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Guided Discovery Learning Integrated with Numbered Heads Together: Effects on Mathematical Problem-Solving and Student Engagement

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Abstract: This study aims to determine the effectiveness of the Guided Discovery Learning (GDL) model integrated with the Numbered Heads Together (NHT) cooperative approach in improving students' mathematical problem-solving ability and student engagement. Effectiveness in this research is operationally defined as a statistically and practically significant improvement in students' scores and engagement compared to conventional learning. This research employed a quasi-experimental design with a nonequivalent control group. The participants were three intact eighth-grade classes at SMPN 1 Jetis Bantul, selected via cluster random sampling. Class VIII A was taught using GDL combined with NHT, class VIII D used GDL without group work, and class VIII B used conventional learning. Data were collected through a mathematical problem-solving test and a student engagement scale, both validated by experts and tested for reliability. Data analysis used One-Way ANOVA followed by a Tukey HSD post hoc test and Kruskal–Wallis to determine significant mean differences among the three groups, complemented by effect size interpretation to assess the magnitude of differences. The results revealed that the GDL with the NHT model was significantly more effective than conventional learning in improving students' mathematical problem-solving ability ($p < 0.05$, Cohen's $d = 0.86$). However, there was no statistically significant difference between the GDL+NHT and GDL-only groups, indicating that adding NHT did not confer a significant cognitive benefit beyond the GDL model alone. In contrast, the NHT component meaningfully enhanced students' engagement, particularly in participatory aspects such as oral, visual, and motor engagement, compared to both the GDL-only and conventional groups. The GDL model, either with or without NHT, effectively enhances students' mathematical problem-solving ability and student engagement compared to conventional instruction. Future studies are recommended to examine this integration in broader contexts and longer durations to explore its potential for sustainable classroom engagement.

Keywords: effectiveness, problem-solving ability, student engagement, guided discovery learning model with numbered heads together, quasi-experimental design.

▪ INTRODUCTION

In the 21st century, mathematics education in schools faces complex challenges due to global demands and technological developments, which require students to master higher-order thinking and problem-solving skills (Maulida & Trimurtini, 2024). One of the global educational priorities is achieving Sustainable Development Goal 4 (SDG 4), which emphasizes inclusive and quality education to improve learning outcomes for all (Tasliah et al., 2024). Quality education by 2030 requires specific skills, including mathematical problem-solving. Quality education in the SDG context requires students to possess strong mathematical problem-solving skills as part of critical thinking competencies needed to face global challenges (Lafuente-Lechuga et al., 2020). Problem-solving ability is an important skill that enables students to apply mathematical knowledge in various real-world contexts (Siswanto & Meiliasari, 2024). This aligns with the view of Manurung et al. (2022) that problem-solving should be developed and taught in schools as part of meaningful mathematics education. The low level of students'

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mathematical problem-solving ability is often related to classroom learning that focuses only on delivering material and exercises, without providing opportunities for students to explore concepts independently (Eismawati et al., 2019)

The low problem-solving ability of students is evident in the results of the 2022 PISA survey. PISA results in 2022 showed that Indonesia ranked 66th out of 81 countries participating in the PISA test in mathematics with an average score of 366 (OECD, 2023). Previous research also revealed that students' mathematical problem-solving skills were low, as evidenced by their inability to meet the indicators of problem-solving ability and a lack of interest in solving mathematical problems (Laila et al., 2021). This research shows that mathematics learning results in suboptimal problem-solving skills. Low problem-solving ability is a problem in probability material, as students often do not understand how to solve them (Mutiah et al., 2023). Research conducted by Putri et al. (2024) shows that students have difficulty solving problems, especially in probability material. The cause of students' difficulty solving mathematical problems is that they are still confused about interpreting the story.

Probability material is essential to master because, in everyday life, there are events with uncertain outcomes that help inform decision-making (Anggrayni et al., 2021). The low problem-solving ability in probability material contradicts the importance of mastering this material for students (Irwana et al., 2023). Given the importance of mastering probability material, students need strong mathematical problem-solving skills. Learning mathematics not only requires cognitive abilities, but also requires affective or attitudinal abilities. One form of attitude or affective domain is student engagement. Student engagement manifests through active involvement, which Kanza et al. (2020) describe as physical or mental activities designed to emphasize understanding of problems encountered in the learning process. This is supported by the opinion of Talelu et al. (2022) that student activities in mathematics learning are essential for creating an interactive and active learning process, thereby achieving maximum learning outcomes. The habit of students who are less active in learning also leads to lower problem-solving skills (Sriwahyuni & Maryati, 2022). Therefore, student engagement in learning mathematics needs to be improved to develop high problem-solving skills.

Problem-solving ability and student engagement both depend on the classroom learning process, which is influenced by how the teacher delivers the material. Mathematics learning is often considered effective when the material is explained as a whole, followed by practice problems, but this approach makes students less active. This type of mathematics learning is called an expository approach. Expository learning makes students less active in the learning process because it remains teacher-centered. This makes students less skilled in math. Meanwhile, in the 21st century, students are expected to possess seven skills, including collaboration, communication, creativity, critical thinking, information literacy, problem-solving, and socio-emotional skills (Varod et al., 2019). So that with these demands, it is necessary for students to face problems in life today or in the future. This situation highlights the need for innovative learning approaches that emphasize discovery and active participation in problem-solving.

