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Mapping the Layers of Understanding: An Analysis of Mathematical Comprehension in Literacy Questions using the Pirie-Kieren Theory

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Abstract: The Pirie-Kieren theory provides a dynamic framework that explains how mathematical understanding develops in layers, starting from initial introduction to reflection through eight layers of understanding. The eight layers of understanding are Primitive Knowing, Image Making, Image Having, Property Noticing, Formalizing, Observing, Structuring, and Inventising. This study aims to analyze students' mathematical understanding in solving literacy problems based on Pirie-Kieren's theory. This study is a qualitative descriptive study, involving 15 tenth-grade students at SMA Negeri 2 Kupang Barat, Indonesia. The research instruments used were literacy tests and interviews. In-depth interviews were conducted with student representatives who had reached each layer of understanding. Student representatives were selected based on purposive sampling. Data analysis in this study was carried out in four stages, namely data reduction, data presentation, conclusion drawing, and triangulation. The literacy test data were analyzed based on Pirie-Kieren's eight layers of understanding. The eight layers of understanding are. The results show that 73.33% of students reached the image having a layer of understanding, 13.33% reached the formalizing layer, 6.67% reached the image-making layer, and 6.67% reached only the primitive knowing layer. No students reached the observing, structuring, or inventing layers. The dominance of students in the image, having a level of understanding, shows that most students have only reached the initial stage. These results indicate that students' mathematical understanding of literacy questions remains at a basic level and has not developed into a reflective understanding.

Keywords: mathematical literacy, literacy questions, mathematical understanding, Pirie Kieren theory.

▪ INTRODUCTION

Current curriculum development requires a simple, practical design that is adaptable to students' learning needs and necessitates a learning process that involves interaction between students, between students and educators, and learning resources in a specific learning environment (Nopa et al., 2019). In line with this principle, the Education Standards, Curriculum, and Assessment Agency emphasizes that the development of operational curricula in educational units must be student-centered, contextual, and easy to understand and implement in accordance with the characteristics and learning needs of each educational environment (BSKAP Kemendikbudristek RI, 2022).The need for a simple and practical curriculum aligns with global demands that emphasize the importance of mathematical literacy as a basic competency for students, as well as recognizing existing mathematical understanding and expanding or developing new understanding (Slaten, 2013). Curriculum development at the national and

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international levels emphasizes the importance of mathematical literacy as a basic competency students must possess to address real-life problems.

Mathematical literacy emphasizes students' ability to analyze and effectively solve mathematical problems in various contexts, as well as their ability to form mathematical structures from a problem after analyzing and recognizing its context (OECD, 2009; Malatjie & Machaba, 2019). Mathematical literacy is defined as the ability to recognize, understand, and interact with mathematics, as well as make informed judgments about the role of mathematics in personal, professional, social, and civic life (Öksüz et al., 2022).According to Saka (2023), literacy is a form of questions presented in context to measure students' ability to apply mathematical skills in real life. Mathematical literacy is very important for individuals and society, because the ability to read and interpret quantitative information and understand everyday phenomena is essential to being part of society and improving the ability to use mathematics in everyday life (Tarim & Tarku, 2022)(Walkington et al., 2015)Chen (2022)states that mathematical literacy is the ability to abstract real problems mathematically, express problems in mathematical language, and construct models using mathematical methods to solve problems. Low mathematical literacy is associated with increased unemployment and decreased economic well-being (Parsons & Bynner, 2005). Mathematical literacy broadly encompasses social, cultural, political, psychological, economic, historical, and societal dimensions (Vithal & Bishop, 2006). This condition requires a theoretical framework that explains the stages of mathematical understanding in depth and in dynamic ways, especially in the context of solving literacy problems. Therefore, a theoretical framework that considers the stages of mathematical understanding in an iterative and layered manner is needed. Kieren Pirie's theory offers a relevant conceptual framework to explain the growth of mathematical understanding through eight layers that interact dynamically and non-linearly.

The Pirie-Kieren theory provides a dynamic framework that explains how mathematical understanding develops in layers, from initial introduction to reflection and the basic connection between community mathematics and the mathematics learned by children in school (Kieren, 1994). According to Martin & LaCroix (2008), this theory consists of eight basic layers, namely, *Primitive knowing, Image making, Image having, Property noticing, Formalizing, Observing, Structuring, and Inventing.* Pirie (1994) emphasizes that the use of categories to classify understanding is a dynamic, holistic process rather than a single achievement. Pirie-Kieren's innovation lies in its alignment with a non-linear, recursive model of mathematical understanding, which differs from the dominant linear learning model (Irvine, 2023). As a theory of mathematical growth, Pirie-Kieren is used to analyze continuous development processes, not limiting understanding to situational categories alone (Zawawi et al., 2023). This model is also practical when applied to students (Gokalp & Sharma, 2010) and is grounded in constructivist principles (Wright, 2014). Dynamic and non-linear processes are at the core of analysis based on this model (Rexhepi & Makasevska, 2024a). The eight layers of understanding are interrelated, ranging from 'primitive knowledge' to 'discovery', describing the evolution of an individual's understanding of a particular mathematical concept (Borgen, 2006; Sengul & Argat, 2016). According to Yao (2020), each layer of the Pirie-Kieren model encompasses all previous layers and is part of the next layer, thereby making the process of mathematical understanding continuous, dynamic, and recursive. According to

Gonzales (2022), in the Pirie-Kieren model, shifts and repetitions occur in children's understanding during problem-solving.

