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Newman's Error Analysis of Trigonometry: Critical Thinking Perspective

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Abstract: Trigonometry is a fundamental topic in secondary mathematics, yet many students still experience difficulties that lead to various errors. This study applies Newman's Error Analysis (NEA) to analyze trigonometry errors and qualitatively link each error stage to critical thinking indicators. The findings show that fewer Newman's errors correspond to stronger critical thinking performance, as S1 made minimal errors across all stages and correctly answered nearly all items aligned with critical thinking indicators. Participants were 29 grade XII students from a senior high school in Majalengka, West Java, Indonesia, selected purposively. The study employed a descriptive, qualitative design, supported by quantitative item analysis. Critical thinking skills were assessed through five open-ended trigonometry questions, aligned with reasoning, inference, clarification, and problem-solving indicators, and validated by experts for content and construct accuracy. Semi-structured interviews involved three students representing high, medium, and low ability levels. The interviews revealed that high-achieving students mainly struggled to express conclusions, while medium- and low-achieving students had broader difficulties in applying concepts and reasoning. The most frequent errors occurred in encoding (up to 100%), followed by process skills (82.7%–96.5%), moderate transformation (≤96.5%), comprehension (34.5%–100%), and fewer reading errors (86.2%). The findings indicate that NEA is effective in diagnosing students' specific cognitive barriers and mapping their weaknesses in critical thinking. The findings show that each of Newman's stages corresponds to critical thinking weaknesses, with reading and comprehension exhibiting weak clarification, transformation showing weak inference, process skills demonstrating weak reasoning and logical evaluation, and encoding displaying poor evaluation and difficulty in expressing conclusions. The study concludes that mathematics instruction should focus on strengthening process skills and training students in clear mathematical communication to minimize encoding errors. It also recommends integrating visual or manipulative learning media that address these errors, as many process and encoding mistakes stem from students' difficulty visualizing angle–side relationships in trigonometric problems.

Keywords: newman's error analysis, critical thinking, trigonometry, student errors.

▪ INTRODUCTION

Trigonometry is one of the essential topics in secondary mathematics that requires mastery of abstract reasoning and conceptual understanding, such as trigonometric ratios and their application to right triangles (Kamber & Takaci, 2018). However, many students still struggle to understand this trigonometry material (Nurjailam et al., 2021). Frequent errors occur not only in calculations but also in understanding information, selecting the right formula, and drawing conclusions (Kranz et al., 2023; Obeng et al., 2024). These student errors are influenced by several factors, including a less conducive learning environment (Juan & Chen, 2022; Marder et al., 2023), less creative and less effective learning models and methods (Schreiber & Ashkenazi, 2024), and a lack of parental involvement in student learning activities. Consequently, several factors contribute to these student errors (Fiskerstrand, 2022; Guzmán et al., 2023). Consequently, students face challenges in mastering basic mathematical concepts, particularly in trigonometry, applying these concepts in everyday life, and performing accurate calculations. These

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issues underscore the need for an in-depth examination of student errors and misconceptions, particularly in the context of trigonometry learning.

Students' errors in trigonometry are not only procedural but also conceptual in nature (Obeng et al., 2024; Tambychik & Meerah, 2010). Many students struggle to identify known and unknown elements, transform the problem context into a mathematical model, and implement systematic solution steps (Lenz et al., 2024; Valdez & Taganap, 2024). Therefore, error analysis has emerged as an effective diagnostic tool for identifying specific learning difficulties in mathematics (Elagha & Pellegrino, 2024). By recognizing common error patterns, teachers can develop targeted interventions to address student misconceptions. This emphasizes the importance of structured error analysis for more accurate diagnosis of student misconceptions.

To systematically identify the types and sources of students' misconceptions, a structured analytical framework is required. One effective and structured framework for diagnosing such errors is Newman's Error Analysis (NEA). The NEA facilitates the identification of specific cognitive stages at which errors occur during problem-solving activities (Muntazhimah et al., 2023). This framework classifies errors into five cognitive stages: 1) reading, 2) understanding, 3) transformation, 4) process skills, and 5) answer writing (Rohmah & Sutiarso, 2017). Each category reveals insights into students' reasoning and the depth of their critical thinking. Through the NEA, researchers and educators can identify the stages at which students encounter obstacles in problem-solving, thus providing an in-depth picture of students' thinking processes and critical thinking abilities. Therefore, NEA is critical in this study as it provides a systematic means to diagnose students' cognitive barriers in trigonometry and to connect these barriers with specific aspects of critical thinking.

To strengthen the theoretical foundation of this study, the five stages of NEA are explicitly linked to critical thinking skills. Errors in the reading and comprehension stages indicate weaknesses in clarification; transformation and process skill errors reflect poor inference and reasoning; and encoding errors reveal students' limited ability to communicate conclusions coherently. This integration aligns the NEA framework with critical thinking indicators, including clarification, inference, reasoning, and evaluation (Anderson & Krathwohl, 2001; Arisoy & Aybek, 2021). Through this alignment, NEA can serve not only as a diagnostic tool but also as a framework for mapping students' critical thinking deficiencies.

Several previous studies have explored the relationship between student error analysis and higher-order thinking skills (HOTS) (Tanujaya et al., 2021; Zhang, 2025) and have also applied NEA to arithmetic, algebra, and geometry (Nuritasari & Aini, 2023). However, the approaches used in these studies tend to be general and lack detail in systematically mapping important thinking processes (Coffey et al., 2022; Suseelan et al., 2022). Thus, previous studies have not been able to illustrate how each type of error specifically reflects students' cognitive and critical thinking weaknesses. Therefore, a more in-depth and structured investigation is needed to systematically link students' error types with critical thinking indicators. In this context, the present study employs the NEA framework as an analytical tool to identify the cognitive stages at which errors occur and map their connections to students' critical thinking skills. Furthermore, research specifically examining student errors in trigonometry is still limited (Taamneh et al., 2024). Previous research has primarily focused on procedural errors, including substitution, calculation errors, or incorrect formula selection. Furthermore, few studies have attempted to link students' trigonometry errors to higher-order or critical thinking skills, resulting in a lack of a comprehensive understanding of the cognitive factors underlying these errors. This prior research suggests that the research gap in trigonometry lies not only in the paucity of research but also in the absence of systematic analysis linking specific error types to students' cognitive and critical thinking weaknesses. Based on this, this study aims to examine the relationship between student errors in solving problems and critical thinking skills in trigonometry material, which has a research gap in error analysis using the NEA systematically. In contrast to previous studies that tend to generalize mathematical errors, this study specifically focuses on student error patterns in trigonometry material, thereby contributing to the improvement of models and methods in mathematics learning. Grounded in the theoretical framework that links cognitive error stages in NEA with dimensions of critical thinking, this study proposes that the types of errors students make correspond to specific weaknesses in their critical thinking processes. Accordingly, this study aims to address the following research question: What errors do high school students make in solving trigonometry problems, and how do these errors reflect weaknesses in their critical thinking skills? By triangulating data from written tests and interviews. This study aims to provide a comprehensive understanding of the types of errors that frequently occur, as well as offer contributions for teachers in designing more effective learning strategies to minimize student errors.