Problems and challenges must be faced with careful preparation, namely, with meaningful learning, namely, the need to choose an appropriate learning model. The discovery learning model is an innovative approach that teachers can use to develop problem-solving skills and encourage active student participation. There are two types of discovery learning models: free discovery and guided discovery. The Guided Discovery Learning (GDL) model is one of the recommended approaches rooted in constructivist theory. In the context of learning, the guided discovery learning model is more suitable for helping students who are not used to learning independently. In addition, this model can help students develop their problem-solving skills through structured teacher support. According to Rahman et al. (2021), GDL helps students build understanding through teacher-guided exploration. Maharani and Kusumah (2022) also revealed that GDL can improve students' reasoning and representation skills in mathematics.

The application of the guided discovery learning model is expected to foster students' independent learning, leveraging its advantages. However, the guided discovery learning model also has disadvantages that can hinder the learning process. This will affect learning success if not handled properly. It is therefore necessary to consider guided discovery learning, with the teacher as a facilitator, to minimize these shortcomings. However, several studies reported that discovery learning sometimes lacks collaborative interaction and student engagement during group tasks (Hwang & Chen, 2021). As a result, some students tend to be passive or rely on more dominant peers, causing uneven participation. In addition, teachers also need to collaborate with cooperative or group-based learning methods. However, the selection of the group method needs to be considered so that all students are actively involved and able to take responsibility for themselves and their group members.

A method that can support the learning process is Numbered Head Together (NHT). Numbered Head Together is a type of cooperative learning that emphasizes a specific structure to influence student interaction patterns and improve academic mastery (Hosnan, 2014). Learning with NHT encourages students to be active in solving problems together, so that students do not perceive various questions as heavy (Erfan et al., 2020). Problem-solving occurs in group discussions, and students play a central role in learning. Students have the freedom to express opinions, exchange ideas, and actively discuss to find solutions to problems. Each group is responsible for its members, as each member is assigned a number to encourage all students to try to solve the existing problems.

To overcome this limitation, the GDL model can be strengthened by integrating cooperative learning strategies such as NHT. NHT allows students to discuss and share ideas in small groups before presenting answers, thereby encouraging active participation and mutual responsibility among members (Kurniawan et al., 2020). Several international studies (Alhazmi & Nyeem, 2022; Hwang & Chen, 2021) also highlight that cooperative structures like NHT enhance students' motivation and social accountability, leading to better engagement and learning outcomes.

From a theoretical perspective, the integration of GDL and NHT aligns with Vygotsky's social constructivism theory, especially the concept of the Zone of Proximal Development (ZPD). Within this framework, learning is most effective when students receive scaffolding through peer interaction and teacher guidance. The discovery process in GDL provides the cognitive challenge needed for deeper understanding, while NHT supplies the social scaffolding necessary for cooperative knowledge construction. Thus, the combination of GDL and NHT is expected to promote both individual reasoning and group participation simultaneously. Empirical evidence regarding this integration, however, remains limited in the Indonesian context. Previous studies mainly examined GDL or NHT separately, and few have investigated their combined effects on problem-solving and engagement in junior high school mathematics. In addition, the effectiveness of GDL+NHT compared to GDL alone has not been consistently proven. Hence, this research aims to fill this gap by examining whether combining GDL and NHT provides a statistically and practically significant advantage over applying GDL without cooperative grouping.

This study was conducted at SMPN 1 Jetis, Bantul, involving three groups: one taught using GDL+NHT, one taught using GDL individually, and one taught using conventional instruction. The focus was on analyzing improvements in mathematical problem-solving and student engagement as indicators of instructional effectiveness. The results are expected to contribute not only to empirical evidence on the integration of discovery-based and cooperative learning but also to practical insights for teachers seeking to enhance classroom engagement and reasoning. Based on the background above, the research questions of this study are formulated as follows:

RQ1: Is the Guided Discovery Learning (GDL) model integrated with Numbered Heads Together (NHT) more effective than conventional learning in improving students' mathematical problem-solving ability?
RQ2: Is the Guided Discovery Learning (GDL) model integrated with Numbered Heads Together (NHT) more effective than conventional learning in improving students' engagement?

▪ METHOD

Participants

The participants of this study were eighth-grade students from SMP Negeri 1 Jetis Bantul. The school is located in a semi-urban area, and the students generally have medium academic performance and moderate learning motivation. The population consisted of all students enrolled in the 2024/2025 academic year. Three intact classes were selected through cluster random sampling to minimize selection bias. Class VIII A was designated as the experimental group I (GDL combined with NHT), class VIII D as the experimental group II (GDL without group work), and class VIII B as the control group (conventional learning). A total of 96 students participated, with similar average mathematical abilities as indicated by the school placement test results.

