Figure 2. The framework of the

innovative metacognition-metaphor learning model for the Kalimantan rare tree single-path prototype. Initially, each component was organized separately, progressing through its own learning stages. The learning model paradigm was developed theoretically, including a description of the metacognitive model, types of rare trees, rare tree metaphors, and hands-on activities. Then the syntaxes develop prescriptively, explaining the learning orientation model, the learning syntax model, and the learning implementation model.



**Figure 2.** The learning model paradigm

The learning model framework has been presented in Figure 3. The learning model is equipped with learning tools, learning activities, and testing results evaluating the effectiveness of learning on mathematics learning outcomes and learning motivation.

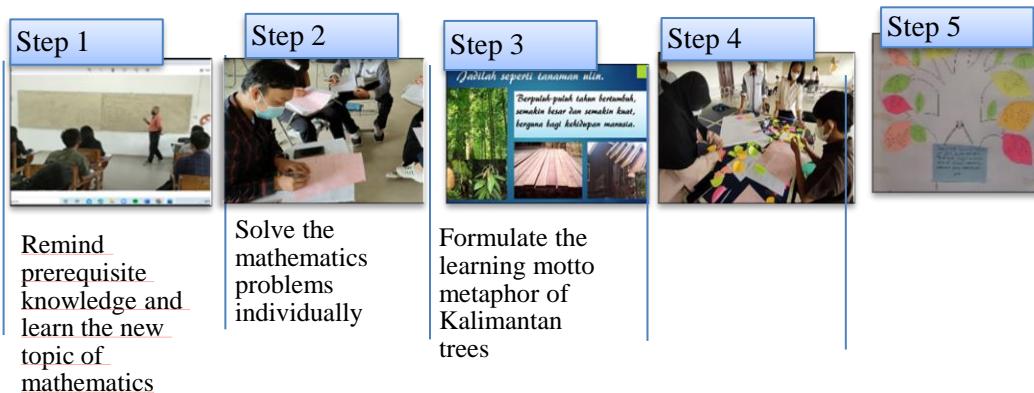


**Figure 3.** The framework description of the learning model

### **Development**

After expert and small-group validation, changes were made to the model framework, and at this stage, the learning model reached its alpha version. The framework

changes are made by integrating metacognition and metaphor components in a hands-on activity.



**Figure 4.** The syntax of the learning model

The syntaxes of the learning model are presented in Figure 4. The following describes the learning stages in the context of learning logarithms. Step 1: Educators invite students to recall the prerequisite material for logarithms, namely, exponential equations, explain new material for logarithmic equations, and explore examples of problems and their solutions.

Step 2: Students solve the mathematical problems individually. Educators prepare the logarithmic equation question, apply the metacognitive questioning strategy, and justify the difficulty level as easy, medium, or difficult. The example of the logarithmic equation problem is presented in Table 2.

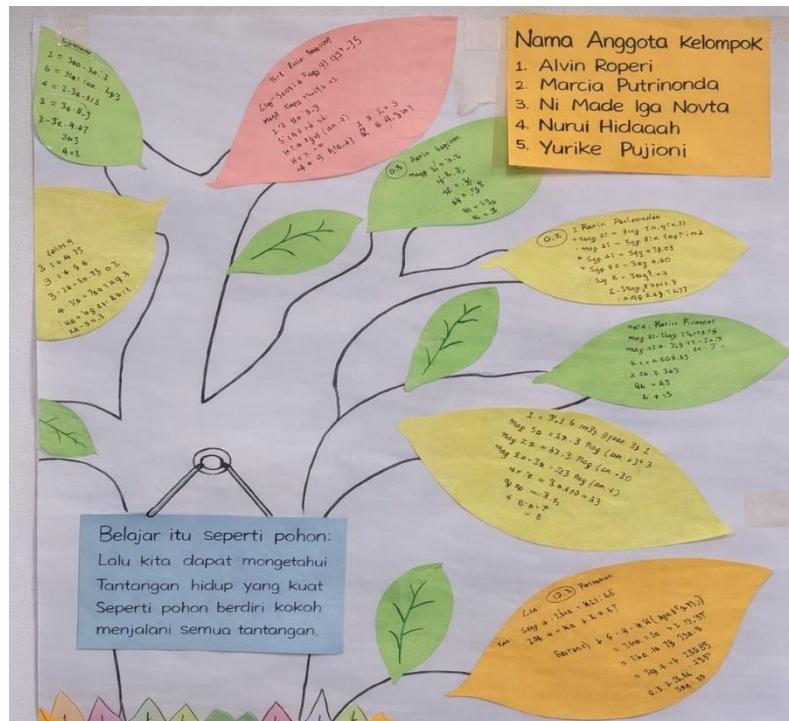
**Table 2.** The example of questions for individual practice with metacognition questions