▪ METHOD

Participants

The participants in this study were 12th-grade students from a public high school in Majalengka Regency, West Java, Indonesia. These participants consisted of students who had studied trigonometry in class. Using a purposive sampling technique, one class of 29 students was selected to participate in this study by completing a written problem-solving test. The class was purposively chosen because it was considered representative of the overall population. This consideration was based on academic and demographic characteristics. Specifically, the selected class had an average mathematics score from the previous semester that was within ±5% of the overall grade-level mean, indicating a balanced composition of students with high, medium, and low abilities. In addition, the class shared a similar gender ratio and learning background with other twelfth-grade classes, ensuring that it reflected the general characteristics of the student population. Based on their performance, students were categorized into three levels of mathematical ability: high, medium, and low using classification criteria adapted from Indrawati et al. (2019). Students were categorized into three levels of mathematical ability: high, medium, and low using the tertile classification method based on their total test scores. The upper tertile (students with the eight highest scores) represented the high-ability group, the lower tertile (students with the eight lowest scores) represented the low-ability group, and the remaining students constituted the medium-ability group. Then, one student representative from each category was selected for a semi-structured interview, resulting in a total of three interview participants.

Research Design and Procedures

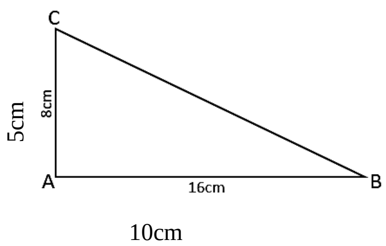
This study employed a collective case study design embedded within a descriptive survey. The descriptive survey involved 29 students to identify general error patterns. In comparison, the collective case study focused on three selected students representing high, medium, and low mathematical abilities to explore their stages of Newman's Error Analysis (NEA) and critical thinking processes in depth, investigating students' errors in solving mathematical problems through the NEA framework. The study was conducted in January 2025 with a structured sequence of procedures. The first stage involved the development of a research instrument comprising five open-ended essay questions on trigonometry, structured to assess critical thinking indicators and meet the criteria for higher-order thinking (HOTS) questions. The instrument was then subjected to a comprehensive validation process, including content validity, construct validity, and instrument pilot testing to ensure its clarity, relevance, and theoretical suitability.

The validated test was then administered to 12th-grade students under standardized conditions. Students' written answers were evaluated using a scoring rubric aligned with Newman's error stages and critical thinking indicators. Students were then categorized into three ability groups (high, medium, and low). One student from each category was selected to participate in a semi-structured interview. The interviews were audio-recorded and transcribed verbatim, then analyzed to identify error patterns and cognitive tendencies. This study also employed data triangulation from test and interview results to obtain a comprehensive understanding of the types of errors, misconceptions, and critical thinking processes that students use when solving trigonometry problems.

Instrumen

The main instrument used in this study consisted of five open-ended contextual essay questions specifically developed to assess students' critical thinking skills in solving problems related to trigonometry. The trigonometry questions are presented in Table 1 below.

Table 1. Trigonometry question instruments

No	Question
1	In a right triangle, there are six basic trigonometric functions: sine, cosine, tangent, cosecant, secant, and cotangent. Explain each of these trigonometric functions and write the formula based on the sides of the right triangle.
2	It is known that the right triangle ABC has a side length of BC of a right triangle as 7 square root of 2 cm. If AC is the hypotenuse of Triangle ABC and the length of AB = 6 cm, what is the length of BC?
3	The teacher gave an assignment to find the value of $cos30°$from various sources (books, calculator apps, and websites). One source says that $cos30° = 0.154251$, while another source gives the value of $cos30° = \frac{1}{2}\sqrt{3}$. Compare the results and determine which is more accurate, and explain your reasoning.
4	 $\sin\ \theta = \frac{\sqrt{5}}{5}$

	From the information given above, is it sufficient to determine sin $\theta = \frac{\sqrt{5}}{5}$ If it is sufficient, determine the truth that the value of sin $\theta = \frac{\sqrt{5}}{5}$; if it is not sufficient, find the missing element from the information above! After determining sin $\theta = \frac{\sqrt{5}}{5}$, determine cos $\theta =$
5	An airplane takes off at an elevation angle of 30° and flies 500m diagonally. Can you determine the effective step? Determine the vertical and horizontal distances of the airplane from the runway….

The item development was based on two complementary theoretical frameworks. The critical thinking indicators and item design were aligned with Bloom's revised taxonomy (Anderson, L. W., & Krathwohl, 2001). To ensure quality, the instrument was validated through expert validation involving a mathematics education lecturer and an experienced high school mathematics teacher. An expert validation process was conducted in two stages, involving a mathematics education lecturer and an experienced high school mathematics teacher.

The first validation, conducted by the lecturer, focused on theoretical and pedagogical aspects, particularly the alignment between trigonometric problem contexts and critical thinking indicators. Based on the expert's feedback, items 3 and 5 were revised to enhance conceptual clarity and consistency with the targeted critical thinking indicators. For item 3, the validator suggested adding an explicit comparison task and justification prompt to strengthen evaluative reasoning. For item 5, the revision aimed to eliminate ambiguity by specifying the vertical and horizontal distances more accurately, thereby better reflecting the application of trigonometric ratios. These revisions improved the clarity, cognitive demand, and construct validity of the instrument.

After implementing these revisions, the revised version of the instrument was submitted to the second validator (the high school mathematics teacher) for practical validation. The teacher confirmed that the instrument was clear, contextually appropriate, and suitable for classroom implementation, without suggesting further changes. Following expert validation, the instrument underwent a pilot test with students from another class to examine its clarity and internal consistency. The reliability analysis yielded a coefficient (r_{11}) of 0.66, which falls into the moderate (acceptable) reliability category, indicating that the instrument demonstrated sufficient consistency in measuring students' problem-solving and critical thinking performance. Each item was explicitly mapped to a specific critical thinking indicator, which is summarized in Table 2.