Mathematical understanding refers to the types of mathematical actions, thinking, and learning that we can observe (Martin & Towers, 2009). According to Syafiqoh et al. (2018),In the learning process, individuals often review their previous level of understanding as a step toward strengthening and developing understanding at a higher level, a process called *folding back*. Research conducted by (Utami et al., 2025) highlightsthat the frequency of *"folding back"* is directly proportional to the depth of students' mathematical understanding. *"Folding back"* acts as a dynamic mechanism in the movement between eight layers and revisiting previous understanding of a concept to expand understanding, Pirie Kieren (Mabotja (2018),Irvine, 2023). The "folding back" process described by Pirie and Kieren involves revisiting previous understanding of a concept to expand that understanding in order to solve problems (Mabotja et al., 2018b). According to George & Voutsina (2024), when students encounter problems, questions, or situations that are difficult to solve, the process of "folding back" occurs, facilitating the expansion of an inadequate or incomplete understanding. Therefore, Hähkiöniemi et al. (2022) emphasize the need for further research on *"folding back"* because this process can deepen mathematical understanding while improving the design of the learning environment.

The results of Rexhepi & Makasevska's (2024)research show that using Pirie-Kieren's theory in everyday-life-based learning, especially in fractions, is more effective than traditional methods. In line with Pirie-Kieren's theory and the findings of Cai & Rott (2024), students who actively formulate or reconstruct problems from situations are considered to have mathematical understanding. Patmaniar et al. (2021) also argue that Pirie Kieren's theory can help teachers detect characteristics of student understanding in solving mathematical problems. However, Pirie Kieren's theory has provided an in-depth framework for explaining mathematical understanding through eight layers of thinking; its application in the context of literacy problem-solving has rarely been studied, thus requiring further investigation. Therefore, this study focuses on analyzing students' mathematical understanding in solving literacy problems, using Pirie-Kieren's layer theory of understanding.

Based on this identification, this study aims to analyze students' mathematical understanding in solving literacy problems using Pirie-Kieren's theory at SMA Negeri 2 Kupang Barat. This location was chosen based on interviews with mathematics teachers, who stated that students' mathematical understanding of literacy problems remained low. The analysis focused on identifying the layers of thinking students went through, the process of forming understanding at each layer, and the obstacles encountered during transitions between layers. The results of this study are expected to provide an overview of students' mathematical understanding in solving mathematical literacy problems in accordance with Kieren Pirie's layers of understanding, as well as practical recommendations for improving mathematical literacy.

▪ METHOD

Participants

This study was conducted in grade ten of SMA Negeri 2 Kupang Barat. In this study, a mathematics literacy test was given to 15 students. The test results were analyzed to determine each student's level of understanding using Pirie-Kieren's theory. From each level of understanding, one student was selected as a representative to be interviewed in depth. The selection was also carried out using purposive sampling, which involves selecting samples based on specific criteria provided by the mathematics teacher.

Research Design and Procedures

This study is a qualitative descriptive study. The purpose of this study is to analyze students' mathematical understanding in solving literacy problems based on Pirie-Kieren's theory at SMA Negeri 2 Kupang Barat. This study was conducted in the odd semester of the 2025/2026 academic year, namely in July 2025. The research was conducted in three stages, namely preparation, implementation, and conclusion. In the preparation stage, the researcher prepared the research instruments. In the implementation stage, the researcher collected test and interview data and analyzed the data. In the conclusion stage, the researcher concluded from the data analysis results.

Instruments

The instruments used in this study were literacy test questions and interview guidelines. The instruments were validated by lecturers and mathematics teachers. Two literacy test questions with combined material were used. These questions are shown in Table 1. The interviews were unstructured, and the questions asked developed spontaneously in response to the subjects' responses. Table 1 presents the literacy questions.