<i>Difficult problem</i>	When you answer, think about the answers to these questions:
<i>Find the solution of:</i>	<ul style="list-style-type: none"> <li>✓ Which properties of logarithms can be used to solve this problem?</li> </ul>
$\frac{^3\log 25 \times ^5\log 81 + \frac{1}{4}\log 2}{^3\log 36 - ^3\log 4} =$	<ul style="list-style-type: none"> <li>✓ Are my choices correct in the form of the question?</li> <li>✓ Are my calculation results correct?</li> </ul>

Step 3, creating a metaphor for the rare trees of Kalimantan, was carried out by forming groups of five students. The educator introduces one of Kalimantan's rare tree species through a PowerPoint template slide, offering a learning slogan based on the tree. Students are tasked with creating a learning motto.

Step 4: Students, in groups, create a fantasy of Kalimantan's rare tree through hands-on activities. Fantasy tree creations require cardboard, colored paper, markers, scissors, and glue. Fantasy tree creations are made by sketching a tree, writing the answer on colored paper in the shape of a leaf, attaching it to the tree, and attaching the learning motto to the fantasy tree creation.

Step 5: Each group displays the fantasy Kalimantan rare tree they created by installing their work on the classroom wall. One or two members wait for their group's work to be displayed; other members may look at the other group's work in turn. This activity encourages students to communicate their work authentically to others and presents the metaphoric meaning of the learning motto at the tree. Educators assess the correctness of the answers and assign scores to those written on the leaves of the fantasy ironwood tree. The metaphor for the motto of learning is depicted as a fantasy tree in Figure 5.



**Figure 5.** Results of student work on the Kalimantan tree fantasy and learning motto

The metaphor of a rare tree as a motto for learning is produced as follows: (a) "The longer the wood of an ironwood tree is submerged in water, the stronger it becomes", (b) "*learn like an ironwood tree which can turn challenges into strength, the more challenges, the greater the strength*", (c) "*be like an ironwood tree, the more you practice solving problems, the stronger your understanding of mathematics will be*".

Field tests proved the effectiveness of the five learning stages. The results provide a holistic description of the learning model, both theoretically and practically. The following presentation presents the results of expert, small-group, and field validation, which brought the learning model to a beta version.

## Expert Validation

Expert validation aims to examine the effectiveness of the components of the learning model, including the impact of useful metacognition questions, Kalimantan rare tree metaphors, hands-on activities, and the integration of metaphors and hands-on activities on mathematical reasoning, interest, and motivation in learning mathematics. Two experts in learning technology and mathematics learning reviewed a prototype of an

innovative learning model based on the rare tree metaphor of Kalimantan. The following description presents the results of the two experts' review of the assessment components mentioned above. Mean and standard deviation of the reviews are shown in Table 3.

**Table 3.** The mean and standard deviation of the experts' review

The learning model components	P1		P2		Number of items
	Mean	Standard Deviation	Mean	Standard Deviation	
1. Effectiveness of metacognition questioning	4.00	0.00	3.75	0.50	4
2. Effectiveness of hands-on activities and metaphors in separate.	3.00	0.00	4.00	0.00	3
3. Effectiveness of hands-on activities and metaphors integrating.	3.75	0.50	3.75	0.50	5

Note: P1 is an expert in mathematics learning; P2 is an expert in instructional technology

### ***The Effectiveness of Metacognitive Questions***

The mean and standard deviation of rating scores are presented in Table 3. The mean and standard deviation of the rating scores from two experts are 4.00 and 0.00, respectively. It means P1 assessed the use of metacognitive questions as effective for developing deductive, inductive, and critical thinking skills, as well as mathematical problem-solving skills. Meanwhile, the mean and standard deviation of expert P2's rating scores are 3.75 and 0.50, respectively. P2 reviewed the use of metacognitive questions as highly effective in developing inductive and deductive reasoning skills, as well as critical thinking skills. However, it was not fully effective in developing mathematical problem-solving skills.

P1 assessed the use of metacognitive questions as effective in developing deductive and inductive reasoning skills, mathematical problem-solving skills, and critical thinking skills. Meanwhile, P2 reviewed the use of metacognitive questions as highly effective for developing inductive and deductive reasoning and critical thinking skills. However, it was not fully effective in developing mathematical problem-solving skills.

