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# The Mechanism of Didactical Obstacles in the Pythagorean Theorem: From Visual Rigidity to Procedural Failure

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**Abstract:** Learning the Pythagorean theorem is a significant challenge at the junior high school level because students often struggle to understand concepts, connect geometric and algebraic representations, and solve contextual problems. Based on previous studies, students' difficulties indicate the presence of learning obstacles. Existing research has addressed students' difficulties, errors, and epistemological obstacles in solving Pythagorean theorem problems and has presented applications of the Pythagorean theorem. Therefore, this study aims to analyze students' didactic learning obstacles to the Pythagorean theorem topic. To achieve this goal, a qualitative case study was conducted. Data was collected through data triangulation: written tests, interviews, and document studies. At the data-collection stage, 30 students and two teachers participated. Based on the written test results, the answers exhibit various characteristics. At the analysis stage, it is performed using ATLAS.ti software. The results show that there is a form of didactic learning obstacles consisting of visual orientation obstacles and formula procedural obstacles. The Visual orientation obstacles include students' lack of understanding of triangle concepts. The procedural obstacles include students' incomprehension of algebraic representations, understanding of problem-solving, understanding of procedures beyond integers, and application of Puythagos' theorem formulas. Visual orientation obstacles cause formula procedural obstacles. The didactic factor that creates obstacles is the way the topic is presented and the teacher's approach to designing learning. Didactic obstacles analysis is an important step in formulating a hypothesis about how a concept should be taught. By knowing the didactic obstacles, teachers or researchers can develop a more accurate Hypothetical Learning Trajectory (HLT). This will lead to the design of learning activities that anticipate common mistakes and misconceptions.

**Keywords:** didactical obstacle, learning obstacle, topic presentation analysis, textbooks, pythagorean theorem.

## ▪ INTRODUCTION

Mathematics plays a role in various disciplines, including the role in problem-solving. According to Bahar, Can, & Maker (2024) "*Mathematical knowledge plays a pivotal role in producing original problem-solving behaviors*". The process of problem-solving requires a clear thought process and understanding. Mathematics can be understood as a process of thinking and understanding, in line with the idea of "ways of thinking and ways of understanding" introduced by Harel (2008). This definition describes a strong relationship between mental activity, thought processes, and mathematical understanding. The pedagogical implications arising from such definitions are significant because they provide deep insights into how humans use their minds to understand abstract mathematical concepts. Thus, mathematics plays a vital role in problem-solving.

The Pythagorean theorem, as part of mathematics, is a fundamental concept in mathematics in a variety of contexts, including geometry, physics, and engineering (Bheda, 2024). Understanding and applying this theorem is essential for solving

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geometric problems and measuring distances and angles in real-world contexts, including in science. Science learning relies heavily on visual illustrations and visually organized tasks as part of a method of learning and proof-of-concept that requires the Pythagorean theorem (Ayyaswamy et al., 2025; Due, 2024). The Pythagorean theorem has ties to various fields and is often used as a prerequisite for advanced subjects such as trigonometry and calculus. Therefore, the Pythagorean theorem is an important topic for students to study.

Although important, the Pythagorean theorem has a hierarchical and abstract structure, so it is often taught only as a procedural formula, i.e., c^2=a^2+b^2. The teacher does not have a special design for introducing the Pythagorean theorem to students. In fact, this theorem is epistemologically situated between geometry and algebra, so its understanding requires the coordination of various representations, whether geometric, symbolic, or relational. When the concept of the Pythagorean theorem became knowledge to be taught, there was a didactic transposition process, which is a theorem that is historically rooted in Euclid's geometric structure (Huylebrouck, 2025) transformed into definitions, formulas c^2=a^2+b^2, sample problems, and specific types of exercises. According to Bruner (1960) that students go through the stages of mental representation, including the iconic, before entering the symbolic stage. When students are given a problem with an image as an illustration, they store the information in visual form. This means that students cannot directly receive information in symbolic form.

The Pythagorean theorem is indirectly accessible to students without the help of a proper didactic context (Brousseau, 2002a). According to Suryadi (2023), didactics is the epistemology of knowledge diffusion and acquisition in society where the diffusion and acquisition of knowledge need to be formalized as knowledge. Knowledge diffusion is a transposition of knowledge from scholarly knowledge into knowledge to be taught, then to taught knowledge, and finally to learned knowledge received by students (Chevallard, 2019). In addition to the diffusion of knowledge, the acquisition of knowledge needs to be formalized as knowledge. Knowledge acquisition includes action situations, formulation, validation, and institutionalization (Brousseau, 1997). Giving students the space to actively participate in the introduction of new knowledge through their own independent discovery is one of the demands of pedagogical theory and curricular documents (Novotná & Hošpesová, 2013). Therefore, it is important to create the right didactic design to help students build knowledge.

In building knowledge, there is a series of processes from scholarly knowledge to learned knowledge through diffusion and knowledge acquisition that is not an easy journey. The learning situation does not guarantee that it will always be relevant in helping students build their knowledge, so students are vulnerable to learning obstacles (Job & Schneider, 2014). Learning obstacles occur when didactic transposition simplifies concepts too far, while learning situations do not support essential exploration of concepts (Artigue & Winsløw, 2010). The Anthropological Theory of the Didactic (ATD) framework views school mathematics as a set of praxeology consisting of a combination of task types, completion techniques, explanatory discourse, and underlying theories, which are always conditioned by institutional and curriculum contexts (Chevallard & Bosch, 2020). Therefore, important to pay attention to how the praxeological component is presented during diffusion and acquisition when conveying knowledge to students.

Based on a literature study conducted by Bariyah, Sufyani, & Jarnawi (2024), many students have difficulty solving problems involving the Pythagorean theorem. This fact is reinforced by field findings that students have difficulty understanding images, operating with algebraic forms, and solving contextual problems that apply the Pythagorean theorem. Based on student and teacher interviews, students understand what the teacher taught, but do not understand the part of the topic that the teacher did not teach. This means that teachers play a crucial role in shaping students' understanding. When the teacher was interviewed, the teacher said that what was conveyed was very dependent on what was in the mathematics textbook. This indicates that there are obstacles to didactic learning. According to Brousseau (2002b) that didactical obstacles are related to learning that does not pay attention to the stages of thinking in sequence and the hierarchy of knowledge. An ineffective learning process can disrupt the educational process because the structural and/or functional relationships developed are not always grounded in an analysis of student characteristics.