Research Design

This research is a quasi-experimental research with a nonequivalent control group design. According to Sugiyono (2019), the nonequivalent control group design is a research design in which the experimental and control groups are not randomly selected. The independent variable in this study is the Guided Discovery Learning (GDL) model with Numbered Head Together (NHT), and the dependent variables are problem-solving ability and students' engagement. The design can be illustrated as follows:

Table 1. Research design

Group	Pretest - Prescale	Treatment	Posttest-Postscale
Experimental 1	O_1	X_1 (GDL+NHT)	O_2
Experimental 2	O_3	X_2 (GDL only)	O_4
Control	O_5	X_3 (Conventional Learning)	O_6

The learning process was conducted over four meetings (each 2 × 40 minutes) following the school's schedule. Each group was taught the same mathematical topic, namely "Line Tangents to Circles." The experimental class used the GDL model integrated with NHT cooperative learning; the experimental class II used GDL individually; and the control group used direct instruction. The learning activities in the GDL stages consisted of problem orientation, data collection, hypothesis formulation, verification, and generalization. In the GDL+NHT group, students discussed each step collaboratively, while in the GDL-only group, they worked individually with teacher guidance. The teacher provided scaffolding by asking guiding questions, clarifying misconceptions, and individually confirming students' reasoning to ensure instructional equivalence with the GDL+NHT group. The researcher acted as the classroom teacher, while an external observer monitored the learning process to minimize bias. Pretests and posttests were administered before and after treatment to assess improvements in problem-solving ability and student engagement.

Instrument

The instruments used in this study were problem-solving ability test questions, consisting of a pretest and posttest of 4 questions each, and a student engagement scale of 25 statements. The problem-solving test consisted of 4 open-ended items covering indicators based on Polya's stages: understanding the problem, devising a plan, carrying out the plan, and reviewing the result. The student engagement observation sheet consisted of 25 items grouped into eight dimensions: visual, oral, listening, writing, drawing, motor, mental, and emotional activity. In this study, the KPM test questions and student engagement scale were consulted with mathematics education lecturers and eighth-grade mathematics teachers to determine their validity. The results of the assessment indicated that the math problem-solving ability test questions and the student engagement scale were valid, allowing the instrument to be tested for reliability. The reliability trial was conducted in class IX, which had taken the opportunity material in class VIII. After calculating the reliability of the problem-solving ability test instrument and the student engagement scale, the Cronbach's Alpha values for the pretest and post-test questions on mathematical problem-solving ability were 0.621 and 0.740, respectively, based on 4 questions. The Cronbach's Alpha results on the student engagement scale (prescale and postscale) were 0.769 and 0.635, respectively, from 25 statements. All instruments used show Cronbach's Alpha > 0.60, indicating reliability and suitability for this study.

Data Analysis

The data analysis technique used in this research was the mean difference test. To analyze improvement in mathematical problem-solving ability, N-gain was used, while gain scores were analyzed for student engagement. The testing technique uses a one-way ANOVA followed by a Tukey HSD post hoc test to identify specific group differences. The test was conducted to determine whether the GDL learning model with NHT was effective in mathematical problem-solving skills and student engagement. The One-Way ANOVA test can be used for a normally distributed population with homogeneous variance. However, if the data are not normally distributed or homogeneous, they can be analyzed using a nonparametric test. The nonparametric test to use is the Kruskal-Wallis test. Based on this statement, before conducting the One-Way ANOVA test, it is necessary first to conduct prerequisite analyses of the research data, namely normality and homogeneity tests.

▪ RESULT AND DISSCUSSION

Implementation of Learning Using Guided Discovery Learning (GDL) Model with Numbered Head Together (NHT)

The Guided Discovery Learning (GDL) model, combined with Numbered Head Together (NHT), was implemented in experimental class 1, with the researcher serving as the teacher. The learning process began with a pretest on mathematical problem-solving skills and a pre-scale of student engagement, followed by four learning sessions, and concluded with a posttest and post-scale. At the beginning of each lesson, the teacher greeted the students, checked attendance, explained the learning objectives, and introduced the steps of the GDL model combined with NHT. Students were divided into six heterogeneous groups consisting of 4–6 members, and each member was assigned a code (e.g., 1A, 1B) to ensure accountability, since any student could be randomly selected to present the group's work.

The six stages of GDL were integrated with NHT activities. During the stimulation stage, students observed contextual probability problems presented in the worksheets (e.g., dice games and wheel-of-fortune-style games). During the problem statement stage, students identified the given problems, while the teacher provided guiding questions when necessary. During data collection and processing, students gathered information, discussed it within their groups, and analyzed it to develop relevant concepts and strategies. The NHT structure encouraged all members to participate actively, as every student was responsible for understanding the group's findings. In the verification stage, students tested their findings on similar problems, while in the generalization stage, they summarized their conclusions and wrote them on the worksheet. The teacher then randomly drew student codes, and the selected students presented their group's results. Other students were encouraged to provide feedback or ask questions, and the teacher offered clarification and reinforcement of key concepts.

During the first meeting, several challenges were observed. Some students, who were more accustomed to teacher-centered instruction, initially waited for direct explanations instead of engaging in discovery activities. However, through consistent guidance and encouragement, they gradually became more confident in expressing ideas and participating in group discussions. The NHT structure played a crucial role in ensuring equal participation, since the possibility of being called to present motivated students to remain attentive and responsible.

As the sessions progressed, the classroom atmosphere became increasingly interactive and collaborative. Students who were usually passive in conventional lessons began to ask questions, share opinions, and respond to their peers' explanations. This indicates that the combination of GDL and NHT not only supported cognitive aspects of problem-solving but also nurtured social skills such as cooperation, responsibility, and communication. The observations confirmed that when discovery learning was reinforced with cooperative interaction, student engagement and problem-solving performance improved significantly. These observations are consistent with Vygotsky's social constructivist principles, in which peer interaction serves as scaffolding for cognitive development. The cooperative structure of NHT allowed social mediation to strengthen conceptual understanding during discovery activities.