Mathematics learning and learning technology experts assess that the metacognition question strategies are effective in developing inductive reasoning, deductive reasoning, mathematical problem-solving, and critical thinking skills. Note of a mathematics education expert that "*effectiveness is achieved if learning pays attention to the learner's prior knowledge*". The expert notes that the teacher reminds the prerequisite knowledge and presents new material. This stage emphasizes mastering prerequisite material in learning new mathematical material. The research results of Rach & Ufer (2020) and Salsabila (2019) show that mastery of initial mathematics knowledge influences the ability to understand advanced mathematical concepts.

The use of metacognitive question strategies strengthens mathematical reasoning. Providing metacognitive questions helps students choose the right mathematical rules or formulas to solve problems. Students are asked questions that encourage them to think about the relationships between ideas when explaining their answers to mathematical questions. The results of this research are in line with the opinions of Verschaffel et al. (2019), Vorhölter (2025), Yıldız & Öztürk (2023), Kallio et al. (2018), García et al.

(2016), Ohtani & Hisasaka (2018), and Tachie (2019), who found that metacognition skill influences students' success in understanding mathematics.

#### ***Effectiveness of Hands-on Activities and Metaphors***

The experts' review of the effectiveness of metaphor and hands-on activities is shown in Table 3. The mean and standard deviation of the expert's rating score for P1 are 3.00 and 0.00, respectively. It means the expert P-1 assessed the metaphor of a rare Kalimantan tree as quite effective in fostering appreciation and attitude persistence, thereby strengthening the memory capacity for mathematics learning. Meanwhile, expert P-2 assessed that constructing appreciation and attitude persistence strengthened the capacity for memory as very effective, with a mean of 4.00 and a standard deviation of 0.00. The experts assessed the learning model for constructing appreciation and attitude persistence and strengthening memory capacity as quite effective for P1 mathematics learning experts and as very effective for P2 instructional technology experts.

#### ***Effectiveness of Hands-On Activities and Metaphors in Integrating***

Table 3 also shows that the mean and standard deviation of the experts P1 and P2 are 3.75 and 0.50, respectively. Refer to Figure 8 shows that both experts assessed that hands-on activities integrated with metaphors were quite effective for activating creative mathematical thinking. However, the metaphor of a rare Kalimantan tree, integrated with hands-on activities, was deemed highly effective in developing collaboration and communication skills, as well as interest and motivation in learning mathematics. Learning technology and mathematics learning experts assessed that metaphors and hands-on activities used separately were quite effective in supporting persistence and strengthening the students' memory capacity. Meanwhile, the mathematics learning experts found that metaphors, when used separately from hands-on activities, can support persistence in learning mathematics and enhance memory capacity.

The hands-on activity, through the assignment to create a creative drawing of a Kalimantan tree, is a fun stage for students. The use of hands-on activity strategies has been shown to increase interest, motivation, and learning outcomes in mathematics, as demonstrated by Atteh et al. (2020). The implementation of the Kalimantan tree metaphor and the creation of tree fantasy, integrated into hands-on activities, to strengthen motivation to learn mathematics. The growth process of this Kalimantan tree and its utilization can be linked to the meaning of perseverance in studying mathematics. This anchoring motivates students to persist in working on math problems. Porter et al. (2000) stated that the use of metaphors is intended to attach positive associations to learning. This advantage is in line with the research results of Rodney et al. (2016), Mason (2016), Hendriana et al. (2017), Thibodi (2017), Olsen et al. (2020), and Gomez (2021), who show that metaphors can increase interest and motivation in learning mathematics.

#### ***Prediction of Learning Models' Success***

The success of the metacognition-metaphor learning model for Kalimantan's rare trees is evident in its impact on mathematical abilities. According to learning technology expert assessors, success in mastering mathematics and motivation to learn are predicted to reach 75% each, while mathematics learning experts predict 85% for both. The argument given by the learning expert is as follows: "The development of this model

should be aimed more at developing metacognitive skills to achieve what is called a habit of mind, which requires a long time."

The results of expert validation indicate that components of the learning model, such as the provision of metacognitive questions, can foster mathematical reasoning. Likewise, using metaphors and hands-on activities increases interest and motivation in learning mathematics. It is an interesting note that good mastery of prerequisite knowledge and consideration of time allocation are references for revising the innovative learning model based on the rare Kalimantan tree metaphor.

The core team revised the draft learning model by adding syntax prescriptions for Step 1, material orientation. The syntax prescription for Step 1 is added with the sentence: "*The teacher reminds students of the prerequisite knowledge regarding the material to be studied,*" and in the model orientation sub-chapter, the sentence needs to be added: "*The need to calculate the adequacy of time allocation with the level of difficulty of making metaphors and creating fantasy trees*". Expert validation results show that the components of the metacognition question strategy learning model are effective in developing mathematical reasoning, problem-solving, and critical thinking skills. Integrated metaphors and hands-on activities can foster interest and motivation in learning mathematics; the predicted achievement rates for the mathematics learning outcomes model are 76% and 85%. The small-group validation showed that the mastery score for logarithmic equations ranged from 76% to 100%. Learning steps can be easily implemented and facilitate mathematical reasoning and motivation to learn.