Previous research by Lipowsky et al. (2009) examines how the three basic dimensions of instructional quality affect students' development of understanding of the Pythagorean Theorem. It was not studied how students encountered difficulties. The difficulty in solving the Pythagorean theorem was studied by Hutapea, Suryadi, & Nurlaelah (2015), who identified the epistemological obstacles students encounter in solving it. The study's results indicate a misunderstanding of the concept. There was no study on how the mechanism of misunderstanding of the concept occurs. Furthermore, Voštinár (2018) presented an application of the Pythagorean theorem. The study examined the development of applications to make it easier for students to understand the topic without examining students' difficulties. Then, Rudi, Suryadi, & Rosjanuardi (2020) studied the difficulties students face in understanding and applying the Pythagorean theorem through an onto-semiotic approach. The results showed that students had difficulty understanding definitions, describing the symbols or notations of mathematical objects, and interpreting mathematical objects. In contrast, when solving problems involving the Pythagorean theorem, students clearly described procedures, algorithms, and problem-solving techniques. There was no study of how the mechanism underlying such difficulties occurs. Further, the research by Rahmi, Yulianti, & Prabawanto (2022) examines the existence of learning obstacles, namely ontogenic obstacles, epistemological obstacles, and didactical obstacles, without examining more deeply how the mechanisms of these obstacles can occur. Moreover, AlSalehi & Borkar (2024) examine expansive mapping and relationships in the Pythagorean theorem without touching the didactic area. Existing studies have not examined in depth how didactic obstacles to the Pythagorean theorem are. The mechanism by which didactic obstacles occur has not been examined.

Although learning obstacles have been documented, recent research emphasizes the need to identify how didactic design choices causally create such obstacles. Previous research has rarely systematically linked textbook praxeology to student-specific errors, leaving gaps in understanding the causal mechanisms underlying obstacle formation. To bridge the gap, the study adopted a robust framework, namely Théorie des Situations Didactiques (TSD) (Brousseau, 2002b) and Anthropological Theory of the Didactic (ATD) (Chevallard & Bosch, 2020b). TSD, with its core concepts of milieu, contrat didactique, situation a-didactique, is excellent for analyzing didactic obstacles because it provides a framework for engineering and analyzing the conditions under which knowledge is supposed to be built. TSD allows researchers not only to record errors, but also to track how the failure of the didactic system to create a rich milieu directly leads to students' failure to take cognitive responsibility (situation a-didactique). ATD complement TSD by providing a precise analytical tool: the concept of praxeology.

Praxeology, comprising types of tasks, techniques, technologies, and theories, enables researchers to conduct an in-depth analysis of textbook documents. This framework allows for the specific identification of how the organization of knowledge in textbooks creates flawed praxeology that is directly a source of didactic obstacles. This framework is superior to other frameworks because it provides detailed units of analysis to explain the causal relationship between learning design and learning outcomes. Therefore, this study aims to examine in depth the didactic obstacles that cause learning difficulties in the Pythagorean theorem topic. Therefore, the research question is: What are the forms of didactic obstacles in solving the problem of the Pythagorean theorem? What are the didactic factors as a source of obstacles?

Didactic obstacles analysis is an important step in formulating a hypothesis about how a concept should be taught (Gravemeijer & Cobb, 2006). An in-depth understanding of the didactic obstacles to this topic of the Pythagorean theorem is crucial for pedagogical progress because it identifies critical conditions for situations that can be presented to students and in which students will engage in activities that will allow them to build a specific meaning and understanding of a particular concept (Laborde, 2014). Didactic theory-study and use case studies of ATD emphasize that a focus on knowledge structures (not just procedures) reduces systematic misconceptions (Brousseau, 2002b), and analysis of didactic obstacles can effectively improve the didactic design designed by teachers to achieve the expected learning objectives (Rezat & Sträßer, 2012).

## ▪ METHOD

### Research Design

In this study, the TSD by Brousseau (1997) was used. It served as an operational framework for designing analytical instruments and procedures to identify students' didactic obstacles to the Pythagorean theorem and to apply the praxeology framework outlined by Chevallard (2019) to analyze textbook content. In textbook analysis, the milieu is realized through the analysis of representation and the task context: how concepts are presented (e.g., the use of symbols, graphs, or the problem context). The goal is to determine whether the milieu presented limits on students' strategies, thereby preventing the emergence of effective didactic situations. Didactic contracts are implemented through task-structure analysis, such as the order of presentation, coherence between tasks, and directive guidance. The goal is to uncover the implications of the didactic contract that exists in the book, for example, whether the book encourages students to memorize (a bad didactic contract) rather than construct a concept (an ideal didactic contract). The ad hoc approach is used in analyzing self-practice problems: determining whether a task or problem actually demands problem-solving in the absence of direct clues from the book's text or previous examples. The goal is to assess the potential of textbooks to create situations in which students must take responsibility for their own strategies without the textbook's direct intervention.

This study aims to examine in depth the didactic obstacles that lead to learning difficulties. This research uses a qualitative case study to explore students' experiences solving Pythagorean theorem problems, teachers' experiences teaching Pythagorean theorems, and textbook presentations of the Pythagorean theorem. According to Brousseau (1997), exploring the didactic situation can be done by analyzing the presentation of textbooks. The textbook used was a mathematics textbook for grade VIII of junior high school, based on the Merdeka curriculum, published by the Ministry of Education and Culture in 2022. This textbook was used by teachers when teaching the Pythagorean theorem.

The research was conducted in May 2025 at a junior high school in Bandung, Indonesia. The selection of this school was based on the finding that students had difficulty solving the Pythagorean theorem problem, and this school used the books in question in its instruction. Given the importance of solving the Pythagorean theorem, this study aims to examine in greater depth how didactical obstacles arise that cause learning difficulties.

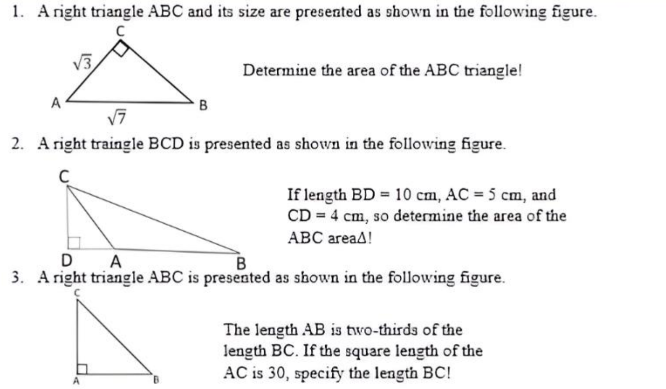
### Instruments

Each type of research requires instruments used during the research process. In this qualitative research, the main instrument was the researcher. The additional instruments used were test instruments and non-test instruments. Both types of instruments were required to answer the research questions. The test instrument included a series of questions on the Pythagorean theorem to uncover research questions about didactical learning obstacles.

The writing test was specifically designed as an experimental environment to trigger the suspected learning obstacles identified through textbook analysis. In this study, milieu was applied in the design of test questions. The consistent mistakes of students in this milieu show a learning obstacle. Didactic contracts were implemented during the analysis of student responses. If students can answer only questions that resemble textbook examples, it indicates excessive adherence to the textbook's didactic contract. The goal is to assess the extent to which didactic contracts limit students' ability to adapt and build new knowledge. The a-didactic situation is applied to analyze the quality of students' answers. The goal is to identify learning obstacles when students fail to function a-didactically, for example, they fail to validate their own solutions or simply use procedures without conceptual understanding.