Overall, the implementation of the GDL model with NHT in experimental class 1 proceeded effectively, although students initially needed time to adjust to discovery-based learning. With sufficient guidance, they became more engaged, active in discussions, and responsible for their group's understanding. The implementation was consistent with the designed teaching modules and validated by observation, indicating that the combination of GDL and NHT effectively enhanced both student engagement and mathematical problem-solving skills. This practical observation confirms that structured guidance and social collaboration create a balanced learning environment where both cognitive and affective goals can be achieved (Hwang & Chen, 2021; Alhazmi & Nyeem, 2022).

Problem-Solving Skills

Based on calculations from students' test results on mathematical problem-solving skills in experimental class 1, experimental class 2, and the control class, the results of the data analysis are as follows.

Table 2. Description of data on test results of mathematical problem-solving skills

Class	Data Description	*Pretest*	*Posttest*	*N-gain*	*Effet Size*		
					EC1	***EC2***	***CC***
Experiment Class 1	N valid	32	32	32	-	0.336 (small)	1.078 (large)
	Mean	28.711	70.312	0.581			
	Minimum	0.00	31.25	0.000			
	Maximum	50.00	93.75	0.909			
	Std. Deviation	10.513	16.574	0.248			
Experiment Class 2	N valid	32	32	32	0.336 (small)	-	0.89 (large)
	Mean	33.984	67.578	0.508			
	Minimum	0,00	37.50	0.000			
	Maximum	68.75	93.75	0.833			
	Std. Deviation	19.760	15.013	0.181			
Control Class	N valid	32	32	32	1.078 (large)	0.89 (large)	-
	Mean	17.969	45.703	0.346			
	Minimum	0.00	18.75	0.000			
	Maximum	37.50	81.25	0.727			
	Std. Deviation	13.258	18.338	0.183			

Table 2 presents the descriptive statistics of students' mathematical problem-solving skills. To provide a clearer visualization of these results, the comparison of mean pretest and posttest scores among the three groups is presented in Figure 1. Figure 1 illustrates the comparison of mean pretest and posttest scores across the three groups, showing the improvement trend driven by different learning models.

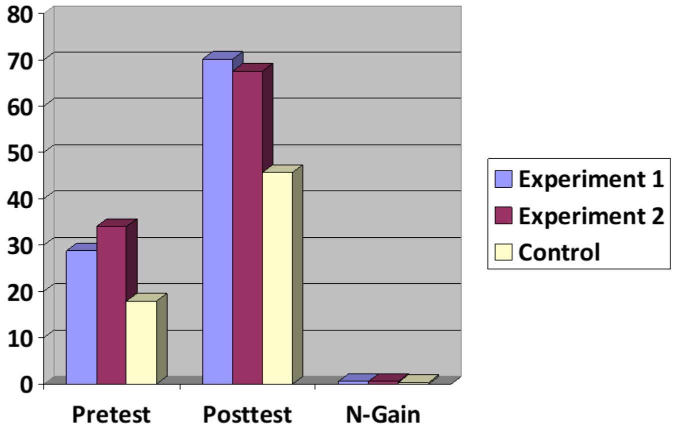

Figure 1. Comparison of average scores of mathematical problem-solving skills

As shown in Figure 1, the mean posttest score of the GDL + NHT group was the highest, followed by the GDL-only and control groups. This indicates that discovery-based learning, particularly when integrated with cooperative strategies, produced greater improvement in students' mathematical problem-solving skills. The visual trend also confirms the large effect size reported in the statistical analysis, emphasizing the practical significance of combining GDL and NHT to enhance students' reasoning and conceptual understanding. The graph shows that students in the GDL+NHT group achieved the highest posttest mean scores, followed by the GDL-only and control groups. This upward trend demonstrates that discovery-based instruction, particularly when supported by cooperative interaction, substantially improved students' mathematical reasoning and conceptual understanding.

The normality test of the pretest data on problem-solving ability indicates that the data are not normally distributed. As a result, the homogeneity test of the pretest data on problem-solving ability for the experimental class 1, the experimental class 2, and the control class cannot be performed. Therefore, the pretest analysis used a nonparametric test, namely the Kruskal-Wallis test, which showed a difference in the group means (sig. = 0.003 < 0.05).

The normality test of N-gain data for problem-solving ability indicates that the data are not normally distributed. As a result, the homogeneity test of N-gain data of problem-solving ability cannot be done. Therefore, the N-gain analysis used a nonparametric test, namely the Kruskal-Wallis test, which showed a difference in means between groups (sig. = 0.000 < 0.05). This finding statistically confirms that the type of learning model used had a meaningful effect on problem-solving ability across groups. So it can be concluded that there are at least two average N-gain scores that differ across students' mathematical problem-solving skills in the experimental class I, the experimental class 2, and the control class. It can be said that learning using the Guided Discovery Learning model, the Numbered Head Together learning model, and the conventional learning model shows a significant difference in N-gain scores for students' maalign with Langi' et al. (2023), who found that the guided discovery learning model is effective for students' mathematical problem-solving skills, and with Silitonga (2024), who found that numbered heads together improvesconcluded that between the experimental class 1, which is given the treatment, namely the GDL learning model with NHT, and the control class, which is given the treatment, namely the conventional model, there is a significant difference (Sig.). which is 0.000, then H_0 is rejected. This means that the guided discovery learning model with numbered heads together is more effective than the conventional model in improving students' mathematical problem-solving ability. These results are in line with the results of research by Langi' et al. (2023), namely, the guided discovery learning model is effective on students' mathematical problem-solving skills, and in line with research by Silitonga (2024), namely, numbered heads together influences improving students' mathematical problem-solving skills.