### **Small-Group Validation**

Small-group validation assesses the implementation of the learning model within small groups of students. The validation assesses learning outcomes and students' responses after the learning process. Small-group validation activities include designing lesson plans, implementing learning activities, and analyzing learning outcomes, as described below. The scores of 10 students on solving logarithmic equation problems ranged from 76 to 100, with an average of 92.3. The percentage of mastery of the concept of logarithmic equations is above 76%.

Student responses to the implementation of the learning steps were as follows: of the ten students, (a) six felt that Step 1 was easy and four difficult, (b) five students said Steps 2 and 3 were easy and five difficult, and (c) eight students said Step 4 was easy and not easy for two students. Step 1: Encourage students to recall prerequisite knowledge related to logarithms. This step is quite challenging, as it requires a strong understanding of logarithms. Steps 2 and 3 prompted students to think aloud to understand the teacher's explanation and used metacognitive questioning to solve the logarithm problem. Students become tired, and their interest can decrease. This obstacle was overcome in Step 4 by stating the learning and creating fantasy tree motto and engaging in a hands-on group activity.

Moreover, students' responses when creating a metaphor for the learning motto of the rare trees of Kalimantan varied as follows: "*It is felt that making metaphors makes students enthusiastic in solving logarithmic equation problems.*" "*With the motto metaphor for learning, which originates from the growth of the ironwood tree, I realized that learning goes through a long process to grow so that it can be useful.*" "*The existence of a learning motto metaphor is a reference for increasing learning motivation*". The

students' responses show that the hands-on activity can foster students' motivation. This response aligns with Holstermann et al. (2010) and Demitra & Dewi (2021), who found that hands-on activities increase students' interest and motivation. The use of metaphor is also found in other research that can mediate students' thinking and motivation (Olsen et al., 2020; Gomez, 2021; Hendriana et al., 2017).

Students also stated that "*having metacognition questions as a guide could help them solve logarithmic equation problems from easy to difficult*". The student responses indicate that the metacognition questions mediated students' reasoning in solving the logarithmic problems. Thi-Nga et al. (2024) stated that metacognition is a higher-order thinking skill that guides students in developing solutions and in controlling their cognition to ensure the solutions they create are appropriate. Azevedo (2020) said that metacognition is needed during learning activities such as problem-solving or repetitive practice of the teacher's demonstration.

### Field Validation

#### *The Effect of Learning Models on Mathematics Learning Outcomes*

The main material taught when applying the learning model in experimental and control classes is a system of linear equations in two variables. The selection of teaching material in field validation differs from that in small-group validation, because the validation was conducted in a different class context. The small-group validation involves students taking the Algebra course. Meanwhile, the field validation should be implemented in a school context, and the teaching material should be selected from junior high school mathematics materials. The selection of materials is tailored to the student's context, and the developed learning model has the potential to be applied across various mathematics learning contexts, including school levels and varying mathematical materials.

The basic competencies achieved are (a) explaining a system of linear equations in two variables and its solution concerning contextual problems, and (b) solving problems related to a system of linear equations in two variables. The learning activities for implementing learning are presented in Table 4.

**Table 4.** The learning syntaxes for the experimental and control groups

Metacognition-Metaphor of the Kalimantan rare tree	Classical instruction
<i>Step 1:</i> <i>Remind of prerequisite knowledge and learn the new topic of the two-variable linear equation system:</i>	<i>Step 1:</i> <i>Remind of prerequisite knowledge and learn the new topic of the two-variable linear equation system</i>
✓ Ask the students to review the knowledge prerequisite of the two-variable linear equation system, such as a linear equation with two variables, and the number operations.	✓ Ask the students to review the knowledge prerequisite of the two-variable linear equation system, such as a linear equation with two variables, the number operations are involved.
✓ Presented the new material on the two-variable linear equation system to students	✓ Presented the new material of the two-variable linear equation system to students.
<i>Step 2:</i> a. <i>Solve the problems of the two-variable linear equation system individually:</i>	<i>Step 2:</i>
✓ Presented the procedure for solving the problem of two variables and the number operations carefully.	
✓ To assign the students to solve the problem of the two-variable linear equation and the number operations	