Meanwhile, non-test instruments included praxeology analysis guidelines, student interview guidelines, and teacher interview guidelines. The praxeology analysis guidelines involve four elements: task types, techniques, technology, and theory. Student interview sheets were used to identify the various difficulties they face when completing assignments. The keywords in the questions were students' understanding of the content, how to solve it, and the difficulties they face. Meanwhile, the interview sheet for teachers aims to explore teachers' views on the presentation of the topic, the explanation of the difficulties students encounter, and the classroom implementation of the Pythagorean theorem.

### Test Instrument


1. A right triangle ABC and its size are presented as shown in the following figure.

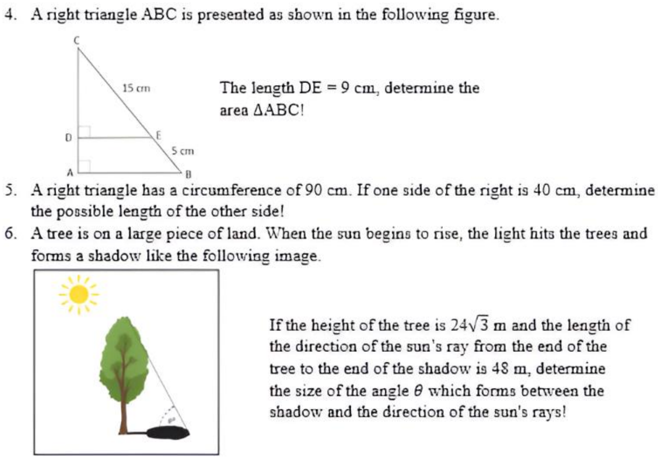
Determine the area of the ABC triangle!

2. A right traingle BCD is presented as shown in the following figure.

If length BD = 10 cm, AC = 5 cm, and CD = 4 cm, so determine the area of the ABC areaΔ!

3. A right triangle ABC is presented as shown in the following figure.

The length AB is two-thirds of the length BC. If the square length of the AC is 30, specify the length BC!


4. A right triangle ABC is presented as shown in the following figure.

The length DE = 9 cm, determine the area ΔABC!

5. A right triangle has a circumference of 90 cm. If one side of the right is 40 cm, determine the possible length of the other side!
6. A tree is on a large piece of land. When the sun begins to rise, the light hits the trees and forms a shadow like the following image.

If the height of the tree is $24\sqrt{3}$ m and the length of the direction of the sun's ray from the end of the tree to the end of the shadow is 48 m, determine the size of the angle $\theta$ which forms between the shadow and the direction of the sun's rays!



**Figure 1.** Test Instrument

**Participants**

This study involved 30 students during a written test. Of the 30 students, 19 were selected for an interview. A total of 19 students had criteria: 1) had different answer characteristics, both true, wrong, or non-answering, 2) if there were the same answer characteristics, then students who had good communication skills were selected according to the teacher's recommendation. According to Creswell (2013), selecting participants who can explain the reasons behind the problem-solving process and errors is crucial to uncovering the root causes of cognitive. The two teachers who taught the Pythagorean theorem were selected for an interview.

The school in this study is located in Bandung Regency, Indonesia, and is state-run. The school has a total of 1,024 students: 434 male and 608 female. The school obtained an A accreditation. Academically, every year, representatives of this school rank among the top five at the district level in government-organized Olympic activities. The academic condition of the students sampled varies. According to the teacher, 19 students: three have above-average abilities, 13 have average abilities, and three have below-average abilities.

**Data Collection**

Data collection in this study was carried out through data triangulation, namely tests, interviews, and documents. The test was conducted on 30 students. The test content was prepared by the researcher based on the results of analyzing the topic presentation in the textbook, including a triangular image that varies as shown in Figure 2 below.

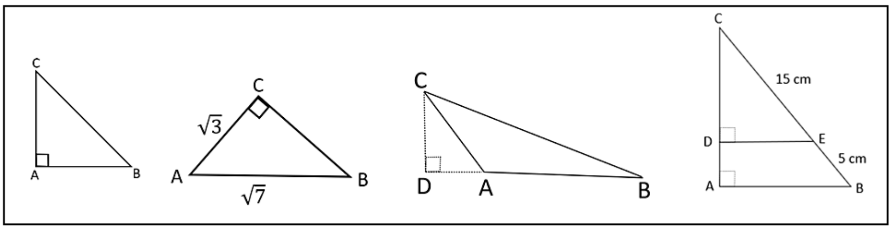



**Figure 2.** Triangle variations

In addition to presenting varied triangle images, the test also included a combination of contextual and non-contextual questions. The proportion of contextual and non-contextual questions is 1:5, based on the findings of textbook analysis. Furthermore, the test questions focus on procedural questions. The conceptual questions were given as follow-ups during the interview, covering the rationale for the procedure used, understanding the Pythagorean theorem, the algebraic representation, and the parts of a triangle. The problem starts with applying the Pythagorean theorem to determine the area of a triangle. Next, a different variation of the triangle is presented, with the same question: determine the triangle's area. The third question contains the usual triangle and story questions that test students' algebraic skills. The fourth question tests students' understanding of the Pythagorean theorem and also applies another concept: cohesion. The fifth question tests students' understanding of translating story problems into mathematical models that include algebraic representations, and the sixth question tests their understanding of solving angular problems using the Pythagorean theorem. The test is given to examine the possibility of didactic obstacles students may encounter.

Furthermore, the interview was conducted after the researcher observed the students' test results on the Pythagorean theorem. This interview was conducted with teachers and students to obtain direct information from participants about the learning activities for the Pythagorean theorem topic and to clarify the researcher's observations of students' test results, including the challenges or difficulties faced, from both the teachers' and students' perspectives. Interview questions for students are organized by the questions they are working on. The documentation instrument in this study collects data in the form of documents that support the completeness of research information. The instrument includes written documents from the mathematics textbook for grade VIII students of the Merdeka Curriculum, published by the Ministry of Education and Culture of the Republic of Indonesia, focusing on data on the presentation of the Pythagorean Theorem. The document was analyzed using a praxeological framework comprising tasks, techniques, technologies, and theories.

In this study, the triangulation process was carried out to identify specific didactic obstacles in the Pythagorean theorem. This process begins by analyzing the textbook, identifying topic that has the potential to cause learning obstacles, and then compiling test instruments based on the results of the textbook analysis. The test was administered to students to identify didactic obstacles. Furthermore, interviews with students and teachers were conducted to confirm the cause. Furthermore, associate the written test findings with student interviews. The initial hypothesis of didactic resistance is a consistent pattern of errors in test results. For example, students make mistakes when writing the Pythagorean theorem formula, depending on the representation of a right triangle. An example of a right-wing triangle representation is in the discussion section. Then, the student interview serves as a confirmation tool. The researcher asks the student who made the mistake to explain the step and the reason (in a think-aloud). Once the student's obstacles are confirmed, the researcher proceeds to the source-tracing stage through teacher interviews. Teacher interviews are used to understand the didactic situation in the classroom: does the teacher emphasize technique without supporting technology? Do teachers teach the Pythagorean theorem just to give formulas?