The use of the guided discovery learning model with numbered heads in this study runs quite effectively. Using the guided discovery learning model with numbered heads together involves students actively in their learning activities. The improvement is supported by the GDL structure, which allows learners to build their knowledge through a guided discovery process. At the same time, NHT provides students with space to collaborate and take an active role in shaping group outcomes. This aligns with the theoretical perspective of Kirschner et al. (2006), who argue that guided instruction provides the necessary structure for effective problem-solving and, when integrated with cooperative learning, enhances both cognitive processing and social support.

The findings indicate that integrating cooperative learning into the discovery framework enables students to construct knowledge through guided exploration while maintaining active engagement and accountability (Slavin, 2019). These results are consistent with studies by Hwang & Chen (2021) and Alhazmi & Nyeem (2022), which reported that combining discovery and cooperation enhances students' reasoning and engagement in mathematics.

Theoretically, this result is in accordance with Bruner's view in discovery learning theory, emphasizing students' active involvement in exploring concepts, and with Vygotsky's idea that social interaction scaffolds cognitive development. The NHT structure encouraged peer dialogue and shared responsibility, while GDL guided students toward systematic reasoning. Meanwhile, NHT, as a cooperative strategy, creates learning conditions that encourage full participation by all group members. When students are assigned responsibilities through numbering, they are more motivated to understand and solve problems because each individual is potentially appointed to represent the group. This finding aligns with the research of Sunita et al. (2021) and Riansyah et al. (2023), which showed that NHT can increase individual responsibility in group work, thereby enhancing problem-solving skills.

The effectiveness of this model is inseparable from the characteristics of GDL, which emphasize students' active involvement in concept discovery under teacher guidance, and from the NHT structure, which encourages interaction and individual responsibility within the group. In learning, students are guided through problem-based LKPD to explore the concept of opportunity and apply problem-solving steps systematically. Structured group discussion through member numbering in NHT ensures that each student is active and responsible, ultimately improving conceptual understanding and critical thinking skills.

These findings support the notion that meaningful learning occurs when learners interact socially to negotiate meaning and receive feedback from peers (Vygotsky, 1978). In the present study, this synergy allowed students to better understand probabilistic reasoning and apply problem-solving steps more effectively. In contrast, the control class using the conventional approach showed lower improvement. Teacher-centered learning tends to limit students' exploration of mathematical concepts, thereby supporting less the development of higher-order thinking skills. This aligns with the findings of Pratiwi et al. (2023), who stated that the conventional approach is less effective at improving problem-solving skills due to a lack of student involvement.

Based on the results of the Post Hoc test, it can be concluded that the experimental class 2, which is given the GDL learning model, differs significantly from the control class, which is given the conventional model, H_0with a p-value of 0.008; thus, H0 is rejected. This means that the guided discovery learning model is more effective than the conventional model in improving students' mathematical problem-solving skills.

The effectiveness is evident in students' success in solving problems through the six stages of GDL: stimulation, problem statement, data collection, data processing, verification, and generalization. Although not done in groups, the learning process remains active and learner-centered. The stages of GDL were found to align with Polya's problem-solving indicators, including understanding the problem, planning strategies, implementing strategies, and re-examining results. This finding is supported by Putri's (2025) research, which confirms that each stage of GDL contributes to indicators of problem-solving skills.

The Guided Discovery Learning model encourages learners to think independently, build understanding through guided discovery, and formulate hypotheses naturally (Rini et al., 2021; Mutmainnah, 2020). Compared to conventional learning, which tends to be teacher-centered and lacks interaction, GDL creates a meaningful learning environment and fosters the exploration of learners' ideas. This explains why students' problem-solving ability in the experimental class 2 improved more than in the control class. Thus, the GDL model is proven to be a more effective learning strategy for improving students' mathematical problem-solving skills, especially in probability.

Based on the results of the Post Hoc test, it can be concluded that between the experimental class 1, which is given the treatment, namely the GDL learning model with NHT, and the experimental class 2, which is given the treatment, namely the GDL learning model, there is a significant difference (Sig.). which is 0.676, then H_0 fails to be rejected. This means that using the guided discovery learning model with numbered heads together is not more effective than the guided discovery learning model for students' mathematical problem-solving skills.

Although there are differences in the delivery technique of experimental class 1, using group discussion through NHT, while experimental class 2 is done individually, both still apply the six core stages of the GDL model consistently. This is in line with the findings of Prasetyowati et al. (2022); Aprilia et al. (2024); Indriati & Siagian (2024), who stated that learning outcomes were not significantly different as long as the GDL stages were fully implemented. The NHT method focuses on group cooperation and responsibility, while individual learning allows for more independent exploration of ideas. However, they both hone problem-solving skills according to Polya's indicators, such as understanding, planning, implementing, and reviewing. This finding is reinforced by Rahayu et al. (2018), who showed that cooperative models such as NHT did not always yield greater improvements than other models. Thus, the difference in approach (group or individual) in applying GDL does not have a significant effect on students' mathematical problem-solving skills.

Based on the results of the Kruskal-Wallis test and the Post Hoc test, it is known that experimental class 1 has a significantly higher average math problem-solving ability than the control class, but is not significantly different from experimental class 2. Therefore, based on the definition of effectiveness used in this study, the Guided Discovery Learning model with Numbered Head Together is not effective at improving mathematics problem-solving skills because it does not outperform the two comparison groups.