Table 1. Mathematical literacy questions

Question
A new restaurant is trying to put together an affordable lunch menu package. The menu consists of: Appetizer: soup or salad Main Course: chicken, fish, or vegetarian Dessert: ice cream or cake Customers can choose one item from each category. The restaurant wants to create a tablet-based automated ordering system and needs to know the various possible combinations and tailor them to customer needs. a. If there is a vegetarian customer who avoids all animal products, including meat and fish, but can still consume cake or ice cream, how many menu combinations can they choose from? b. If the restaurant wants to create three vegetarian-friendly menu packages and three menu packages for general customers, how should they choose the menu combinations? c. Create recommendations in the form of a menu table and provide mathematical and logical reasons in terms of variety and possible choices!
The school cafeteria offers a breakfast menu package that includes rice, eggs, crackers, and drinks. The rice consists of white rice, yellow rice, and fried rice. The eggs are served as omelets, fried eggs, and boiled eggs. Crackers consist of fish crackers and shrimp crackers. Drinks include water and hot tea. Here are the prices for each menu item:

- Food Menu
  - White rice
  - Yellow rice
  - Fried rice
  - Eggs (omelet, sunny-side up, boiled)
  - Crackers
- Drinks
  - Water (200 ml bottle)
  - Tea (200 ml glass)

a. If a student is trying to reduce their sugar intake, how many menu combinations are still available for them to choose from?
b. If the student only has Rp. 10,000, how many menu options can they purchase without exceeding their budget?
c. Create a budget-friendly menu suggestion for other students based on the results of the combination analysis and budget constraints!

Data Analysis

Data analysis in this study was conducted in four stages, namely data reduction, data presentation, conclusion drawing, and triangulation. In the reduction stage, the researcher selected, focused, and organized the data from the tests and interviews. In the data presentation stage, the researcher presented the reduced data. In concluding, the researcher interpreted the relationship between the test and interview results. Next, the researcher performed method triangulation, which is comparing the test and interview results linked to Pirie Kieren's layers of understanding indicators. Table 2 presents the indicators for the understanding layer based on Kieren Pirie's theory.

Table 2. Understanding layer indicators based on pirie-kieren's theory

Understanding Layer	Indicator
Primitive Knowing (Initial knowledge)	Understanding the meaning of vegetarianism, food categories, and rules for choosing foods by category
Image Making (Initial stage of idea formation)	Imagining combinations of menu categories
Image Having (basic idea outline)	Arranging possible initial combinations based on understanding
Property Noticing (identifying concepts from ideas)	Recognizing limitations (vegetarians do not eat chicken/fish)
Formalizing (ideas are formed systematically)	Systematically organizing combinations
Observing (reviewing the ideas that have been formed)	Re-evaluating the selected combinations
Structuring (building structured and complex ideas)	Comparing combinations and arranging them in tabular form
Inventising (creating new ideas based on experience)	Providing logical reasons and evaluating combination strategies

▪ RESULT AND DISSCUSSION

An analysis of students' understanding in solving mathematical literacy problems, based on the layers of understanding in Pirie and Kieren's theory, was conducted with 15 students using two mathematical literacy questions. Students' answers were categorized based on the indicators of understanding levels in Pirie-Kieren's theory.

Mathematical ability comprehension in this study refers to students' understanding of mathematical literacy questions grounded in the theory of Pirie and Kieren. Figure 1 shows the results of grouping students' answers into eight levels of Kieren Pirie's comprehension.

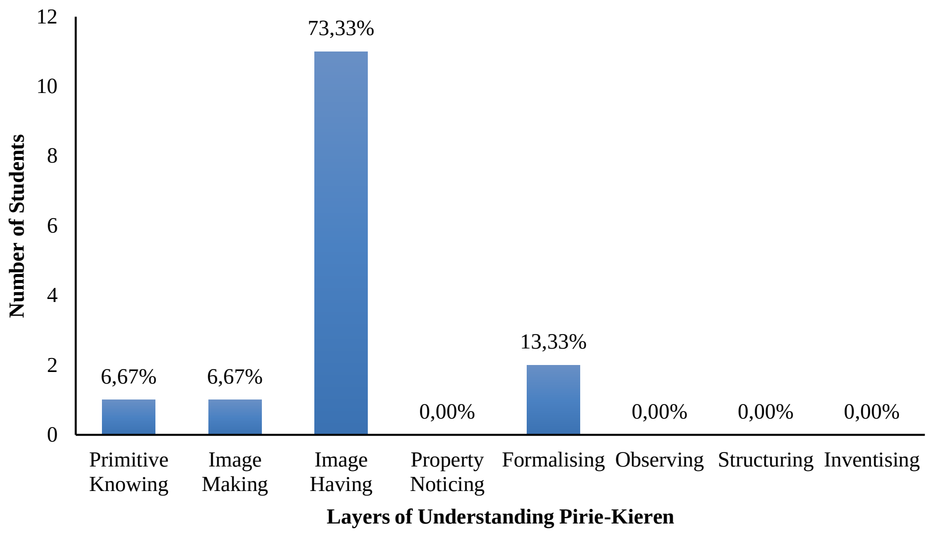

Figure 1. Percentage of student comprehension achievement based on pirie-kieren's layers of understanding

Figure 1 shows that, of the 15 students, 11 students (73.33%) reached the Image Having level of understanding, two students (13.33%) reached the Formalizing level of understanding, one student (6.67%) reached the Image Making level of understanding, and one student (6.67%) only reached the Primitive Knowing level of understanding. No subjects reached the Observing, Structuring, and Inventing levels of understanding.