### Data Analysis

Data analysis was carried out concurrently with data collection and writing the findings. To analyze data, refer to the opinions of Miles and Huberman (1984), which consist of data reduction, data presentation, and conclusions.

**Data Reduction**

Data reduction was used to select, focus on, and eliminate irrelevant data, ensuring that the data obtained aligns with the research objectives. This activity was carried out using the ATLAS.ti software, to maintain reliability, that is, related to reliable research results, ensuring that research does not depend only on the researcher in analyzing information sourced from the research subject. The use of this tool facilitates detailed documentation and a thorough review of all stages in the research process.

**Data Presentation**

Data presentation activities involved organizing the reduced data into tables, matrices, or diagrams to enable clear visualization and facilitate analysis. In this study, the results of the data reduction process were obtained using ATLAS.ti software was presented in the form of a table. The table provided the items included in the didactical obstacle. This format allowed for a clear understanding of how each type of didactical obstacle. The tabular representation ensured the findings were accessible, easy to interpret, and effective for drawing meaningful insights.

**Coding**

The coding in this study came from raw data, namely, test responses confirmed through interviews and textbook descriptions. In relation to test response, to make it easier to describe students' answers, the characteristics of the correct students' answers are given the symbol of the letter R, and the characteristics of the incorrect answers are given the symbol of the letter W, and the non-answers are given the symbol of the letter Z. Furthermore, there are number symbols before and after the letters. The number symbol before the letter indicates the problem number, and the number symbol after the letter indicates the diversity of answer types. If there is no number symbol after the letter, it means that there is no diversity of answers or that there is only one type of answer. For example, if there is a type 1R symbol, it means that (1) shows problem number 1, then (R) shows the correct answer to the problem. No number after the letter R indicates that there is no diversity of answers or that there is only 1 type of answer. Another example, for example, there is a 2W3 symbol, which means that (2) shows question number two, (W) shows the incorrect answer, and (3) shows the third variation of the student's answer type.

From this data, the data was then reduced to narrower categories, namely, the location of student errors. From the location of students' mistakes, they were categorized as learning obstacles. At this stage, the data in the table were transformed into a more abstract, conceptual form to uncover deeper meaning through interpretation and to correlate the findings with existing theories. Based on this, conclusions were drawn to produce new findings.

## ▪ RESULT AND DISSCUSSION

**Forms of Didactical Obstacles**

To ensure students experience learning obstacles, the researcher developed an instrument for the Pythagorean theorem. The questions were organized into six items and tested with students. Based on the characteristics of the students' answers, the categorization is used to identify the location of the difficulties. The results of categorization with the help of ATLAS.ti software, are presented in Table 1 as follows.

**Table 1.** Result of analysis

| Category | Item | Frequency of Occurrence | Examples of Empirical Evidence |
|---|---|---|---|
| Visual orientation obstacles | Misunderstanding of the concept of triangles (type A) | 17 | Student's answer to problem 2: "I do not know about number two because I do not know the formula. In the triangle ABC, it is the base side is BC, the AC is the height side." |
| Formula procedural obstacles | Misunderstanding of algebraic representations (type B) | 7 | Student's answer to problem 3: "I was confused about finding the length of the base. That is two-thirds of the length of BC, while the length of BC is unknown." |
| | Misunderstanding of the problems (type C) | 37 | Student's answer to problem 4: "I did not do it because I forgot how to do it, and I also forgot the formula." |
| | Misunderstanding of operating procedures other than integer operations (type D) | 6 | Student's answer to problem 1: "I do not know about number one. Moreover, the length of the roots is used, I do not understand either..." |
| | Misunderstanding of the application of the Pythagorean theorem formula in problem solving (type E) | 28 | Student's answer to problem 6: The height of the triangle using the root makes it difficult for me, then it is θ, which has never been obtained, confused about what to look for first, so it seems to use the Pythagorean theorem first. However, I am not sure about the answer..." |

Based on Table 1, two didactical obstacles were identified in solving the Pythagorean theorem problem: the visual orientation obstacle and the formula procedural obstacle. The difficulty for students with visual orientation challenges is that they do not understand the concept of triangles. Then, the difficulties students face in procedural obstacles to formulas include not understanding algebraic representations, not understanding problems, not understanding procedures beyond integers, and not understanding the application of Pythagorean theorem formulas. The elaboration of each didactical obstacle is as follows.

### *Visual Orientation Obstacles*

In a didactic context, visual orientation obstacles refer to students' cognitive limitations in interpreting visual representations, not due to eye disturbances, but rather due to rigid didactic exposure. For example, students fail to recognize the triangle's height and hypotenuse. Based on Table 1, many students do not understand the concept of triangles. The study found that students did not understand the parts of a triangle when the right triangle was not in its usual position, as it is often presented in students' textbooks. For example, the book lists the right triangle ABC as follows:

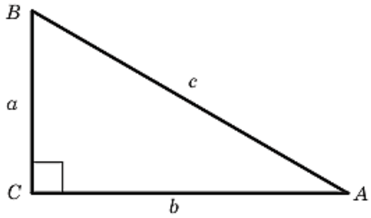



**Figure 3.** Triangle ABC

Based on Figure 3, students are used to writing: $c^2 = a^2 + b^2$, with $\overline{CA}$ as the base and $\overline{BC}$ as the height, then $\overline{AB}$ as the hypotenuse. When students are asked to specify the base, height, or even hypotenuse part of the following ABC triangle in problem 1:

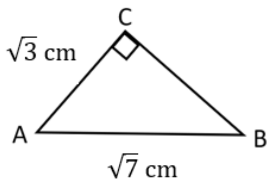



**Figure 4.** Triangle on problem 1

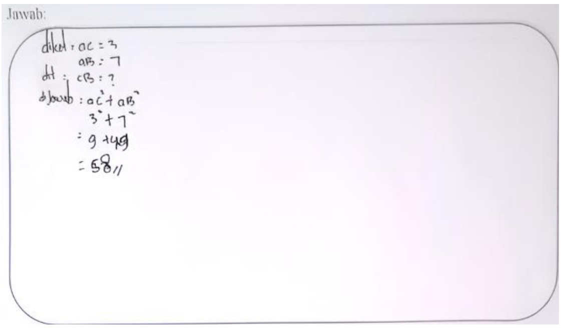



**Figure 5.** Sample answers for type 1W1 students

Type 1W1 students replied that $\overline{AB}$ is the base side and $\overline{BC}$is the height side. Then, almost all of the students, represented by 1W2, 1W3, 1W4, 1W5, 1W6, and 1Z, answered that $\overline{AB}$ is the base side and $\overline{AC}$ is the height side. Moreover, 1W7 answered that $\overline{AC}$ is the base side and $\overline{BC}$ is the height side, or $\overline{AB}$ is the base side, and C is the height side. Next, student 2W9 expressed his doubts in determining the parts of the triangle. He said, $\overline{AB}$ is the base side. It is just that he has doubts about its height.