Nevertheless, observations during the learning process showed a change in students' behavior when solving mathematical problems, especially in experimental class 1. Based on observation sheets, students in experimental class 1 showed progress in their thinking when faced with more structured problem-solving tasks, especially in understanding the problem and evaluating the results of the solution. Learners in experimental class 1 gradually became able to identify important information from the problem independently, propose a solution strategy through group discussions, and demonstrate a reflective attitude by rechecking the results of their solutions.

Compared to the experimental class 2, these behavioral changes were not always reflected in higher test scores, but were evident in how learners discussed, responded to problem challenges, and solved problems collaboratively. The NHT method in experimental class 1 provided more space for learners to share strategies and check each other's understanding, thereby fostering more active and open mathematical thinking. However, it did not have a significant impact on learners' final problem-solving results.

Although the GDL+NHT group did not significantly outperform the GDL-only group, the cooperative setting promoted higher engagement and peer regulation. This suggests that cognitive benefits from GDL are comparable across individual and group settings, but NHT provides additional socio-emotional and motivational advantages (Gillies, 2016; Sunita et al., 2021). Hence, the integration primarily strengthens students' participation rather than producing drastically higher cognitive scores. From classroom observations, it was evident that the GDL+NHT environment fostered more reflective problem-solving behaviors, particularly in understanding and evaluating steps. Even without significant numerical differences, the observable improvement in collaborative reasoning demonstrates the practical relevance of integrating discovery and cooperative learning. Beyond cognitive outcomes, this study also examined students' engagement levels as an affective dimension of learning effectiveness, presented in the following section.

Student Engagement

Based on calculations from the student engagement scale results in experimental class 1, experimental class 2, and the control class, the results of the data analysis are as follows.

Table 3. Description of data on student engagement results

Class	Data Description	*Prescale*	*Postscale*	*Gain*	*Effet Size*		
					EC1	*EC2*	*CC*
Experiment Class 1	N valid	32	32	32	-	1.299 (large)	1.236 (large)
	Mean	62.981	71.915	8.934			
	Minimum	45.624	50.178	-0.935			

	Maximum	82.485	101.114	21.398			
	Std. Deviation	10.142	11.226	6.175			
Experiment Class 2	N valid	32	32	32	1.299 (large)	-	0.034 (trivial)
	Mean	70.651	70.926	0.275			
	Minimum	52.087	49.236	-13.072			
	Maximum	88.866	93.969	14.726			
	Std. Deviation	8.699	9.322	7.120			
Control Class	N valid	32	32	32	1..236 (large)	0.034 (trivial)	-
	Mean	69.200	69.214	0.014			
	Minimum	60.819	57.536	-16.366			
	Maximum	80.922	89.444	15.079			
	Std. Deviation	4.959	7.588	8.126			

Table 3 presents the descriptive data on student engagement. Figure 2 displays the mean prescale and postscale scores across the three groups to visualize changes in students' participation levels.

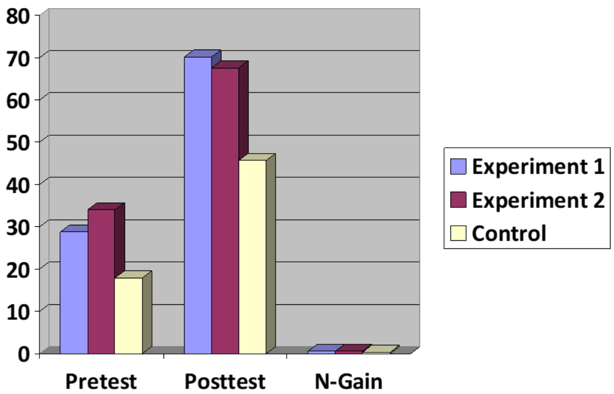

Figure 2. Comparison of average scores of student engagement

Figure 2 shows that students in the GDL + NHT group experienced the largest mean increase in student engagement, followed by the GDL-only group, while the control group showed minimal change. This visual evidence supports the quantitative results that integrating cooperative interaction within discovery learning significantly enhances classroom engagement and student participation.

Student engagement was analyzed across eight observed dimensions, namely visual, oral, listening, writing, drawing, motor, mental, and emotional. The post-scale mean scores for each dimension in the three treatment groups are presented in Figure 3. To illustrate the multidimensional profile of student engagement, the data were visualized using a Radar Chart (Figure 3). Each axis represents one dimension of student engagement, while the colored areas indicate the mean post-scale scores of each treatment group.