Table 2. Indicators of critical thinking skills assessed in the test

No	Critical Thinking Skills Indicators	Question Number
1	Students analyze the relationships among sides and angles in right triangles to explain the meaning and formulas of trigonometric functions (sin, cos, tan).	1
2	Students apply analytical reasoning using the Pythagorean Theorem to determine unknown sides or angles in right triangles and justify each step of their calculations.	2
3	Students evaluate and compare trigonometric results from various sources to determine the most accurate value and provide reasoning for their decision.	3

4	Students identify assumptions and infer missing elements in trigonometric problems to construct logical connections between given and required quantities.	4
5	Students design and create a systematic and efficient problem-solving strategy to solve contextual trigonometry problems related to everyday life.	5

Validated aspects included content, construct, and language. Based on their feedback, several questions were revised to become HOTS. The validated written test was administered to 29 twelfth-grade students in a regular classroom setting under standardized conditions. Students were allotted 60 minutes to complete the test. During the test, the researcher provided consistent instructions and ensured that no external aids (such as calculators or reference materials) were used. Students were required to work individually, and the environment was maintained in a quiet and controlled classroom to minimize distractions. After evaluating the students' work, semi-structured interviews were conducted for representatives of different ability groups. Each interview lasted approximately five to ten minutes and was audio-recorded and transcribed verbatim for analysis. Although the duration was relatively brief, it was deemed sufficient because the interview protocol was semi-structured and highly focused on questioning students' perceptions, difficulties, and the reasoning behind their answers, enabling concise yet meaningful insights into their thought processes. Each question directly referred to students' written responses, allowing the interviewer to elicit concise but meaningful explanations of their reasoning.

Semi-Structured Interview Protocol

To complement the written test data and gain a deeper understanding of students' mathematical thinking, a semi-structured interview protocol was developed to examine the cognitive and metacognitive processes underlying students' problem-solving behavior. The protocol was theoretically based on the NEA model, which categorizes student errors into five levels: reading, comprehension, transformation, process skills, and answer writing (Abdullah et al., 2015). Of the four interview questions used, two questions directly explored students' thinking processes related to the comprehension, transformation, process skills, and encoding stages of the NEA framework, namely Q2: What is the most common difficulty you face when working on math problems, especially trigonometry? Moreover, Q3: Of the problems you worked on, which one did you find most difficult? Why?. Specifically, these questions asked students to describe the types of difficulties they encountered when solving trigonometry problems and to explain which problems they found most difficult and why. The remaining questions, namely Q1: What do you think about math? Moreover, Q4: In your opinion, what learning media is most helpful in understanding trigonometry? Served as supporting prompts to contextualize students' perceptions of mathematics and their chosen learning media, providing complementary information regarding the motivational and representational aspects that influenced their performance across the NEA stages. This framework allows for a systematic investigation of the origins and nature of students' difficulties in solving problems.

Similarly, the interview protocol also incorporates elements of the thinking framework, enabling the exploration of students' logical interpretations of the problem context, justification of chosen strategies, and self-reflection on the results. Interview questions are designed to be open-ended and flexible, allowing for adaptive yet theoretically grounded dialogue. This enables a more in-depth and nuanced understanding of students' responses, reflecting their diverse perspectives and experiences.

The interviews were conducted in a semi-structured conversational format to allow flexibility and encourage student self-reflection. Each session followed a consistent structure: (1) exploring students' general views about mathematics, (2) identifying the most common difficulties they encounter when solving trigonometric problems, (3) discussing which problems they found most challenging and why, and (4) examining what types of learning media or representations they consider most helpful in understanding trigonometric concepts. This structure enabled the researcher to obtain rich qualitative data regarding students' perceptions, learning difficulties, and reasoning strategies related to their trigonometric error patterns, as framed by Newman's Error Analysis (NEA) and critical thinking skills.

Data Analysis

This study employed a data collection technique to assess students' mathematical problem-solving abilities in trigonometry. These tests were collected and analyzed after the learning activities and assessed by the researcher as student outcomes. Afterward, student work was grouped into high, medium, and low ability levels (Webb-Williams, 2021). This was followed by semi-structured interviews based on Kallio et al. (2016) to represent the groups by ability level. Each interview lasted approximately five to ten minutes and was audio-recorded and transcribed verbatim for analysis. Although the interview duration was relatively brief, this decision was made to keep students engaged and reduce cognitive fatigue. To compensate for the short duration, probing questions were strategically aligned with each of Newman's Error Analysis (NEA) stages, allowing the researcher to capture in-depth insights into students' cognitive and metacognitive reasoning processes within a limited timeframe. The students' written answers and interview transcripts were then coded according to the five stages of Newman's Error Analysis (NEA): reading, comprehension, transformation, process skills, and encoding. A coding rubric was developed to define each error type and ensure consistency in classification. Specifically, comprehension errors also included cases in which students failed to state or identify what was being asked in the problem, reflecting an incomplete understanding of the question's objective. The coding framework is summarized in Table 3.

Table 3. The coding framework

No	NEA Stage	Coding Criteria
1	Reading Error	The student fails to recognize or correctly read key information, symbols, or terms in the question.
2	Comprehension Error	The student reads the problem but misunderstands the meaning or relationships between given quantities, or fails to identify what is being asked.
3	Transformation Error	The student understands the problem but applies an incorrect formula, fails to model the situation mathematically, or constructs a wrong equation.
4	Process Skills Error	The student selects the correct formula but makes calculation or algebraic manipulation mistakes.

5	Encoding Error	The student obtains the correct result but fails to communicate it properly or omits the logical conclusion.

After data collection through the tests and interviews, the next stage was qualitative data analysis to gain a deeper understanding of student errors. Methodological triangulation between written tests and interviews played a crucial role in ensuring that the findings were both empirical and theoretically enriched, in accordance with the interpretative depth required in qualitative educational research. In this study, triangulation was not limited to using interview data as supporting explanations for test results; rather, it was applied to cross-check and interpret the consistency or divergence between the two data sources. For instance, students who made similar written errors were compared in terms of their verbal reasoning during interviews to confirm whether the same cognitive barriers were evident across methods. Conversely, when discrepancies arose, such as when students provided correct written answers but expressed conceptual confusion during interviews, these differences were analyzed to uncover hidden misconceptions or surface-level procedural understanding. This analytical approach strengthened the validity of the interpretation by allowing both convergence and divergence of findings to inform the discussion.