<p>carefully. The teacher distributed a problem written on colored paper. The difficulty level problem was written on the red paper. The medium-level problem was written on the green paper. The easy-level problem on the yellow paper.</p> <ul style="list-style-type: none"> <li>✓ The teacher let the students choose which problems they could solve, at challenging, medium, or easy levels, and asked them to answer that problem.</li> </ul>	<p><i>Solve the problems of a two-variable linear equation system individually.</i></p> <ul style="list-style-type: none"> <li>✓ The teacher taught the students the method to solve the two-variable linear equation system.</li> <li>✓ The teacher assigns the students to solve some of the problems about a two-variable linear equation system.</li> </ul>
<p><i>Step 3: Formulation of a learning motto metaphor of Kalimantan's rare trees.</i></p>	<p><i>Step 3:</i> <i>Ask a student to write his solution on the board.</i></p>
<ul style="list-style-type: none"> <li>✓ The teacher asks students to sit in groups, introduces Kalimantan's rare trees using media such as YouTube or PowerPoint slides, and then asks the group to discuss what the learning motto is based on the growth of Kalimantan's rare trees.</li> </ul>	<ul style="list-style-type: none"> <li>✓ The teacher will point to one student to write their answer on the whiteboard. As the student works, the teacher will observe their answers and assess their answer, correct or incorrect.</li> </ul>
<p><i>Step 4:</i></p>	<p><i>Step 3:</i> <i>Ask a student to write his solution on the board.</i></p>
<p><i>Students, in groups, create a fantasy about Kalimantan's rare trees through a hands-on activity.</i></p>	<ul style="list-style-type: none"> <li>✓ The teacher reflects the students' answer and explains to the students if they do not understand.</li> </ul>
<ul style="list-style-type: none"> <li>✓ The students agreed to sketch a rare Kalimantan tree, which would be used as a motto on the cardboard, until a complete sketch of the tree trunk, branches, and leaves was formed.</li> <li>✓ Each student rewrites their solution on colored paper and cuts it into a leaf of a tree fantasy. Then stick the paper on the fantasy tree.</li> <li>✓ Students stick up written learning mottos to motivate them and add ornaments according to their ideas to beautify their work.</li> </ul>	<p><i>Step 4:</i> The teacher is to assign homework to the students.</p>
<p><i>Step 5:</i></p> <p><i>To assess the solution of mathematical problems and the creation of the Kalimantan's rare tree fantasy</i></p> <ul style="list-style-type: none"> <li>✓ The group displays the results of their fantasy tree work.</li> <li>✓ Students stick up written mottos to motivate them and add ornaments according to their ideas to beautify their work.</li> <li>✓ Teacher assessed the solution and their creation.</li> </ul>	<ul style="list-style-type: none"> <li>✓ The teacher selects the questions with varying levels of difficulty and asks students to work on them at home.</li> </ul>

Learning is carried out in the context of Mathematics learning in Junior High School, which is supported by learning tools such as lesson plans, student worksheets, videos about rare trees in Kalimantan, mathematics problem sheets equipped with metacognitive question strategies, and materials for making fantasy trees (cardboard, colored paper, scissors, glue, markers). The students' worksheet uses metacognition questions presented in Figure 6, and their learning product, Keruing (*Dipterocarpus spp.*) tree fantasy, presented in Figure 7.

The homogeneity test uses Levene's test result on the mathematics achievement of  $F = 6.276$ , at significance of  $p = 0.15$  ( $df = 1; 54, p > 52$ ), and motivation of  $F = 0.699$  at significance of  $p = 0.407$  ( $df = 1; 52, p > 0.05$ ). The result showed that mathematics achievement and motivation scores were homogeneous.

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1. Jumlah keloreng Indra dan Handi 60 butir, sedangkan selisih keloreng Indra dan Handi adalah 16 butir. Buatlah model matematika dengan sistem persamaan linear dua variabel untuk penyelesaiannya.  
Jawaban:  
 Apa simbol variabel dari keloreng Indra menurut idemu?  
  
 Apa simbol variabel dari keloreng Handi menurut idemu?
2. Bagaimana bentuk persamaan matematika dari kalimat "Jumlah keloreng Indra dan Handi 60 butir? Coba tuliskan!  
Jawaban:
3. Bagaimana bentuk persamaan matematika dari kalimat "Sedangkan selisih keloreng Indra dan Handi adalah 16 butir? Tuliskan!  
Jawaban:
4. Dengan bentuk persamaan matematika dari kedua kalimat di atas, sistem persamaan linear dua variabel untuk kedua persoalan tadi? Carilah banyak keloreng Indra dan Handi.  
Jawaban:
5. Jangan lupa, setelah menemukan banyak keloreng Indra dan Handi, periksa