*"That is the answer, ma'am. I do not know the formula for the area of a triangle. For the base AB, for the height BC."* Furthermore, in problem 2, when the triangle ABC is repositioned as follows.

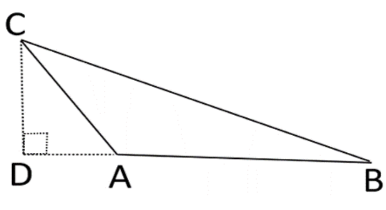



**Figure 6.** Triangle on problems 2

The students' answers for determining which side of the base and which side of the triangle ABC is the hypotenuse are diverse. In students with type 2Z mentioned that the $\overline{BC}$ as a base side and $\overline{CD}$ as a high side. On the type of student 2W1, 2W6, and 2W7 mentioned the side of the base $\overline{AB}$ and $\overline{AC}$ as a high side. In the 2W2 student type, it is said that $\overline{BD}$ is the base side and $\overline{CD}$ is the high side. Students with type 2W3 said that $\overline{BC}$ is the base side and $\overline{AC}$ is the high side. The type of 2W4 student said that $\overline{BD}$ is the base side and $\overline{AC}$ is the high side.

*"I do not know about number three because I do not know the formula. In the triangle ABC, it is the base BC, if the AC is high."*

Students' obstacles in identifying the parts of a triangle are related to their semiotic representation. According to Duval (2017), mathematical understanding requires the ability to operate in various semiotic registers, be it verbal, algebraic, or graphic, and perform conversion and treatment between these registers. Treatmen is the transformation of a representation within the same register. In a geometric context, this means that students must be able to mentally manipulate the object being seen. When students are taught the Pythagorean theorem only through standard visual models (right-right triangles standing upright with the oblique side facing right), they form rigid mental representations. As a result, when the triangle is rotated, the student fails to perform the mental treatment (turning the triangle back to an upright position) to verify the position of the right angle and hypotenuse. They are tied to the property's shape and position. This is a rigid visual scheme that hinders flexible problem-solving (Routledge, 2016).

Cognitively, students often associate hypotenuse (c) as the visual "longest side," or the positional "hypotenuse side" rather than as the side directly opposite the angle 90^0. The standard orientation reinforces the false visual bond that states that the hypotenuse side is always on the right. When the triangle is rotated, these visual ties become obsolete knowledge that the student is unable to adapt to the new situation.

Visual orientation obstacles are not caused by student error. The Pythagorean theorem, which presents only a standard model (for example, all textbook problems have the same orientation), fails to create an adequate didactic situation. Didactic situations are supposed to force students to reinvent knowledge, that is, solve problems without explicit instruction. As a result, Students are never faced with contradictions that encourage them to adapt, so they do not develop flexible knowledge. Monotonous didactic choices form a didactic contract that is wrong in the minds of students. These findings corroborate several studies that say that students experience learning obstacles in understanding the concept of triangles (Aprizal Bintara & Prabawanto, 2024). In fact, triangular matter is the prerequisite topic for the Pythagorean theorem.

### *Formula Procedural Obstacles*

Formula procedural obstacles refer to learning difficulties that arise when students can only perform basic arithmetic computations with the formulas of the Pythagorean theorem without understanding their conceptual meaning, algebraic structure, or the context of their application. The majority of students are not yet able to turn problems into mathematical sentences. This can be seen in the test results for problems 3 and 5. In problem 3, most students' answers are incorrect.

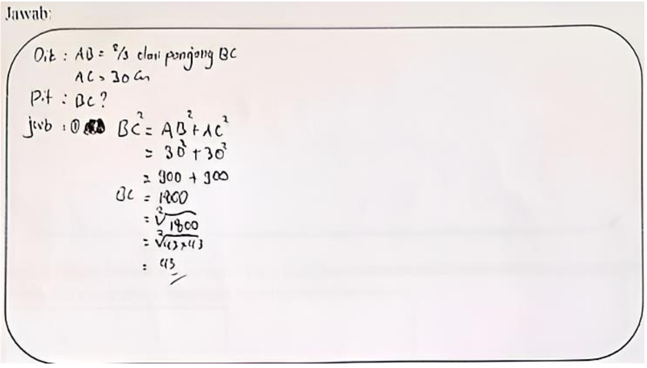


**Figure 7.** Sample answers for type 3W7 students

Type 1W1 students replied that $\overline{AB}$ is the base side and $\overline{BC}$ is the height side. Then, almost all of the students, represented by 1W2, 1W3, 1W4, 1W5, 1W6, and 1Z, answered that $\overline{AB}$ is the base side and $\overline{AC}$ is the height side. Moreover, 1W7 answered that $\overline{AC}$ is the base side and $\overline{BC}$ is the height side, or $\overline{AB}$ is the base side, and C is the height side. Next, student 2W9 expressed his doubts in determining the parts of the triangle. He said, $\overline{AB}$ is the base side. It is just that he has doubts about its height.

*"I was confused about finding the length of the base. That is two-thirds of the length of BC, while the length of BC is unknown."*

In problem 5, for example, the student types 5W1, 5W3, and 5W4; write the circumference directly on the triangle, but do not write it as an equation. After that, they were wrong in answering. Furthermore, the 5Z type does not write down the answer at all. Based on the interviews, students said they had never encountered the same type of problems. This type of problem is also not presented in textbooks.

*"I have never done a problem like this before, so I do not understand it yet. This I do perfunctory."*

Many students are not yet able to work with algebraic expressions. This is evident in the students' answers to problems 4 and 6. In problem 4, of the 30 students who answered, there were 8 types of student answers. One type is a blank (unfilled) answer (4Z) and seven types are incorrect answers (4W1, 4W2, 4W3, 4W4, 4W5, 4W6, and 3W7). All of these incorrect answer types do not indicate the operation of algebraic forms. Furthermore, in problem 6, most students did not answer the questions. Based on in-depth interviews with student representatives, students do not understand how to do it. The reason given is that they forgot the formula and have never encountered this type of problem. Problem 6 that was tested on the student was also not presented in the textbook. Therefore, the presentation of the topic does not provide students with learning experiences related to problems 4 and 6, so they experience learning obstacles, namely didactical obstacles. The difficulty of students in converting an event into a mathematical sentence and operating the algebraic form corroborates previous research that stated that students have difficulties in the concept of the equal sign, the notion of variable, algebraic expression, operation in algebra, and mathematization (Utami & Prabawanto, 2023).