In this study, the data analysis method used was qualitative data analysis. The qualitative analysis process aims to organize data, group it into manageable parts, synthesize it, identify patterns, reveal significant and valuable insights, and determine the narrative to be conveyed to others. This data analysis process includes data reduction and data presentation. The steps taken in the data reduction process include simplifying the data by summarizing it, highlighting important aspects, focusing on its essence, identifying patterns, and eliminating irrelevant data. The steps taken in the data presentation process include presenting the data in the form of concise narratives, graphs, interconnections between categories, flowcharts, and other visual aids (Younas et al., 2022). In the final stage, a thematic synthesis was conducted to link participants' verbal responses to specific stages of error development and dimensions of critical thinking. This process allows researchers to make interpretive inferences regarding students' conceptual understanding, procedural fluency, and self-regulatory awareness in problem-solving. To increase the rigor and credibility of the analysis, the coding process was conducted by a single researcher, supported through researcher triangulation in the form of peer discussions and repeated reviews, to ensure consistency and minimize subjective bias. Specifically, two additional researchers with backgrounds in mathematics education independently reviewed and coded a subset of students' written responses and interview transcripts. The independent coding results were then compared and discussed collaboratively with the main researcher to reach a consensus on the interpretation of student errors and their corresponding critical thinking indicators. This process functioned as a verification step to ensure consistency in data interpretation, minimize subjective bias, and strengthen the credibility of qualitative findings through inter-researcher agreement.

▪ RESULT AND DISSCUSSION

The analysis, facilitated by the Newman's Error Analysis (NEA) framework, revealed that each type of student error was closely related to specific critical thinking indicators. Errors at the reading and comprehension stages indicate weaknesses in the ability to clarify and interpret information. For example, several students were unable to restate what was known or asked in the problem, and some directly attempted calculations without identifying key elements. These response patterns explicitly demonstrate students' difficulties in focusing on the question, analyzing the given arguments, and asking or answering clarification questions, three key aspects of the elementary clarification dimension of critical thinking. At the same time, errors at the transformation stage reflect weak inference and decision-making abilities. Furthermore, process skill errors indicate limitations in logical evaluation and the selection of solution strategies. In contrast, encoding errors are related to the inability to communicate results coherently, reflecting students' weak ability to construct coherent mathematical arguments.

Twenty-nine students were given five trigonometry problem-solving problems of varying difficulty. Each problem was designed to represent a specific stage of critical thinking, ranging from basic problems involving the identification of triangle elements to complex problems based on everyday applications.

Analysis of Student Errors

The distribution of student achievement on each question is shown in Table 4.

Table 4. Percentage of student errors at each newman stage

	Reading	Comprehension	Transformation	Process Skills	Encoding
Q1	86.2%	41.4%	37.9%	41.4%	44.8%
Q2	13.8%	37.9%	13.8%	37.9%	93.1%
Q3	31%	34.5%	62.1%	96.5%	96.5%
Q4	37.9%	41.4%	37.9%	82.7%	100%
Q5	100%	100%	96.5%	96.5%	100%

To observe the variation in students' error patterns across Newman's stages more clearly, the data are presented per question. Therefore, the data from Table 4100% stacked bar chart, as shown in Figure 1. It should be noted that the data in this figure are presented differently from the table, as they have been adjusted to a 0–100% scale to illustrate the proportional distribution of errors at each Newman stage for every question.

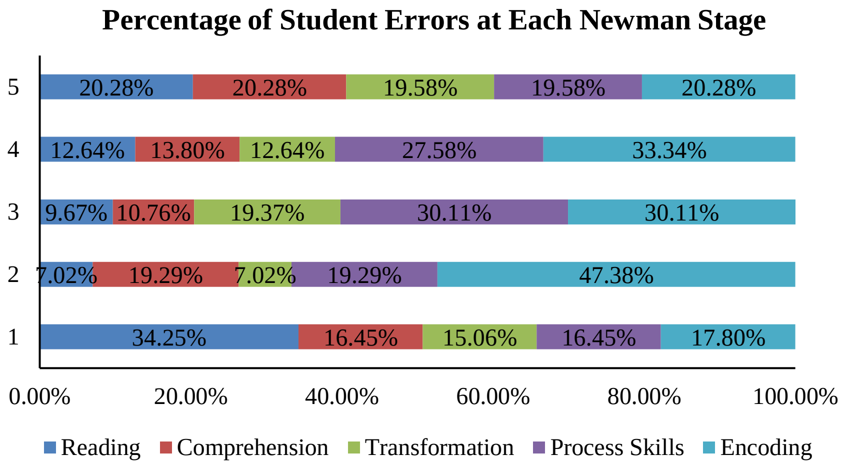


Figure 1. Percentage of student errors at each newman stage (100% stacked bar chart)

Figure 1 presents a 100% stacked bar chart illustrating the proportion of student errors across the five stages of Newman's Error Analysis (Reading, Comprehension, Transformation, Process Skills, and Encoding) for each trigonometry question (Q1–Q5). Each bar represents one question, while the colored segments indicate the relative percentage of errors within each stage. The figure shows that the Encoding and Process Skills stages occupy the largest portions, particularly for Questions 4 and 5, where nearly all students made errors. This finding suggests that most students struggled with performing accurate calculations and presenting their final answers correctly. In contrast, the Reading and Comprehension stages display relatively smaller portions, suggesting that only a few students struggled to understand the questions or identify known and unknown quantities. Nevertheless, noticeable variation can still be seen in the Transformation stage, where students often failed to translate contextual information into appropriate trigonometric representations.

Table 4 presents the percentage of students who made errors at each stage of Newman's procedure for every trigonometry problem. The data reveal variations in error patterns across questions, indicating that students' difficulties shifted depending on the cognitive demand of each task. At the Reading stage, 86.2% of students made errors on Q1, suggesting initial difficulty in understanding the problem statement. In contrast, the percentage decreased sharply in Q2 (13.8%), indicating that students became more familiar with the task format. Errors at the Comprehension stage ranged from 34.5% to 100%, showing that several students struggled to interpret what was being asked and to relate it to relevant mathematical concepts. At the Transformation stage, errors varied moderately, with the highest error rate (96.5%) in Q5, where students failed to convert contextual information into appropriate mathematical models. The Process Skills stage showed consistently high error rates, particularly in Q3–Q5 (82.7%–96.5%), revealing students' weaknesses in procedural fluency, computational accuracy, and logical sequencing of steps. Finally, the Encoding stage showed the highest and most persistent errors across almost all questions, reaching 100% in Q4 and Q5, suggesting that while many students could perform calculations, they struggled to communicate final answers coherently or to draw valid conclusions. These findings indicate that as problem complexity increases.