**Figure 6.** The example mathematics problems on the worksheet and the students' work



**Figure 7.** Students' creation of a rare Kalimantan tree fantasy of Keruing (*Dipterocarpus spp.*)

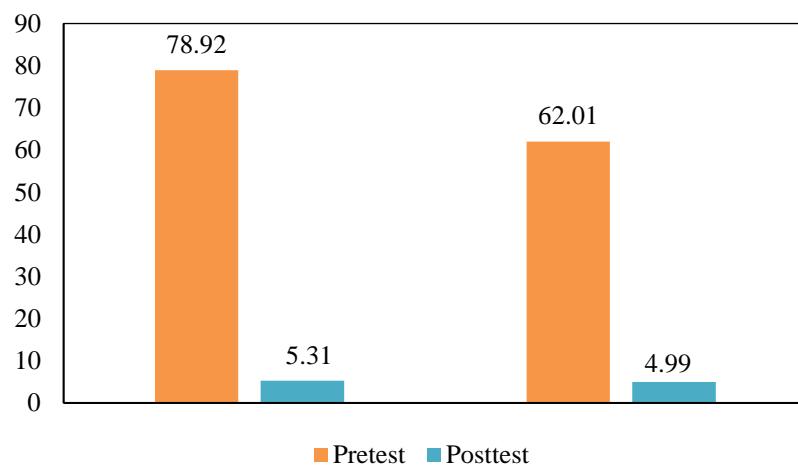
#### ***Homogeneity and Normality Assumption***

The normality test for mathematics achievement and motivation scores using the one-sample Kolmogorov-Smirnov (KS) test. The normality test of mathematics achievement scores of the experiment group of pretest with  $KS = 0.76$  at significance of

$p = 0.61$ , post-test with  $KS = 1.15$  at significance  $p = 0.15$  ( $df = 25$ ,  $p > 0.05$ ), and the control group of pretest with  $KS = 1.03$  significance at  $p = 0.24$ , post-test with  $KS = 1.19$  at significance at  $p = 0.12$  ( $df = 29$ ,  $p > 0.05$ ). The normality test of motivation scores of the experiment group of pre-test with  $KS = 0.69$  at significance of  $p = 0.73$ , post-test with  $KS = 1.15$  at significance  $p = 0.62$  ( $df = 25$ ,  $p > 0.05$ ) ( $df = 25$ ,  $p > 0.05$ ), and the control group of pre-test with  $KS = 0.91$  significance at  $p = 0.38$ , post-test with  $KS = 0.82$  at significance at  $p = 0.52$  ( $df = 29$ ,  $p > 0.05$ ). The results show that the mathematics achievement and motivation scores for both the experimental and control groups in the pretest-posttest fulfill the assumption of normality.

### ***The Effect of the Learning Model on Mathematics Learning Outcomes***

The average and standard deviation of mathematics learning outcomes are presented in Table 5 and Figure 8. The initial condition of students' abilities in the experimental and control groups, as indicated by the average and standard deviation of pretest scores, is relatively similar. However, the post-test shows a high increase in the average. Based on the increase in average scores, the N-Gain of the experimental group was 0.77, higher than the N-Gain of the control group of 0.60. The learning model of metacognition-metaphor of the rare Kalimantan tree, is more effective than the classical instruction. The N-Gain of the experimental group of students who learned using the metacognition-metaphor model of the rare Kalimantan tree. It means the mathematics learning outcome increased by 77%. The improvement in students' mathematics learning outcomes has exceeded the predictions of instructional technology experts (75%) but remains below those of mathematics education experts (85%).



**Figure 8.** Mean and N-Gain of pretest-posttest of mathematics learning outcomes

**Table 5.** The average and standard deviation values

Statistic	Experiment		Control	
	Pretest	Posttest	Pretest	Posttest
Mean	5.31	78.92	4.99	62.01
Std. Deviation	3.44	8.79	3.02	19.03

**Table 6.** The result of the analysis of the variance of mathematical learning outcomes

Tests of Between-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	4620.23 <sup>a</sup>	3	1540.08	6.87	.00
Intercept	60876.47	1	60876.47	271.37	.00
LM	967.89	1	967.89	4.32	.04
Pretest	776.23	1	776.23	3.46	.07
LM * Pretest	.52	1	.52	.00	.96
Error	11216.58	50	224.33		
Total	279195.07	54			
Corrected Total	15836.81	53			

a. R Squared = .29 (Adjusted R Squared = .25)