Students also do not remember the Pythagorean theorem on problem 1. For example, type 1W1 wrote the Pythagorean theorem $\overline{CB} = \overline{AC}^2 + \overline{AB}^2$. Then, in type 1W2 wrote $\overline{BC}^2 = \overline{AB}^2 - \overline{AC}^2$. Then on problem 2, students also do not understand the Pythagorean theorem. For example, type 2W1 and 2W2 mentioned that $\overline{BC} = \overline{AC} + \overline{AB}$, then type 2W2 wrote $\overline{BC} = \sqrt{(\overline{BD} + \overline{AD})^2 - \overline{BC}^2}$. Meanwhile, type 2W3 and 2W6 had correct answer, that was $\overline{AD}^2 = \overline{AC}^2 - \overline{CD}^2$. Then type 2W4 and 2W5 did not mention the formula of the Pythagorean theorem, type 3W7 wrote $\overline{CB}^2 = \overline{AB}^2 + \overline{AC}^2$. Supposedly, the formula of the Pythagorean theorem used is $\overline{AB}^2 = \overline{BC}^2 + \overline{AC}^2$. The results of student interviews in the Pythagorean theorem section stated that students forgot the formula of the Pythagorean theorem.

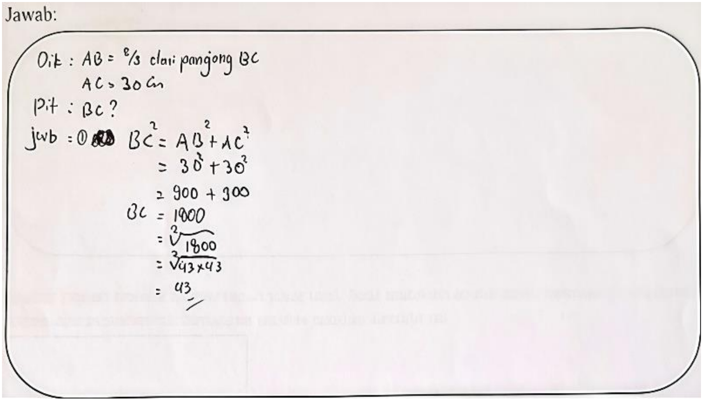
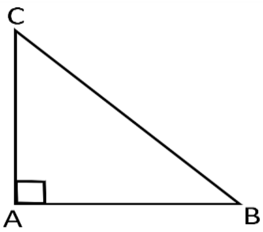



**Figure 8.** Sample answers for type 1W2 students

In problem 3, where students are asked to specify the length of the $\overline{BC}$ with the length of $\overline{AB}$ is two-thirds of the length of $\overline{BC}$ and square of the length of $\overline{AC}$ is 30 cm of the following triangle of ABC.




Students' test answers show mixed results. Regarding the Pythagorean theorem that should be used to find the length of $\overline{BC}$ , the type 3W1 wrote $\overline{BC} = a^2 + b^2$, with a = 30 cm, and b = $\frac{2}{3}$ cm. On type 3W2 and 3W3, the students did not write down the theorem. On type 3W4, Students wrote $\overline{BC} = \overline{AB}$ X $\overline{AC}$. On type 3W6 wrote $\overline{BC}^2 = \overline{AC} + \overline{AB}^2$. The other students did not write the answer. Forgetting the Pythagorean theorem formula occurs in problem 4. Type 4W4 and 4W5 did not write the Pythagorean theorem, and type 4W6 wrote $\overline{CD} = \overline{EC} - \overline{DE}$. Then type 4Z did not answer. Then, on the problem 5, students were asked to determine the possible length of the other side if a right triangle has a circumference of 90 cm and one side of the right side is 40 cm. Students of type 5W1, 5W2, and 5Z, which dominate the number of students, did not write down the Pythagorean theorem, while type 5W4 wrote $a^2 = b^2 - c^2$ with a hypotenuse side, b, and c right side. In problem 6, only type 6W4 can write its Pythagorean theorem correctly, and there is only one person of this type. The other students do not write down the theorems. The students' interviews revealed that they had forgotten about this.

*"For this matter, I forgot his Pythagorean theorem. If the side of the base is BD and the height AC. So I wrote the area of the triangle ABC = 1 2 ×BD×AC = 25"*

Learning difficulties arise when students can only perform procedural operations on the Pythagorean theorem formula without understanding its conceptual meaning. This concerns the dissociation between procedural and conceptual knowledge (Contreras, 2025; Hurrell, 2021). Students see the Pythagorean theorem only as a sequence of mechanical steps, with no relation to geometric objects. Students master formulas $a^2 + b^2 = c^2$ as symbolic strings that must be memorized and filled in, but they fail in understanding algebraic representations because they have nothing to do with the meaning of their variables. The squares attached to the variable $a^2, b^2, c^2$ only seen as a multiplication operation, not as a square area. Then variables a, b, c are seen as positional side sign, not as property side sign (upright side or hypotenuse). As a result, when it comes to asking for an upright side search $a^2 = c^2 - b^2$. Students are not able to perform correct algebraic transformations because they are bound to procedural structures and fail to understand the structure of relationships (the hypotenuse side is the result of the sum of the length of two upright sides).

The difficulty in understanding procedures beyond integer operations (specifically the square root, which produces irrational numbers) is an example of a didactical obstacle. In school practice, the majority of examples of Pythagorean theorem problems are presented using Pythagorean triples, e.g., 3, 4, 5 or 5, 12, 13, which produce integer solutions. This creates an implicit didactic contract in students' minds that the correct mathematical solution must produce integers. When an answer is found that is not an integer, the student assumes it is incorrect. Knowledge that was previously valid because it was used to working with integers, became obsolete and hindered the acquisition of new knowledge.

Students' difficulties in understanding the problem and the application of Pythagorean theorem formulas in problem solving are associated with the perspective of mathematical modeling (Lesh & Zawojewski, 2007). Applicative problem-solving demands more than computing; It requires a modeling process. Students fail in the first phase of modeling: translation. They cannot translate contextual situations into geometric and algebraic representations. When a picture of a right triangle is unavailable, students cannot identify which side is the hypotenuse and which is the upright. This failure indicates that the student's procedural understanding of the formula is closed and can only be activated with highly structured inputs (triangles labeled a, b, and c).

**The Relationship between Visual Orientation Obstacles and Formula Prosedural Obstacles**

Based on the results of the study, the inability of students to identify hypotenuse or perpendicular sides when the orientation of the right triangle was changed from the standard position showed that the students were bound by visuality rather than concepts, for example, viewing the hypotenuse as a side with a diagonal position, rather than as a side facing the right angle. This indicates the failure of conceptual abstraction. In other words, there has been a misconception as a result of research by Özerem (2012) which mentioned that students have a misconception of triangles. Study by Haj Yahya & Hershkowitz (2024) stated that reliance on a single image representation interferes with the formation of concepts. In other words, in this study, attachment to the visual led to failures in conceptual transitions.