This integrated interpretation allows NEA to function not merely as an error classification tool but also as a framework for diagnosing unachieved dimensions of critical thinking. Specifically, comprehension and transformation errors frequently observed in Question 1 reflect weaknesses in Elementary Clarification, as students struggled to interpret the problem statement and identify the relationship among triangle elements. Errors in Question 2 indicate deficiencies in Basic Support, where students failed to justify procedural steps or provide valid reasoning for the application of trigonometric formulas. Transformation and process skill errors in Question 3 demonstrate a lack of Inference, as students were unable to draw logical conclusions from the given data or connect results from different sources. In Question 4, frequent process skills and encoding errors signify weaknesses in Logical Evaluation, as students were unable to assess the accuracy of their reasoning or verify the consistency of their solutions. Finally, errors in Question 5 correspond to deficiencies in Strategies and Tactics, as students showed difficulty designing systematic and efficient problem-solving plans for contextual trigonometry problems and failed to communicate coherent conclusions.

Most Frequent Errors

The research results showed that the most frequent errors occurred at the Encoding stage (86.9%), when students had to write a final answer that referred to the question or presented a conclusion. The high number of errors at this stage indicates that although some students were able to perform calculations or solve problems very well, they were not yet accustomed to explaining or concluding their results coherently, clearly, and logically (Ahzan et al., 2022; Ramadhani et al., 2024).

Several factors may explain this phenomenon. First, most students lack mathematical literacy skills, particularly in writing coherent mathematical arguments. This is evident in the large number of answers that stop midway through the process, without a valid conclusion. Second, weak reflection skills prevent students from reviewing their solution steps, often leaving out the final answer or conclusion. Third, encoding errors also arise because students still perceive calculation as the ultimate goal, not the presentation of results (Ahzan et al., 2022; Bye et al., 2024). However, in the context of mathematics learning, the ability to explain or summarize answers plays a crucial role in demonstrating conceptual understanding.

Compared to other stages, errors in the encoding stage were significantly higher in problem-solving. This indicates that even students who correctly read the problem and transformed it into a mathematical model still failed in the skills process (71%) and encoding (86.9%). In other words, encoding remains a major weakness that hinders students' overall success in solving problems.

In conclusion, the Encoding stage is the most critical in the problem-solving process because it serves as a bridge between calculation results and mathematical communication. The high error rate at this stage highlights the need for learning interventions to focus not only on concepts and procedures but also on students' ability to organize, summarize, and draw conclusions from their answers. Teachers need to emphasize the importance of writing down conclusions, for example, by encouraging students to practice problems that require them to include complete solution steps along with statements of results and conclusions.

Analysis of the Case Study of Three Subjects

To gain a deeper understanding of these errors, three students representing high (S1), medium (S2), and low (S3) ability levels were interviewed and analyzed in detail. Table 5 summarizes each student's achievement across Newman's stages.

Table 5. Student achievement at the newman stage

Newman Stage	(S1) High					(S2) Medium					(S3) Low				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
Reading	✓	✓	✓	✓	✗	✗	✓	✓	✓	✗	✗	✓	✗	✓	✗
Comprehension	✓	✓	✓	✓	✗	✓	✓	✓	✓	✗	✓	✓	✗	✗	✗
Transformation	✓	✓	✓	✓	✓	✓	✓	✗	✓	✗	✓	✓	✗	✗	✗
Process Skills	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✓	✗	✗	✗	✗
Encoding	✓	✗	✓	✗	✗	✓	✗	✗	✗	✗	✓	✗	✗	✗	✗

High-Level Student

This highly capable student demonstrated quite good achievement across all Newman stages. In the Reading stage, S1 was able to read questions and correctly recognize the symbols used, although there were still errors in one question at this stage. In the Comprehension stage, S1 was able to determine the information known and asked, although in one question, he still demonstrated a lack of understanding. In the Transformation stage, S1 was always able to transform verbal information into an appropriate mathematical model. In the Process Skills stage, S1 was able to correctly complete calculations on almost all questions. However, in the Encoding stage, S1 still had difficulty presenting the final answer coherently and completely on several questions, especially those that were contextual. The following is the result of an interview with S1.

Q1: *What do you think about math?*
S1: *Math is a challenging and enjoyable subject, especially when you solve problems correctly. So, I love math!*
Q2: *What is the most common difficulty you face when working on math problems, especially trigonometry?*
S1: *The difficulty lies in applying and verbalizing the answers to everyday life.*
Q3: *Of the problems you worked on, which one did you find most difficult? Why?*
S1: *Number 5, because it is a contextual trigonometry problem, it is quite difficult, and the time limit is tight.*
Q4: *In your opinion, what learning media is most helpful in understanding trigonometry?*
S1: *Visual media is good, but conventional media is also good.*

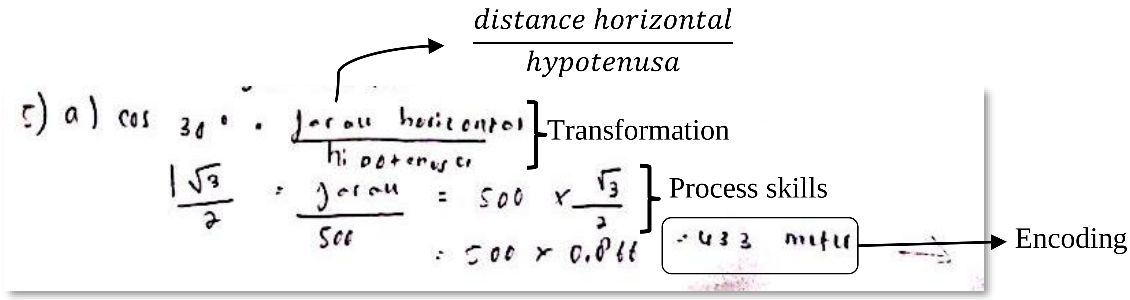


Figure 2. S1 answer (1)

Based on S1's answer in Figure 2, it can be seen that the student was able to perform the transformation by writing the formula from. This indicates that the student was able to transform the problem into a mathematical model. At the process skills stage, S1 also appeared to be able to perform calculations correctly, namely, substituting values. However, at the encoding stage, the student did not write the conclusion in his answer, so the encoding stage was incomplete. In addition, the student did not write the reading and comprehension stages, because he immediately jumped to using the formula without explaining the information known and asked in the problem. These results align with interviews that revealed students found contextual trigonometry problems (Q3) challenging, primarily due to time constraints. This explains why S1 only completed part

(a) of the problem and did not continue to part (b), and tended to neglect the initial stages (reading and comprehension) in order to save time.