Note: LM = Learning Model

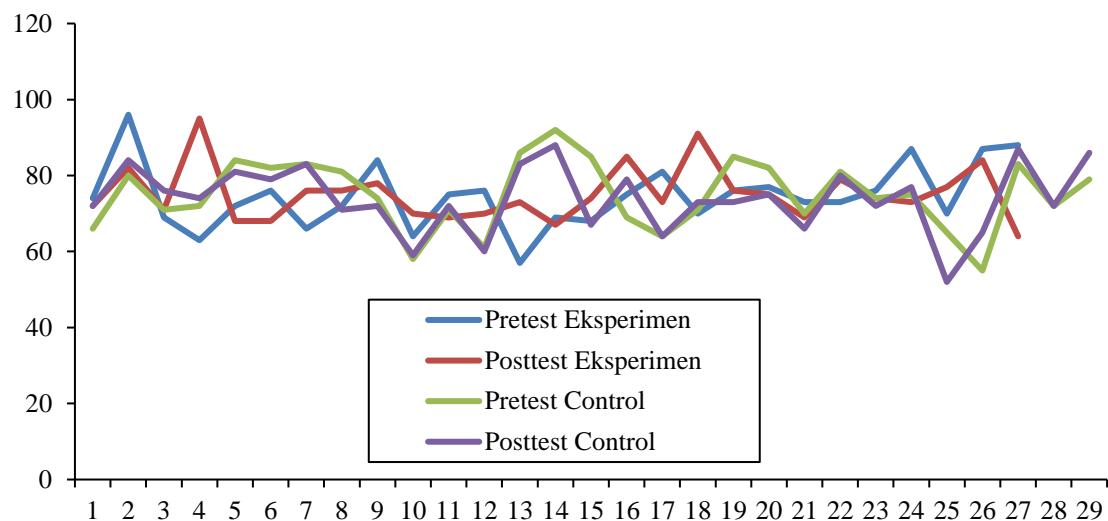
The effect of the learning model on mathematics learning outcomes, as analyzed using ANOVA, is presented in Table 12. The F coefficient in the LM (Learning Model) is 4.32 with a significant level of 0.04 ( $\alpha = 0.05$ ). This means that the metacognition-metaphor learning model of a rare Kalimantan tree is effective for mathematics learning outcomes. The use of metacognitive question strategies in the learning model is effective for mathematical learning outcomes. Providing metacognitive questions helps students choose the right mathematical rules or formulas to solve problems. Students are asked questions that encourage them to think about the relationships between ideas when explaining their answers to mathematical questions. The results of this research are in line with Tak et al. (2025), Yıldız & Öztürk (2023) that metacognition awareness mediates between attitude and mathematical reasoning. Kallio et al. (2018), Ohtani & Hisasaka (2018), and Tachie (2019) found that metacognition influences students' success in understanding mathematics, can be trained, and that asking questions triggers and self-regulation are important factors. García et al. (2016) stated that metacognition skills encourage students to acquire greater levels of learning control.

#### ***The Effect of the Learning Model on Motivation to Learn Mathematics***

The initial condition of the experimental group students' mathematics learning motivation, of the pretest score, was lower than the post-test score. Those scores show an increase in motivation before and after receiving treatment. Compared with the students in the control group, students' initial learning motivation shows a decrease. The average value and standard deviation of mathematics learning motivation scores are presented in Table 7.

**Table 7.** The average and standard deviation values

Statistic	Experiment		Control	
	Pretest	Posttest	Pretest	Posttest
N	25	25	29	29
Mean	74.59	75.15	74.86	73.86
Std.	8.52	7.24	9.15	8.76
Deviation				



**Figure 9.** Students' motivation for the pretest-posttest in the experiment and control groups

**Table 8.** The result of the analysis of variance

Tests of Between-Subjects Effects						
Dependent Variable: Motivation Posttest						
Source	Type III Sum of Squares	df	Mean Square	F	Sig.	
Corrected Model	1194.38 <sup>a</sup>	3	398.13	8.85	.00	
Intercept	1781.31	1	1781.31	39.59	.00	
LM	583.52	1	583.52	12.97	.00	
Motivation Pretest	482.05	1	482.05	10.71	.00	
LM * Motivation	562.48	1	562.48	12.50	.00	
Pretest						
Error	2339.61	52	44.99			
Total	314199.00	56				
Corrected Total	3533.98	55				

a. R Squared = .34 (Adjusted R Squared = .30)

Note: LM = Learning Model

The results of the Analysis of Variance in Table 8 show that the main effect of LM (Learning Model) on motivation is significant ( $F = 12.97$ ), and there is a significant interaction between initial motivation and the learning model ( $F = 12.50$ ;  $0,00 < 0.05$ ). The field validation results show that implementing the learning model in the school context fosters motivation. The finding that the interaction between initial motivation and the learning model significantly influences student motivation during learning. The metacognition-metaphor learning model of rare Kalimantan trees fosters awareness and increases motivation during mathematics learning. The initial motivation influences the growth of students' motivation improvement during the learning process. The metacognition-metaphor learning model of the rare Kalimantan tree is an external factor that fosters intrinsic motivation and eliminates the struggle in mathematics learning. This finding aligns with Lo et al. (2022), who found that motivation and learning experience

are key factors in determining cognitive outcomes. Learning experiences will occur through the use of learning models that contain components of motivational strengthening strategies.