Based on Figure 1, four students representing the achievement of Pirie Kieren's stages were selected as subjects. The selected subjects were one student who reached only the Primitive Knowing level, one who reached the Image Making level, one who reached the Image Having level, and one who reached the Formalizing level. The selection was also based on the teacher's recommendation, namely, students who could communicate well. Furthermore, to simplify the writing, the four selected students were referred to as Subject A, B, C, and D.

Subject A

Figure 2 shows Subject A's worksheet for literacy question number 1. In Figure 2, it can be seen that Subject A did not answer the question in part (c) and only answered the questions in parts (a) and (b). Subject A's answer in part (a) shows that Subject A answered the combination that could be chosen, but did not mention the right menu for vegetarians. Subject A correctly mentioned the numerical calculation result of 4, but did not detail the process. Subject A understood the vegetarian rules but did not understand

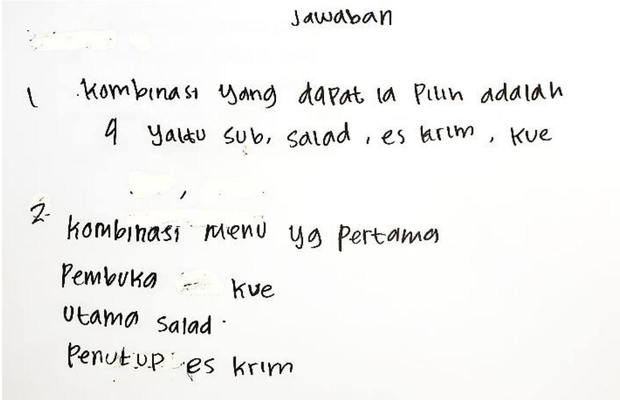

Figure 2. Subject A's answers to the literacy question number 1

the combination of appetizer, main course, and dessert categories. Subject A's answer in section (b) was not correct. Subject A mentioned a menu combination, but it did not match the requests for vegetarians and general customers. Based on the interview results, Subject A explained that he did not answer part (c) because he did not understand the instructions. When asked why the combination was 4 in part (a), Subject A replied that four was to explain that the right foods for vegetarians were soup, salad, ice cream, and cake. In section (b), Subject A stated that they were confused about how to make combinations. Based on triangulation between the worksheet and the interview, it can be said that Subject A understood the meaning of vegetarian but did not understand the food categories or the rules for choosing within each category. Based on Table 2, Subject A was at the Primitive Knowing level of understanding. Next, an analysis was conducted on literacy question number 2. Figure 3 shows Subject A's worksheet for literacy question number 2.

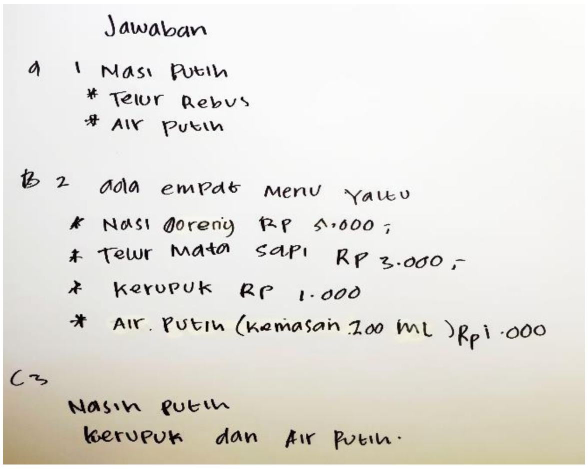

Figure 3. Subject A's answers to the literacy question number 2

Figure 3 shows that Subject A answered all questions (a), (b), and (c). In part (a), Subject A only wrote down one food choice for students who are trying to reduce their sugar consumption, but did not answer the menu combinations that could still be chosen. In part (b), Subject A wrote down one menu option with a budget of Rp. 11,000, whereas the question asked was what menu options would be available with a budget of Rp. 10,000. This can be seen from Subject A's worksheet, which shows fried rice (Rp . 5,000), fried eggs (Rp. 3,000), crackers (Rp. 1,000), and water (Rp. 1,000), while the question specified fried rice at Rp. 6,000. Therefore, Subject A incorrectly selected the menu items for a budget of Rp. 10,000. Subject A did not answer many menu options that could be purchased without exceeding the budget. In part (c), Subject A only wrote down one menu suggestion (white rice, crackers, and water), but there was no explanation as to whether the menu suggestion was in line with the budget. The interview with Subject A explained why the possible food menus were written down so students could eat without referring to the requirements in the question. Based on triangulation between the worksheet and the interview, it was found that Subject A understood the meaning of vegetarian but did not understand food categories or the rules for selecting each category. Based on Table 2, Subject A was at the Primitive Knowing level of understanding.