The missing reading and comprehension stages in S1's responses were primarily due to time constraints, not a lack of conceptual understanding. Evidence from other questions suggests that S1 is capable of completing these stages when sufficient time is available. The incomplete portion of Question 5(b) also supports this interpretation, as the student mentioned in the interview that the time limit was tight. However, the incomplete coding stage, where S1 failed to restate or interpret the final result, appears to stem from a different cause. In several earlier items, S1 repeatedly omitted explicit conclusions even when the calculation was correct. This pattern indicates a limited ability to translate numerical answers back into contextual meaning, suggesting that S1 tends to view calculations as the end of problem-solving rather than as part of a broader reasoning and communication process.

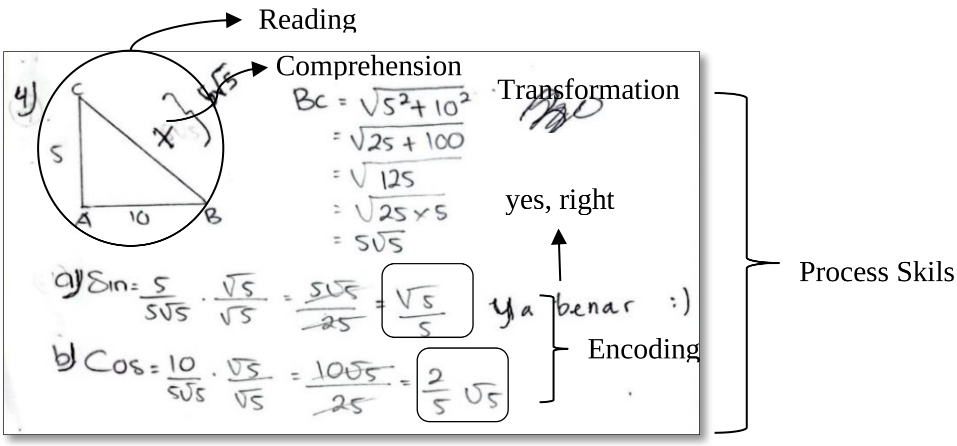


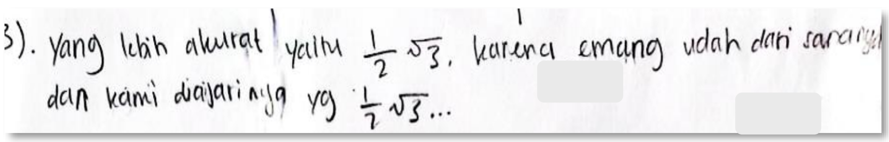
Figure 3. S1 Answer (2)

As seen in Figure 3, S1 demonstrated a more coherent solution process compared to the previous question. In the reading and comprehension stages, the student first drew a triangle and wrote down the known information, indicating that they could identify the elements of the triangle. In the transformation stage, S1 used the Pythagorean Theorem to determine the length of the unknown side, then proceeded to the process skills stage by systematically calculating the sine and cosine values. The calculation results were relatively correct and in accordance with trigonometric procedures. However, in the encoding stage, although the student had written the final answer in the form of a radical fraction, the result was still incomplete because it was not accompanied by a conclusion or units appropriate to the context of the problem. Thus, although S1 generally successfully demonstrated all stages of the solution, weaknesses in the encoding stage remained unchanged from the previous answer.

Intermediate-Level Students

This intermediate-ability student demonstrated varying achievement across Newman's stages. In the Reading stage, S2 was able to read the questions and correctly recognize the symbols used in most of them, although there were still errors in two questions, namely questions 1 and 5. In the Comprehension stage, S2 was able to determine the known and asked information in almost all questions, except for an error in question 5. In the Transformation stage, S2 successfully converted verbal information into a mathematical model in the first three questions. In the Process Skills stage, S2 was able to complete the calculations correctly, but there were also calculation errors. This then impacted the Encoding stage, where only question 1 was written correctly by S2, while the final answers for the other questions were either not written or did not match the question. Overall, S2's main weaknesses were in process skills and presentation of the final answer, especially for application questions. The following is the result of an interview with S2.

Q1: *What do you think about math?*
S2: *Math is a subject that is both easy and difficult, but I do not really like math.*
Q2: *What is the most common difficulty you face when working on math problems, especially trigonometry?*
S2: *The difficulty is applying and combining words into answers.*
Q3: *Of the questions you have worked on, which one did you find most difficult? Why?*
S2: *Number 3 because I can differentiate, but have difficulty using the words.*
Q4: *In your opinion, what learning media is most helpful in understanding trigonometry material?*
S2: *Visual media such as animated videos or triangle props are very helpful. With props*



Reading & Compherension

3) The more accurate one is $\frac{1}{2}\sqrt{3}$, because it's been like that from the start and we were taught $\frac{1}{2}\sqrt{3}$

Figure 4. S2 answer (1)

Based on S2's answer in Figure 3, it appears that the student was able to complete the reading and comprehension stages, as he understood that the question required a comparison of cos 30° values from two different sources and to determine which was more accurate. In the transformation stage, S2 did not write down the calculation process or mathematical proof; instead, they directly wrote down the value of cos 30° from memory. This indicates that the student tended to use a memorization approach rather than comparing two values from different sources. In the process skills stage, the student had not demonstrated further calculation skills to convert the value of cos 30. In the encoding stage, the answer was also incomplete because it only stopped at the statement "which is more accurate" without a more detailed mathematical explanation or conclusion. These results align with the interview (Q3), which showed that S2 found question number 3 the most difficult, not because he could not distinguish the correct value, but because he had difficulty expressing it in words. This reinforces that the student's main weakness lies in his word management skills, namely, explaining mathematical reasons coherently, even though he was actually able to recognize the correct answer.

These findings also demonstrate that S2's predominant process skills errors reflect weaknesses in the Logical Evaluation aspect of critical thinking. Although S2 could perform basic calculations, the inability to verify the correctness of each procedural step or to justify the logical connection between results indicates a deficit in evaluating the coherence of reasoning. This aligns with the Logical Evaluation indicator, where students are expected not only to perform operations but also to assess whether their chosen method and outcome are logically sound within the context of the problem.