Based on this, it can be said that the obstacle starts when the visual orientation obstacle appears. Suppose the student cannot accurately identify which side is the hypotenuse and which is a leg. In that case, the entire subsequent procedure will be invalid, even if the student has memorized the formula correctly. The studies by Park & Kim (2017) and Ramírez-Uclés & Ruiz-Hidalgo (2022) found that attachment to specific examples or cases can affect generalization abilities. This study shows that visual orientation obstacles are manifestations of geometric conceptual abstraction failures, while procedural formula failures are manifestations of failures in transitioning to algebraic formulas. However, because visual failure forces students to rely on unconceptual procedures, visual orientation obstacles directly increase the frequency and severity of formula procedural obstacles when a-didactic situations are encountered.

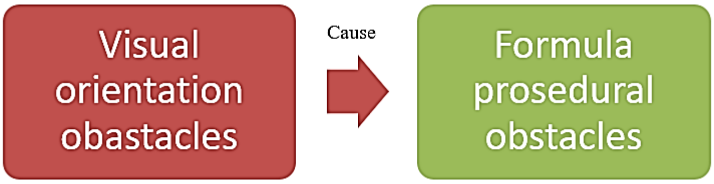



**Figure 9.** The relationship of the obstacles

**The Relationship between Students' Abilities and Types of Didactic Obstacles**

Figure 10 below is a crosstabulation between students' abilities and didactical obstacles. Participants in this study have high, medium, and low abilities. Of the six problems tested for the students, each student experienced several errors related to the type of didactic obstacle, whether visual or procedural. Each obstacle experienced is converted into categorical 1 and categorical 2 data. Category 1 is given if the visual orientation obstacle is more dominant, and category 2 is given if the procedural obstacle formula obstacle is more dominant.

**Student abilties * Didactical obstacle Crosstabulation**

| | | | Didactical obstacle: formula procedural obstacles | Total |
|---|---|---|---|---|
| Student abilties | Low | Count | 3 | 3 |
| | | Expected Count | 3.0 | 3.0 |
| | | % within Didactical obstacle | 15.8% | 15.8% |
| | Middle | Count | 13 | 13 |
| | | Expected Count | 13.0 | 13.0 |
| | | % within Didactical obstacle | 68.4% | 68.4% |
| | High | Count | 3 | 3 |
| | | Expected Count | 3.0 | 3.0 |
| | | % within Didactical obstacle | 15.8% | 15.8% |
| Total | | Count | 19 | 19 |
| | | Expected Count | 19.0 | 19.0 |
| | | % within Didactical obstacle | 100.0% | 100.0% |

**Chi-Square Tests**

| | Value |
|---|---|
| Pearson Chi-Square | .[a] |
| N of Valid Cases | 19 |

a. No statistics are computed because Didactical obstacle is a constant.

**Figure 10**. The relationship between student abilities and didactical obstacles

Based on Figure 10, in each problem tested to students, all dominant students experienced procedural obstacles. This means that there is no relationship between students' abilities and the type of didactical obstacle. This can be seen in the results of the chi-square test, which show a constant value for the didactical obstacle, indicating that no statistics are computed.

**Didactic Factors as a Source of Obstacles**

In this study, to identify didactic obstacles, six problems were prepared. The results of the students' tests show that there are difficulties in solving the problems of the Pythagorean theorem, namely type A difficulties related to incomprehension of the concept of triangles, type B difficulties related to incomprehension of algebraic representations, type C difficulties related to incomprehension of problems, type D difficulties related to incomprehension of number operations other than integers, and type E difficulties related to incomprehension of the application of the Pythagorean theorem. The difficulty students have with each problem is shown in the following Figure 3 Sankey diagrams.

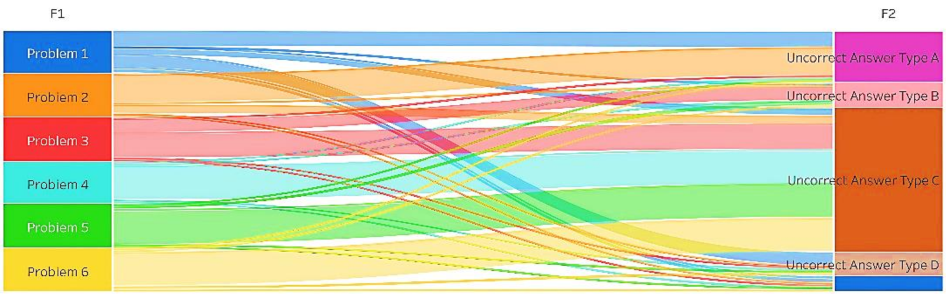



**Figure 11.** Distribution of students' difficulties in each question

Based on Figure 11, the largest flow volume of difficulty occurs in problem 2, with type A difficulty (i.e., students do not understand the concept of triangles). Thus, problem design 2 is the problem design that causes the most visual orientation obstacles. Problem 2 presents a triangle image with a different visualization than usual. The next large volume of flow is in problems 3, 4, 5, and 6, and problem 3, which is difficult for type C students; namely, students do not understand the problem. This category of difficulty refers to students who do not know how to solve problems, meaning they lack a good understanding of triangle concepts, the Pythagorean theorem, algebraic operations, and operations involving non-standard numbers. This means that the design of problems 3, 4, 5, and 6 causes the most procedural obstacles to the formula. Based on this, the most common obstacles students encountered were problems related to non-standard triangle visualization, the use of non-standard triangle length measurements, the application of non-standard Pythagorean theorems, and the use of unusual algebraic forms.

Non-standard visualization of triangles, non-standard use of triangle length measurements, non-standard application of Pythagorean theorems, and unusual use of algebraic form operations are presented, and students are never given them, so they are not used to them. That is, the presentation of the milieu influences, in this case, the textbook and the teacher-created didactic design, which in turn affects the didactic obstacle. Based on the results of the textbook analysis and student and teacher interviews, didactic obstacles were identified, including how the topic is presented in textbooks and how teachers present it in the classroom.

**Presentation of Topic in Textbooks**

Textbooks serve as a physical form of the curriculum, as a means for teachers to convey information and diagnostic tools to find repeated difficulties and mistakes of students (Bittar, 2022). The way the topic is presented in this textbook shapes the milieu. Based on the textbook analysis, 13 types of assignments were analyzed. Some tasks are coherent, and some are incoherent. In general, the presentation of topics in student textbooks is not systematic and tends to be monotonous, with triangle drawings and Pythagorean theorems presented in similar ways. In fact, the systematic presentation of a topic can prevent students from learning obstacles (Fitriani & Widjajanti, 2024). However, several jumps occurred. The existence of a jump in the series of tasks will cause discontinuity in the student's thinking process (Suryadi, 2025). This indicates a learning obstacle. According to Gagne's theory, learning involves a series of skills arranged hierarchically, from simple to complex. This means the series of tasks presented should be coherent, starting with simple tasks that support the next, more complex ones. Indirectly, this series of tasks that are not coherent contradicts Gagne's theory (Driscoll, 1994).