Subject B

Figure 4 shows Subject B's worksheet for literacy question number 1

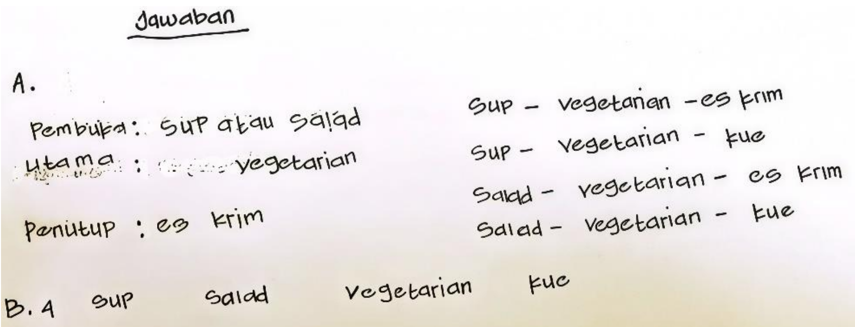

Figure 4. Subject B's answer to the literacy question number 1

In Figure 4, Subject B answered questions in parts (a) and (b) but did not answer questions in part (c). Subject A understood the meaning of vegetarian, as can be seen from the answer that the main menu combination included a vegetarian menu in part (a). Subject A imagined a combination of menu categories for vegetarians, but did not mention the number of combinations. In section (b), Subject A did not write down the menu combinations for the three vegetarian and general customer options. He only wrote down the menus without any clear explanation. In the interview, Subject B explained that he did not pay attention to the question, so he wrote down only the vegetarian menus for students without counting the combinations. Subject A stated that the reason for writing (soup, salad, vegetarian, cake) in section (b) was for vegetarians. However, they did not answer the question about menus for general customers. The reason Subject A did not complete section (c) was that they were confused about how to recommend food menus in table form. Based on triangulation between the worksheet and the interview, it can be said that Subject A understood the meaning of vegetarian and could imagine combinations of menu categories. Thus, based on Table 2, Subject B was at the Image Making level of understanding. Next, an analysis was conducted on literacy question number 2. Figure 5 shows Subject B's worksheet for literacy question number 2.

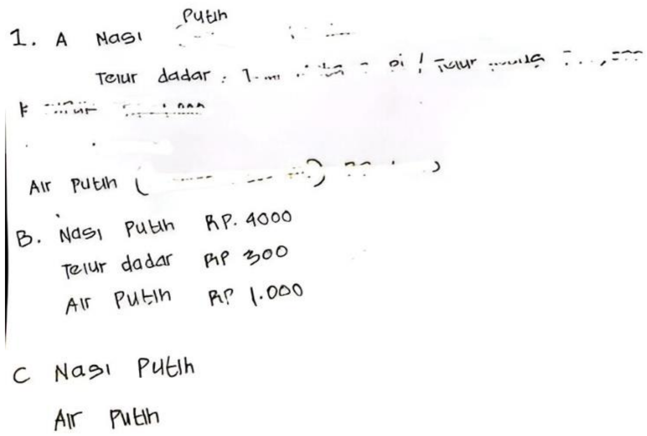

Figure 5. Subject B's answer to the literacy question number 2

Figure 5 shows that Subject B answered questions (b) and (c), while Subject B hesitated to answer question (a). Subject B erased their previous work, leading to the conclusion that part (a) was not completed. In part (b), Subject B recommended a menu with a budget of Rp. 10,000, consisting of white rice (Rp. 4,000), omelet (Rp. 300), and water (Rp. 1,000). Thus, the total budget for Subject B's recommended menu is Rp. 5,300, while the price list for an omelet is Rp. 3,000. It can be concluded that Subject B has imagined a combination of menu categories, even though Subject B did not answer all the questions in part (b), as they wrote down only one menu option. In section (c), Subject B only wrote down one menu recommendation, namely (white rice and water). In the interview, Subject B explained that the most economical menu was (white rice and water) because it would not exceed the budget of Rp. 10,000. Based on triangulation between the worksheet and the interview, it can be said that Subject B was able to imagine a menu combination with a budget of Rp. 10,000. When asked why he deleted his answer in section (a), Subject B explained that he deleted it because his answer was wrong and he wanted to correct it, but before he could do so, the teacher asked for the answer sheets to be collected. Based on Table 2, Subject B is at the Image Making level of understanding.