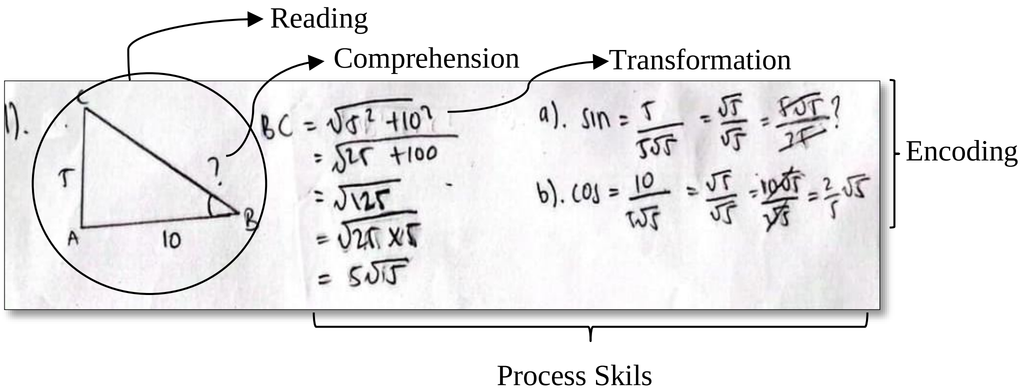


Figure 5. S2 answer (2)

Based on S2's answer in Figure 4, it can be seen that the student performed well in the reading and comprehension stages. In the transformation stage, the student was able to write a mathematical model using the Pythagorean Theorem to find the length of the hypotenuse BC. This demonstrates that S2 is capable of transforming the problem into an appropriate mathematical model. In the process skills stage, the student continued by performing calculations to determine the sine and cosine values. However, in part (a), the student's answer was incorrect because there was an error in simplifying the calculation result of the sine value. Meanwhile, in part (b), the student correctly answered the requested cosine value. This shows that S2 was quite thorough in several calculation steps. In the encoding stage, the student's answer also appeared incomplete because it only stopped at the result number without clearly stating the conclusion.

Low-Level Students

This low-ability student has not consistently achieved all of Newman's stages. In the Reading stage, S3 was only able to read and recognize symbols correctly in a few questions. In the Comprehension stage, S3 was only able to determine the known and requested information in questions 1 and 2, but failed to do so in the following three questions. In the Transformation stage, S3 was also only able to transform information into a mathematical model in a few other questions, but was unable to complete them. The dominant error occurred in the Process Skills stage, where S3 only correctly answered the first question. Consequently, in the Encoding stage, S3 only managed to write the final answer to the first question and failed to provide a correct conclusion in all the other questions. The following is the result of an interview with S3.

Q1: *What do you think about math?*
S3: *Math is difficult, in my opinion.*
Q2: *What is the most common difficulty you face when working on math problems, especially trigonometry?*
S3: *The difficulty is that I am confused about the formulas and how to solve them because they are complicated.*
Q3: *Of the questions you have worked on, which one did you find the most difficult? Why?*
S3: *Number 4 was really tricky, and I forgot the formula.*
Q4: *In your opinion, what learning media is most helpful in understanding trigonometry material?*
S3: *Visual media such as animated videos or props*

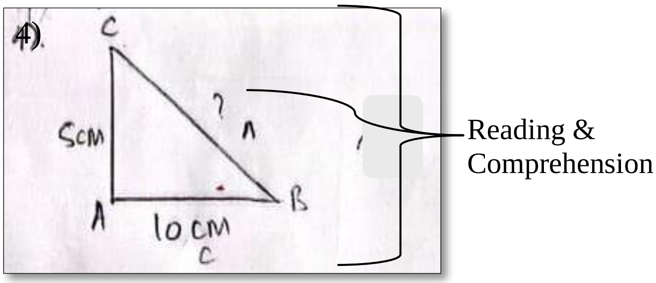


Figure 6. S3 answer

Based on S3's answer in Figure 6, it can be seen that the student only wrote down a picture of a triangle and the known side lengths, namely AC = 5 cm and AB = 10 cm. This indicates that the student has only reached the reading and comprehension stage, namely, being able to copy the basic information from the problem into a picture. However, the student was unable to proceed to the transformation, process skills, or encoding stages. Thus, the student did not use the Pythagorean Theorem or trigonometric ratios to find the length of the hypotenuse, nor did he provide a final answer.

In addition to the transformation and process skill difficulties, S3's responses also revealed persistent weaknesses in logical monitoring and metacognitive awareness. The student did not attempt to verify or recheck the given information or computation steps, indicating that the learning focus was primarily procedural rather than reflective. Compared to S1 and S2, who at least attempted partial transformations or encoded partial conclusions, S3's approach stopped early in the reasoning chain, suggesting that the student's cognitive load was already overwhelmed at the comprehension stage. This aligns with Nurjailam et al. (2021), who found that low-ability students often experience cognitive overload when faced with multi-step trigonometric problems, leading to early-stage disengagement. Furthermore, interview data reinforce the conclusion that S3's difficulties stem from both cognitive and affective factors. Statements such as "Math is difficult" and "I forgot the formula" reflect a lack of confidence and motivation, which may further hinder persistence in the reasoning process. These emotional barriers can exacerbate the observed comprehension and transformation errors. Hence, learning interventions for low-performing students should not only target conceptual understanding but also promote self-efficacy and reflective engagement through gradual exposure to problem-solving steps, accompanied by structured teacher feedback.

From a critical thinking perspective, S3’s failure in the transformation stage corresponds to a deficiency in the Inference dimension. The student was unable to infer missing relationships between the known and unknown quantities or select an appropriate mathematical model, revealing an inability to draw logical connections from partial information. This indicates that the transformation errors are not merely procedural but conceptual, suggesting that the student struggled to construct meaning and draw inferences based on the given data.

The results of this study provide important insights into students' errors in solving trigonometry problems, analyzed using NEA. When linked to critical thinking indicators, it is clear that each stage of the NEA is associated with specific weaknesses in students' problem-solving processes. The overall findings indicate that the encoding stage is the most problematic among all Newman stages, followed by process skills. This pattern reveals that although students can often perform calculations correctly, they struggle to communicate results coherently and draw valid conclusions. Interestingly, this dominance of encoding errors is not unique to this study. Similar findings were reported by Ahzan et al. (2022) in algebra and Siskawati et al. (2023) in geometry, where students also tended to omit conclusions or misstate final answers. This consistency across mathematical domains suggests a broader theoretical issue: Indonesian students often perceive problem-solving as ending once numerical computation is complete, overlooking the metacognitive step of verifying and articulating results. Within the NEA framework, this reflects a deficiency in the Strategies and Tactics dimension of critical thinking; students can execute algorithms but fail to manage or evaluate their reasoning process. As shown in Table 4, the highest error percentages occurred in the process skills (71%) and encoding (86.9%) stages. The high frequency of process skill errors also aligns with Kenney & Ntow (2024) and Sari & Valentino (2017), who found that many students fail to integrate procedural fluency with logical evaluation. However, compared to previous NEA-based studies in other mathematical domains, the dominant type of error differs. For instance, Kenney & Ntow (2024) reported that transformation errors were most frequent in algebraic word-problem solving because students struggled to convert verbal statements into symbolic form. Similarly, Sari & Valentino (2017) found that Indonesian students frequently made transformation and comprehension errors when solving PISA-based mathematical problems, as they often failed to link contextual information with suitable mathematical representations. Unlike those findings, the present study revealed that errors were more dominant at the encoding stage, suggesting that trigonometric problem solving demands not only contextual understanding but also the ability to articulate and verify conclusions coherently. This divergence indicates that trigonometry imposes a higher representational and metacognitive load than algebra. Theoretically, this means that while algebraic problem solving primarily challenges students’ symbolic manipulation skills, trigonometric reasoning requires coordinating multiple representational systems: geometric visualization, numerical computation, and contextual interpretation.