As shown in Figure 3, the GDL+NHT group (Experimental 1) exhibits the largest overall area on the radar chart, particularly on the oral, listening, and emotional dimensions. This suggests that integrating the Numbered Heads Together (NHT) cooperative strategy within the Guided Discovery Learning (GDL) model effectively

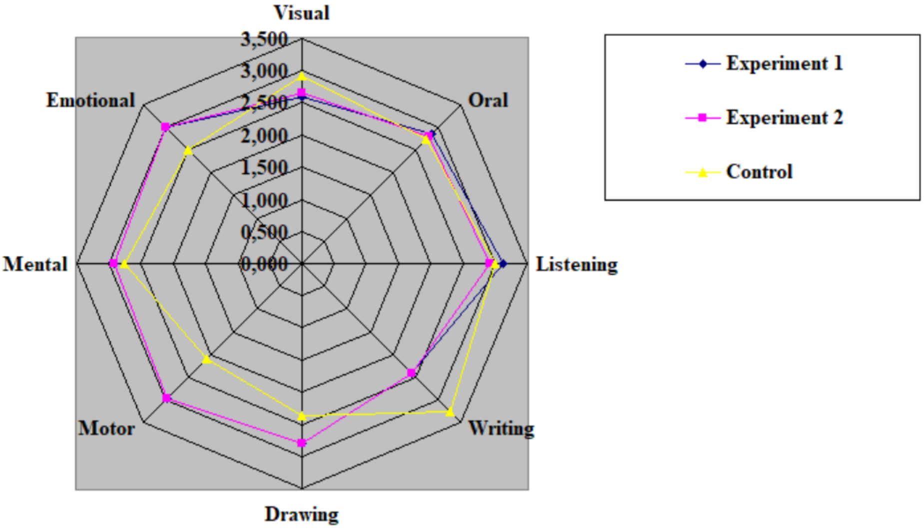

Figure 3. Radar chart of student engagement dimensions across treatment groups

enhances students' participatory and affective engagement through peer interaction and social accountability.

Meanwhile, the GDL group without group work (Experimental 2) shows moderate improvement in the mental and writing dimensions, reflecting cognitive involvement in problem-solving activities but without collaborative reinforcement. The control group shows relatively lower, more uneven scores across dimensions, with slightly higher results only in writing and listening, which typically occur in conventional learning settings focused on note-taking and teacher explanation.

These results indicate that the addition of the NHT component primarily strengthens socially interactive and emotional dimensions of student engagement, consistent with Vygotsky's social constructivist theory. According to this theoretical framework, peer interaction serves as scaffolding that supports learners' progress in the Zone of Proximal Development (ZPD).

Although statistical testing showed no significant difference between the GDL+NHT and GDL-only groups in problem-solving ability, the radar chart provides qualitative evidence that the GDL+NHT model promotes greater engagement in specific types particularly the participatory and emotional dimensions.

The Shapiro-Wilk test for normality of the prescale data on student engagement shows that the sig. The prescale scores for the three classes are all greater than 0.05. This means that the prescale data is normally distributed. Based on the Levene Test homogeneity test results for the prescale data on student engagement, the p-value is less than 0.05 (0.000). This means that the prescale data has an unequal or inhomogeneous variance. Therefore, the test performed is a nonparametric test of mean differences, namely the Kruskal-Wallis test, applied to the prescale data on student engagement.

Based on the results of the Kruskal-Wallis test, the difference in the average prescale data on student engagement across the three classes is 0.007, which is smaller than the significance level of 0.05, so H0 isH_0 rejected. This shows that there are at least two average prescale scores that differ in student engagement between the experimental classes (I and 2) and the control class. Based on the results of the normality test of the

Gain data on student engagement, it was found that the sig. The Gain scores for the three classes were all greater than 0.05. This means that the Gain data is normally distributed. Therefore, the next prerequisite test can proceed, namely the homogeneity test.

Based on the Levene test for homogeneity of variance for the Gain data on student engagement, the p-value is 0.2666, which is greater than 0.05. This means that the Gain data for student engagement have the same variance, or are homogeneous. Based on the normality and homogeneity tests, the Gain data for student engagement were normally distributed and homogeneously distributed. The results of the One-Way ANOVA test show that there is a significant difference in the student engagement gain score between the three classes (sig. = 0.000 < 0.05), so H_0 is rejected. This shows that the GDL learning model, along with NHT and conventional learning models, has different effects on student engagement. Furthermore, Tukey's Post Hoc test was conducted to determine pairs of classes that were significantly different.

Based on the Tukey test, class 1, which received the treatment GDL+NHT, differs significantly from the control class, which received the conventional model. The p-value is 0.000, so H0 is rejected. This means that the guided discovery learning model with numbered heads together is more effective than the conventional model for student engagement. This finding aligns with previous research indicating that the GDL and NHT models are effective in increasing student engagement (Janah et al., 2024; Daulay et al., 2023). The GDL model provides learners with opportunities to actively participate in the concept-discovery process. At the same time, NHT encourages group cooperation and individual responsibility, thereby increasing interaction and participation in learning.

In the discovery stage, learners actively read, pay close attention to the teacher's directions, and are more willing to ask questions when they encounter difficulties. Discussion activities in NHT also foster a collaborative atmosphere that encourages learners, including passive ones, to express opinions and take responsibility for their group's results. This aligns with the views of Fathoni (2022) and Astutik & Wulandari (2020), who emphasize that active student involvement at every stage of learning can increase engagement. In contrast, the conventional learning model in the control class is more passive and teacher-centered, leading to low student engagement. Thus, the GDL learning model with NHT is proven to provide a more active, collaborative learning environment and to encourage equal student involvement.

Based on the Tukey test, it can be concluded that the experimental class 2, which received the guided discovery learning model, differs significantly from the control class, which received the conventional model. H_0The p-value is 0.988, so H0 is accepted. This means that guided discovery learning models are not more effective than conventional learning models in increasing student engagement. This is reinforced by research by Putra (2015) and Rumana (2019), which state that GDL does not always produce higher engagement than conventional models.