Subject C

Figure 6 shows Subject C's worksheet for literacy question number 1.

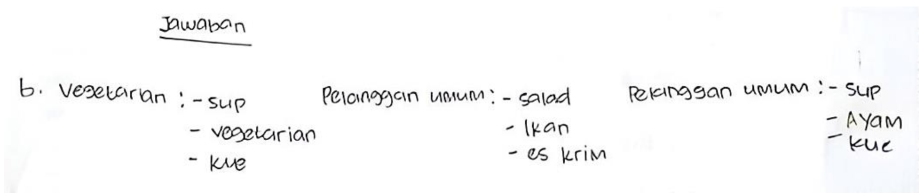

Figure 6. Subject C's answer to the literacy question number 1

In Figure 6, it can be seen that Subject C did not answer questions (a) and (c) and only answered question (b). Subject C understood the meaning of vegetarian, which can be seen from the answer to the vegetarian menu combination. Subject C also imagined combinations of menu categories and compiled initial combinations based on their understanding. However, Subject C only wrote down one vegetarian-friendly menu combination (soup, vegetarian, cake) and two menu combinations for general customers (salad, fish, ice cream; soup, chicken, cake). Subject C did not answer question (b) completely. The interview with Subject C revealed that the instructions in parts (a) and (c) were difficult to understand, so they did not answer these two parts. Subject C also did not understand the menu package options, so in part (b), they only wrote down foods that vegetarians and general customers could eat. Based on triangulation between the worksheet and the interview, it can be said that Subject C understood the meaning of vegetarianism, food categories, and the rules for choosing within each category. Subject C was able to imagine combinations of menu categories and could also compile possible combinations based on their understanding. Based on Table 2, Subject C was at the Image Having level of understanding. Next, an analysis was conducted on literacy question number 2. Figure 7 shows Subject C's worksheet for literacy question number 2.

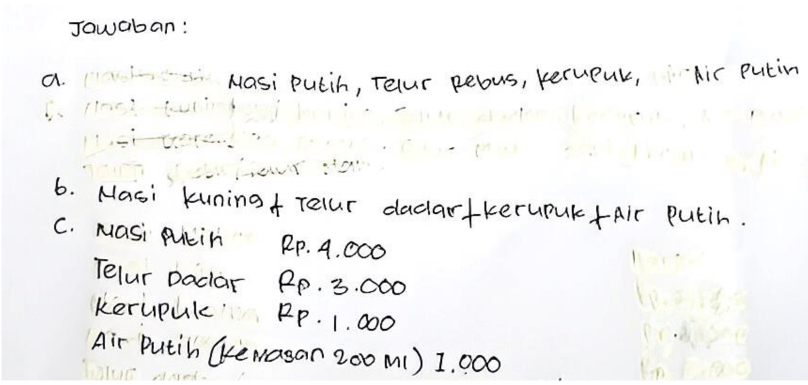

Figure 7. Subject C's answers to the literacy question number 2

Figure 7 shows that Subject C answered all questions (a), (b), and (c). In part (a), Subject C only wrote down one food choice for students who are trying to reduce their sugar intake, but did not answer the combination of menus that could still be chosen. In part (b), Subject C only wrote down one menu option with a budget of Rp. 10,000, but did not answer the many menu options that could be purchased without exceeding the budget. In part (c), Subject C wrote only one example with a price list but did not make recommendations for economical menus based on budget constraints. The interview with Subject C revealed that they only wrote down foods that could be eaten by students trying to reduce their sugar intake and, based on budget constraints, did not write down the combinations. When asked why they deleted their previous answer and gave the answer as written, Subject C explained that they initially wrote down the menu without considering factors such as reduced sugar intake and budget constraints. Subject C also added that the answer given was the result of imitating their friend without understanding the reasons behind the choice. Based on triangulation between the worksheet and the interview, it was found that Subject C understood the food categories and the rules for choosing each category. Subject C was also able to imagine combinations of menu categories and arrange possible menu combinations based on their understanding. Therefore, based on Table 2, Subject C was at the Image Having level of understanding.