At the reading and comprehension stages, errors reflected students’ limited ability to focus on the problem and clarify information dimensions corresponding to the

Elementary Clarification level of critical thinking as they often failed to interpret problem statements accurately or connect symbolic representations to contextual meaning, a finding consistent with (Abdullah et al., 2015), who noted that early misinterpretation of problem language frequently triggered subsequent computational mistakes. Similarly, errors at the transformation stage primarily indicated deficits in Inference, as students struggled to connect known and unknown quantities or select appropriate mathematical representations, aligning with Tambychik & Meerah (2010), who emphasized that reasoning breakdowns occur when students cannot infer relationships between problem data and the underlying concept.

Based on the analysis of the three ability levels, the results show distinct patterns. High-ability students (S1) performed well in the reading, comprehension, and transformation stages, with only a few errors in processing and encoding skills. Their difficulties primarily arose in contextual problems, where students did not write complete conclusions despite correct calculations (Aydın & Özgeldi, 2019; Hughes et al., 2020). Medium-ability students (S2) were able to achieve the initial stages of learning. However, they struggled with processing and encoding skills on most problems, indicating that, despite understanding the problem, they lacked thoroughness in their calculations, writing final answers, and conclusions (Elagha & Pellegrino, 2024). Low-ability students (S3) exhibited weaknesses in almost all stages, particularly in process skills, which subsequently led to consistent encoding errors (Ramadhani et al., 2024). These results confirm that student ability levels significantly influence the types and frequency of errors that occur at each of Newman's stages.

Interview results also support these findings. High-ability students reported that time constraints and encoding were the main obstacles. Intermediate-ability students reported that the main challenge lay in applying mathematical concepts to everyday life and formulating answers verbally, in line with their errors at the encoding stage. Meanwhile, low-ability students reported that trigonometry itself was challenging from the outset, resulting in errors that appeared early on and persisted into subsequent problems. Based on these findings, several context-specific solutions can be proposed. Because the most frequent errors occurred in the encoding stage, teachers should implement reflective verification tasks that require students not only to perform calculations but also to explicitly restate their conclusions in verbal or written form. For example, after completing each trigonometric problem, students can be asked to write a brief statement explaining how their numerical result relates to the geometric context. This activity targets the Strategies and Tactics dimension of critical thinking by strengthening students' ability to translate reasoning into coherent mathematical communication. For process skill errors, particularly those related to logical evaluation, instruction should include error-based learning exercises in which students analyze and correct intentionally flawed trigonometric solutions. This approach encourages metacognitive reflection and logical verification, helping students monitor whether each computational step aligns with the geometric structure of the problem. Overall, these targeted interventions directly address the dominant error types identified in this study, specifically encoding and process skills, while simultaneously fostering the critical thinking dimensions of logical evaluation and strategic reasoning within the context of trigonometry.

▪ CONCLUSION

This study was conducted on students who had previously studied trigonometry at the high school level. Participants were grouped into three categories based on their learning achievement: high, medium, and low. The primary objective of this study was to identify the types of errors most frequently made by students when solving trigonometry problems and to investigate the underlying factors that contribute to these errors. The research findings revealed that the highest and most persistent errors occurred in the encoding stage, with an overall error rate of 86.9%, reaching 100% in several items (Q4–Q5), followed by consistently high process skill errors (82.7%–96.5%). Transformation errors occurred moderately (up to 96.5%). In comparison, comprehension errors ranged from 34.5% to 100%, and reading errors were fewer but more prevalent in Q1 (86.2%), indicating that many students can calculate but struggle to articulate coherent conclusions. This pattern suggests a deeper cognitive implication: students tend to perceive mathematical problem-solving as ending with numerical computation, overlooking the metacognitive step of validating and articulating their reasoning. Such a tendency reflects a procedural orientation rather than a reflective one, highlighting a gap between operational proficiency and conceptual understanding in trigonometric thinking. These errors suggest that many students were able to perform calculations but struggled to draw coherent conclusions, indicating weaknesses in metacognitive monitoring and mathematical communication. This pattern suggests a cumulative reasoning process in which early-stage errors in transformation or calculation propagate to encoding difficulties, indicating the interdependence of cognitive stages in mathematical problem-solving. The study also contributes theoretically by empirically mapping NEA stages to critical thinking dimensions such as Elementary Clarification, Inference, Logical Evaluation, and Strategies and Tactics. Thus, NEA is shown not only as a diagnostic tool for identifying student errors but also as a framework for interpreting unachieved aspects of higher-order reasoning.

The implications of these findings are twofold: practical and theoretical. Practically, teachers can employ NEA serves not only as a diagnostic tool for identifying procedural errors but also as a lens for mapping cognitive and critical thinking dimensions across problem-solving stages, such as reflective verification exercises requiring students to restate conclusions (encoding) and error-based discussions that enhance logical evaluation and metacognitive monitoring (process skills). Theoretically, the dominance of encoding errors in trigonometry, unlike transformation errors commonly found in algebra, reveals the unique cognitive challenge of linking numerical computation with geometric interpretation. This insight extends the theoretical scope of NEA and highlights the need for instruction emphasizing reflective reasoning over procedural repetition. The study's limitation lies in its small, context-specific sample and reliance on a single coder; thus, future research should include more diverse participants and independent coders to improve inter-rater reliability. Despite these limitations, this study provides a meaningful contribution by bridging cognitive error analysis and critical thinking in mathematics education. This approach aligns with the vision of 21st-century education, which places critical thinking skills as a core competency in shaping a generation of learners. Specifically, by revealing students' weaknesses in logical evaluation and encoding, this study provides a concrete roadmap for developing learning strategies that move beyond procedural fluency toward reflective mathematical reasoning thereby operationalizing the ideals of 21st-century learning within real classroom practice.

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