In this study, although the experimental class 2 used worksheets that facilitated independent discovery, the lack of student interaction and limitations in classroom management led to suboptimal student engagement. Learners' involvement in learning is individualized, so those who are struggling tend to be passive. This aligns with the views of Markaban (2008) and Pranoto et al. (2017), who argue that individual GDLs are less able to meet all learners' needs, especially those who require social assistance or peer stimulation. In addition, Maryani et al. (2018) emphasized that social interaction is important in improving learning engagement. This difference in results also shows that the implementation of GDL individually has a different impact compared to GDL in groups, as in Janah et al. (2024), where discussion among learners also encouraged engagement. Thus, implementing GDL without group interaction tends to produce student engagement comparable to that in conventional learning and does not show a significant advantage.

Based on the Tukey test, it can be concluded that the experimental class 1, which is given the guided discovery learning model, has significantly more heads than the experimental class 2, which is given the guided discovery learning model. The p-value is 0.000, so H0 isH_0 rejected. This means that the guided discovery learning model with numbered heads together is more effective for student engagement than the model for student engagement. This finding is consistent with the research by Daulay et al. (2023), which shows that the NHT model increases student engagement.

One contributing factor is the social interaction and group discussion in learning within the NHT setting, which encourages students to actively express ideas, take responsibility for group work, and be directly involved in the problem-solving process. This aligns with the findings of Astutik & Wulandari (2020), who found that NHT can increase student engagement, especially during the discussion stage, which requires each group member's readiness and participation. These results highlight that cooperative learning structures, when integrated with guided discovery, can improve social participation and emotional involvement in learning (Gillies, 2016). The findings also reinforce Vygotsky's concept of the Zone of Proximal Development, where social scaffolding among peers supports not only cognitive but also affective growth.

In contrast, in experimental class 2, GDL was applied individually, so learners did not have space to interact or exchange ideas. Learners who have difficulty understanding concepts independently become less active, as noted by Markaban (2008) and Churniawan et al. (2020), who found that not all learners can follow GDL independently. The absence of social interaction also hinders clarification of understanding, which is usually achieved through discussion, as noted by Maryani et al. (2018), who stated that cooperative learning is more effective at encouraging engagement due to peer support.

Thus, combining the GDL model with NHT not only strengthens the cognitive aspects of learning but also creates a more active, collaborative learning environment, which significantly increases student engagement. Therefore, it can be concluded that the GDL with the NHT model is more effective than individual GDL in improving student engagement in learning mathematics. This integrated approach enhances both engagement and accountability, leading to more balanced and meaningful learning experiences (Hwang & Chen, 2021; Gillies, 2016).

Based on the results of the One-Way ANOVA test and continued with the Tukey test, it is known that experimental class 1 has a significantly higher average student engagement than experimental class 2 and the control class. Therefore, based on the definition of effectiveness used in this study, the Guided Discovery Learning model with Numbered Head Together can be deemed effective at increasing student engagement because it outperforms both comparison groups.

In addition to being inferred from the student engagement scale, this effectiveness is also reflected in changes in learner behavior during the learning process, as observed through the student engagement observation sheet. Observations were made at each meeting by the observer. In experimental class 1, active behavior increased from one meeting to the next. For example, students paid closer attention to the teacher and their friends during discussions, and they began to dare to answer questions and express opinions. In addition, writing activities also increased with active involvement in working on LKPD.

These behavioral changes were not as substantial as those observed in experimental class 2, which used the GDL learning model without NHT. This shows that NHT succeeded in encouraging interaction, social engagement, and individual responsibility in the group, thus strengthening overall student engagement. Therefore, this observational data reinforces the statistical results and shows that the GDL model with NHT not only increases the student engagement score but also changes learners' learning behavior to be more active and positive across various aspects. The limitations of this study include its short implementation period (4 meetings) and its focus on a single school, which may limit generalizability. However, the results still provide important implications for practice: mathematics teachers can integrate discovery and cooperative strategies to balance cognitive and social aspects of learning.

Overall, the findings indicate that integrating the Guided Discovery Learning model with Numbered Head Together offers a balanced instructional framework that supports both problem-solving and student engagement. While the addition of NHT did not yield a statistically significant advantage over GDL alone in cognitive performance, it meaningfully increased student participation and engagement. These results suggest that combining discovery and cooperative elements provides both cognitive depth and social scaffolding, making mathematics learning more interactive, reflective, and effective in the classroom context.

▪ CONCLUSION

Based on the findings of this study, two major conclusions can be drawn. First, the Guided Discovery Learning (GDL) model, either implemented individually or integrated with the Numbered Heads Together (NHT) approach, was proven to be more effective than conventional learning in improving students' mathematical problem-solving ability and student engagement. This indicates that discovery-based instruction allows students to construct their own understanding through exploration and teacher guidance, while cooperative structures, such as NHT, enhance motivation and participation.

Second, although the GDL + NHT class achieved the highest mean score, its advantage over GDL alone was not statistically significant. This suggests that while NHT strengthens interaction and engagement, it does not necessarily yield additional gains in problem-solving performance compared with GDL alone. However, the large effect size indicates that both discovery-based models have practical significance for improving students' mathematical reasoning and engagement.

The results have implications for classroom practice: teachers are encouraged to apply GDL as a foundation for discovery-based learning and, when possible, to integrate cooperative strategies, such as NHT, to foster student engagement. Future research should extend the duration of implementation, involve diverse school contexts, and explore other cooperative models to further understand the synergy between cognitive and social aspects of learning.

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