Subject D

Figure 8 shows Subject D's worksheet for literacy question number 1

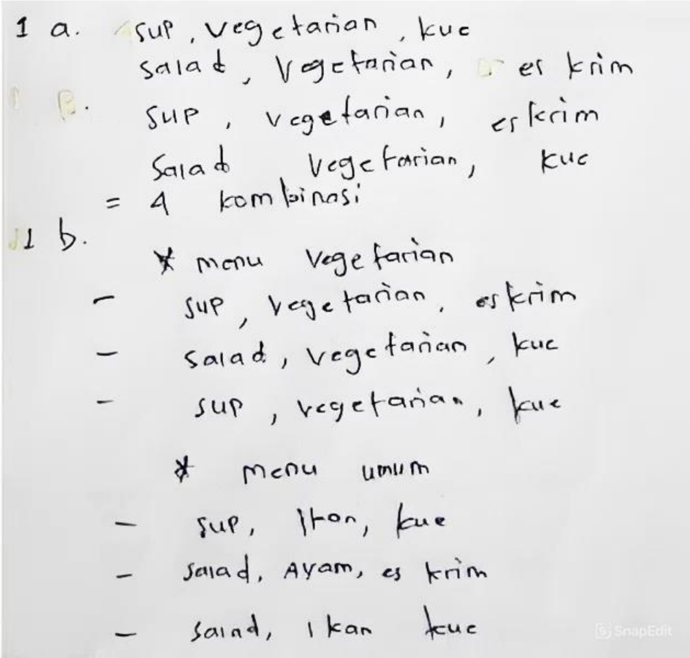

Figure 8. Subject D's answer to the literacy question number 1

Figure 8 above shows Subject D's answers to literacy question number 1. Subject D answered parts (a) and (b) but did not answer part (c). Analysis of Subject D's answer shows that Subject D was able to recommend several vegetarian menus and wrote down four possible combinations. In part (b), Subject D wrote down three menu packages for vegetarians and three menu packages for general customers. Thus, Subject D reached the Formalizing level of understanding, which is the ability to systematically organize combinations in accordance with the requirements of the question. This finding is reinforced by the interview results. When asked, "Why did you write this answer?" the student replied, "I gave that answer because I understood that the question only asked me to compile a menu of food and drinks suitable for vegetarians and general customers." Subject D explained that in part (a), there were four menu combinations that a vegetarian who still wanted to eat cake or ice cream could choose from. In part (b), he mentioned three menu combinations that were recommended for both vegetarian and general customers. As for part (c), Subject D did not provide an answer because he immediately moved on to question number two. When he wanted to return to complete that section, there was not enough time. In addition, Subject D admitted that he did not fully understand the instructions in part (c), which asked for menu recommendations to be compiled in table form. Based on triangulation between the worksheet and the interview, it can be said that Subject D organized the combinations systematically. Thus, based on Table 2, Subject D is at the Formalizing level of understanding. Next, the analysis continued on literacy question number 2. Figure 9 below.

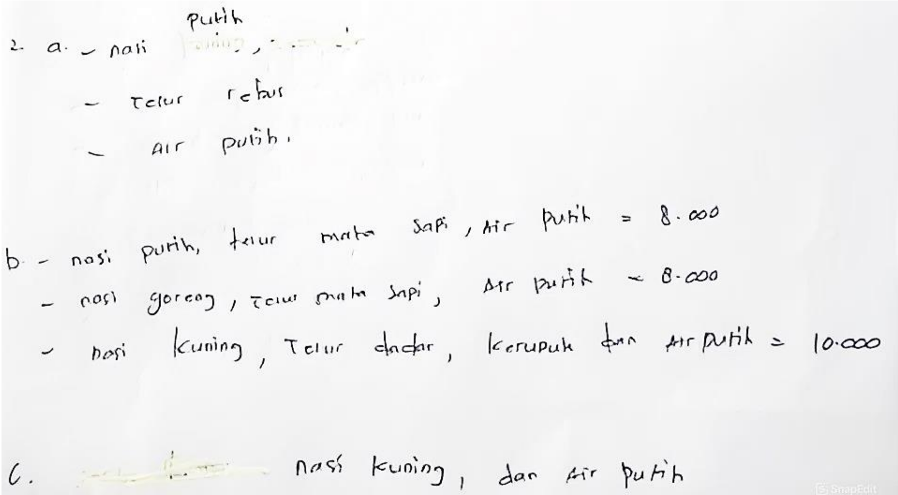

Figure 9. Subject D's answer to the literacy question number 2

Based on Subject D's answer shown in Figure 9, it can be seen that for part (a), they have detailed the menu options if sugar consumption is reduced, but they have not written down the number of possible combinations. Subject D also wrote down three menu choices for a budget of Rp. 10,000, namely white rice, fried eggs, and water for a budget of Rp. 8,000; fried rice, fried eggs, and water for a budget of Rp. 8,000; and yellow rice, omelet, crackers, and water for a budget of Rp. 10,000. From the results recorded for the menu presentation and budget, there is an error in the fried rice, fried egg, and water menu: the total budget is Rp. 10,000, not Rp. 8,000. Subject D did not record how many menu options fit the budget. In part (c), Subject D provided menu recommendations that fit the budget: yellow rice and water, costing Rp. 6,000. During the interview, Subject D admitted to being confused in determining low-sugar menu choices in part (a) because they did not know what types of food were classified as low in sugar. Therefore, they recommended only one menu combination. In section (b), Subject D stated that because there was no specific emphasis in the question, he wrote down only three menu recommendations in accordance with the budget of less than or equal to Rp. 10,000. Furthermore, according to Subject D, the recommended economical menu combination was yellow rice and water, with the reasoning that students could still eat well by ordering yellow rice and stay healthy by drinking water. Based on triangulation between the worksheet and the interview, it is said that Subject D has organized the combinations systematically. Thus, based on Table 2, Subject D is at the Formalizing level of understanding.