The model of metacognition-metaphor of the rare Kalimantan tree, developed through integrated hands-on activities, and the metaphor of the growth of the rare Kalimantan tree, integrate to strengthen students' motivation. This learning model serves as a mediator of students' motivation during their learning of mathematics. The initial motivation can persist through hands-on activity and serve as a metaphor for the learning slogan of Kalimantan's rare trees. This finding is supported by Huang, Spector & Yang (2020); Weigand, Trgalova & Tabach (2024); and Tambunan, Sinaga & Widada (2021), who found that hands-on activities make a strong contribution to student motivation. Scheiner et al. (2022) recommend using metaphors to reconstruct mathematical learning in some national contexts. Hendriana et al. (2022) used ethnometaphors to mediate students' understanding of mathematics, as metaphors grounded in local knowledge enable students to connect mathematics to their daily lives.

The field validation results show that this learning model can reinforce mathematics mastery and motivation to learn mathematics. This result aligns with Demitra & Dewi (2021), who used metacognitive questions, metaphors, and hands-on fantasy tree creation to increase students' interest in studying analytical trigonometry. Teachers can use metacognitive questioning to guide students' thinking as they solve mathematical problems. When the students feel boring during they solve the mathematical problems, teacher should be do something such as (a) invite students to study the growth of rare Kalimantan Trees in groups watching the Kalimantan rara tree, (b) motivates students by inviting them to create metaphors for learning mathematics mottos based on the growth of rare Kalimantan trees, (c) motivate students to solve as many math problems as possible and create Kalimantan fantasy trees with hands-on activities.

## **Limitation**

Overall, the validation of the metacognitive learning model using the rare Kalimantan tree metaphor has successfully demonstrated its effectiveness for mathematics learning outcomes. However, this study has limitations: while analyzing mathematics learning achievement data, it did not specifically examine improvements in learning outcomes according to competency achievement indicators for the two-variable linear equation system. Consequently, the effectiveness of learning cannot be explained by the indicators used to assess the learning model.

## **CONCLUSION**

The learning model of the metacognitive metaphor of a rare Kalimantan tree was developed to support mathematical reasoning and maintain learning motivation. The biggest challenges in learning mathematics are weak reasoning and low motivation. Development of the learning model of metacognition-metaphor Kalimantan rare trees, starting from the single path prototype, alpha version, and reaching the beta version. In the first stage, the learning model was developed as a single-path prototype. The learning model comprised three components: metacognitive questions, metaphors for rare Kalimantan trees, and a hands-on activity to create a fantasy tree, which was still in rough draft form.

The learning model has reached an alpha version through expert and small-group validation. The results of organizing the three strategy components through expert review were considered very effective in the metacognitive question strategy component and the hands-on activity integrated with metaphors. The results of the small-group validation showed that the learning syntax was easy to implement in the classroom. The validation results yielded a learning model that reached the alpha stage, with effective learning strategies and implementable in the context of classroom mathematics learning.

The learning model field validation through testing its effect on mathematics learning outcomes and motivation in the school context. The field validation proved the learning model is effective for mathematics learning outcomes and motivation. The prior motivation-learning model interaction is effective in fostering motivation during students' learning. The model has reached the beta version.

This research has contributed to improving students' reasoning and motivation in mathematics learning. Theoretically, that implies the development of a mathematics learning model that uses elements of indigenous knowledge and integrates them with components of strategy metaphors and hands-on activities. The suggestion for future research is to study the effect of the learning model on other learning indicators, such as the content of mathematics, interest, and resiliency, and to analyze in depth the formation of mathematical reasoning.

Suggestions for teachers in conducting mathematics instruction include asking metacognitive questions to facilitate students' reasoning. The metacognition-metaphor learning model of rare Kalimantan trees can be used to teach students with low motivation or weak reasoning skills in mathematics. This model can be integrated with other learning models, such as problem-based or project-based learning, or with a deep learning approach. These models of learning have no way to strengthen students' motivation and reasoning in mathematics. In this case, the metacognition-metaphor learning model of Kalimantan's rare trees serves as an approach to more effectively support problem- or project-based learning than before.

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