The results of the study show that most students reach only the Image Having level of understanding, with few reaching the Formalizing level and none reaching the Observing, Structuring, or Inventising levels. This pattern of achievement confirms that students' mathematical understanding of literacy questions remains at a basic level and has not developed into more complex, reflective, or conceptual understanding. These findings are in line with Pirie Kieren's theoretical framework, which describes understanding as developing gradually, in layers, and non-linearly.

The dominance of students at the Image Having level shows that most students have only reached the ability to construct an initial picture of menu combinations, but are not yet able to identify important properties such as category boundaries, menu selection rules, or applicable combinatorial principles. This finding is in line with Husband et al. (2023),who emphasize that the Image Making and Image Having stages are important aspects in triggering the construction of understanding, especially through representational activities.

The findings on the four subjects show that the characteristics of the questions strongly influence errors in understanding the context and instructions. These results are supported by the opinion of Gokalp & Bulut (2018) students' question-solving strategies are highly dependent on the structure of the questions, including the clarity of the boundaries and context. This can be seen in Subjects A and B, who misunderstood the rules for selecting categories, and Subjects C and D, who were unable to model the budget context correctly.

Students at a higher level, such as Formalizing (Subject D), appear able to systematically arrange combinations. However, they still struggle to provide logical arguments and connect their answers to the questions. This is reinforced by the findings of Arenas & Peñaloza et al. (2024)that students often fail to formalize mathematical concepts and tend to use strategies without understanding the relationship between the question's context and the mathematical concept. Thus, even though Subject D is at a more advanced level, reflective aspects and mathematical argumentative abilities have not yet been strongly developed.

The fluctuating understanding process in some students also shows symptoms of unsuccessful folding back. Kieren Pirie's theory explains that folding back is an important mechanism when students' understanding is not yet stable, and they need to return to the previous level to improve their concepts. Irvine (2023) and Utami et al. (2025) emphasize that the success of folding back determines the quality of subsequent knowledge reconstruction. In this study, students returned to their initial knowledge. However, they were unable to expand their understanding, so the folding-back process stopped at the reproductive stage rather than the constructive stage.

This limitation in understanding was also evident in the interviews. Some students simply copied their friends' answers (Subject C), were confused by the meaning of categories (Subject A), or were unable to provide mathematical reasons for their choices. This pattern reinforces the view of Codes et al. (2013)that the formation of understanding depends on the interaction between various layers through a mechanism of repetition, which in this case did not occur effectively. This means that the learning experience did not provide enough space for students to internalize concepts through layered interactions. At the Primitive Knowing layer, as seen in Subject A, students do not even have a basic understanding of categories. According to Martin & LaCroix (2008), this stage requires students to apply general knowledge in meaningful activities. However, Subject A was not yet able to do this; therefore, he still needed intensive guidance to develop an initial understanding of mathematical literacy concepts. The results of this study are also aligned with Wright (2014),who states that mathematical understanding can only develop if students truly construct meaning from their learning experiences. Because most students process information superficially without reflection, they fail to reach the layers of Observing, Structuring, and Inventising. Even Subject D, who has reached Formalizing, still shows logical errors and inaccuracies in relating budget concepts. From the perspective of the literacy context, the failure of most students to reach higher levels also reinforces the findings of Rexhepi & Makasevska (2024),that learning that is not grounded in concrete contexts makes it difficult for students to reach higher levels of understanding in the Pirie-Kieren model. In this study, even though the questions were context-based, previous learning experiences may not have been sufficient to encourage students to connect information in context in depth.

Overall, the study's results show that students' understanding development is strongly influenced by their ability to organize information and connect real-world contexts with mathematical concepts. However, most students have not been able to cross the threshold to reflective and structural understanding. This indicates the need for learning interventions that not only involve real-world contexts but also guide students to explore representations, arguments, and mathematical reflections consistent with Pirie Kieren's framework.

▪ CONCLUSION

The use of Pirie Kieren's theoretical framework provides a new perspective on the stages of growth in students' understanding of mathematical literacy questions. The main innovation in this study lies in mapping the layers of student thinking, which shows that the process of mathematical understanding takes place gradually and non-linearly. This study shows that integrating layers of understanding in learning can help identify barriers to understanding, thereby improving mathematical literacy. Thus, the results of this study enrich the understanding of mathematical literacy and open up opportunities for the future development of Pirie Kieren's theory-based contextual learning.

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