A total of 7 of the 13 tasks used techniques fully presented in the book. Thus, students cannot develop their own techniques. In addition, students are not given space to provide reasons or justifications for their techniques. This means that students are not allowed to build their own knowledge. This is not in accordance with the concept of devolution conveyed by Brousseau (1997), which holds that students build their own knowledge while teachers create didactic situations that support them in doing so. In addition, it is also contrary to the theory of learning constructivism supported by Bruner (1960), Piaget & Vygotsky in (Hamilton & Ghatala (1994) that the knowledge students gain is actively built through their interactions with the environment and previous experiences. Constructivism emphasizes that knowledge is actively constructed by individuals through interaction with the environment and experience (Fosnot, 2013). The presentation of assignments that provide techniques has the potential to make students memorize the completion procedure and increase the cognitive load of students without having a good understanding of the concept of the Pythagorean theorem. Learning that is carried out following textbooks becomes meaningless, even learning is meaningful, referring to Ausubel's theory (Ausubel, 1968), can increase the absorption of topics.

**How Teachers Teach**

The results of interviews with teachers indicated that the learning of the Pythagorean theorem and triangles was conducted without a specific method, only through a combination of discussion and lecture. After students were asked to discuss, the teacher did not confirm their answers; student answers were collected immediately afterward, and students were not asked to present the results. Then, when learning is conducted in lectures, the teacher delivers the topic, and students only listen. In addition, teachers provide examples to students when students have difficulty solving problems. Due to less effective learning, the information students obtain is only temporary. The didactic situation created by the teacher prevents students not constructing their own knowledge. This is not in accordance with the concept of devolution conveyed by Brousseau (1997), which holds that students build their own knowledge while teachers create didactic situations that support them in doing so. Moreover, it is contrary to the constructivist theory of learning, as advanced by Bruner, Piaget, and Vygotsky, which holds that knowledge is actively constructed by the student through interaction with the environment and prior experiences. In fact, learning that students can build their own knowledge will enable them to absorb information and incorporate information into their long-term memory (La Usa, 2021). In addition, according to Suryadi (2025), didactic situations that are too easy will cause students' development not to be in accordance with their intellectual capacity. Learning that is too easy can lead to a lack of students' problem-solving skills.

Based on the teacher's interview, the teacher delivered the topic to the students, focusing solely on the procedural aspect. A study by Rittle-Johnson, Fyfe, & Loehr (2016) compared the effects of emphasizing a conceptual vs. procedural topic in a single lesson. It showed that focusing solely on procedure reduced conceptual knowledge development and flexibility. Schneider, Rittle-Johnson, & Star (2011) explained the relationship between types of knowledge and emphasized the importance of procedural flexibility; procedural practice alone does not guarantee flexibility or conceptual understanding. Research by DeCaro (2016) suggests that the task context that encourages the use of quick/shortcut procedures may hinder procedural flexibility and subsequent conceptual understanding. Thus, focusing solely on procedural matters will create didactic obstacles.

**Mechanism of Emergence of Didactic Obstacle**

Based on the description of the form of didactic obstacles and didactic factors as sources of obstacles, there is a mechanism for the process of forming didactic obstacles, as shown in the following Figure 12.

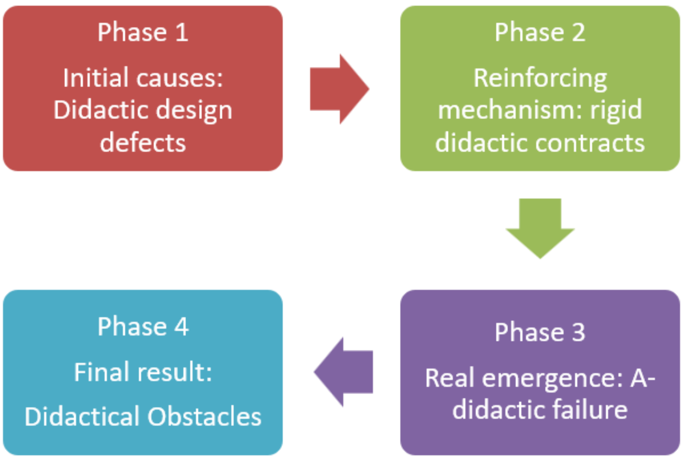



**Figure 12.** Diagram of the mechanism of emergence of didactic obstacle

Based on Figure 12, the mechanism for the formation of didactical obstacles begins with a defective didactic design; for example, the topic is presented in a monotonous, partial manner. As a result, students' knowledge is tied to precise visual/procedural representations. After phase 1, enter phase 2, which is a rigid didactic contract. For example, monotonous presentations and a lack of variety reinforce students' expectations to memorize and imitate procedures, rather than build concepts. The validity of the answer is assumed to be in the teacher/book, not in the self-understanding. As a result, students fail to take epistemological responsibility for their knowledge. Then, in phase 3, there is an a-didactic failure. For example, when faced with non-standard questions (outside the familiar textbook context), the knowledge they have fails to function. The new milieu is not adaptable. As a result, students fail to abstract core concepts, for example, misidentification of the oblique side. In the last phase, namely phase 4, there is a didactical obstacle. This systematic error is institutionalized and believed by students to be true "knowledge." Structured, difficult-to-fix difficulties that are evident and appear to stem from teaching choices are not students' innate difficulties.

In the implementation of this research, it is not spared from limitations. The limitation is to eliminate the characteristics of the same student's answers for students who have poor communication skills. Furthermore, the research can be carried out by including all students who took the test to take the interview to obtain more comprehensive research results.

## ▪ CONCLUSION

Based on the presentation in the results and discussion section, it can be concluded that how the topic is presented in textbooks and how teachers present it can affect the emergence of learning obstacles. The learning obstacle in the Pythagorean theorem topic is evident in the many mistakes and difficulties students experience. A diverse understanding of a concept indicates a weakness in the didactic design created. The results of the study revealed the presence of didactical obstacles in the form of visual orientation and procedural formula obstacles. Visual orientation obstacles cause formula procedural obstacles. Didactical obstacles are identified as a weak understanding of the concept of triangles and of algebraic representations. The mechanism of the occurrence of didactical obstacles starts from the milieu that is defective, continues with a rigid didactic contract, and then fails to create an effective didactical obstacle, so that didactical obstacles arise. The mechanism for the emergence of this didactical obstacle begins with a weak didactic design, namely, the presentation of assignments in textbooks that tend to be incoherent. In addition, there is a weak didactic contract, in which the task-completion technique is fully provided by the book and the teacher, leading to procedural compliance and simply memorizing formulas without understanding the concepts. In other words, the a-didactical situation becomes ineffective.

The findings regarding the mechanism of the emergence of didactic obstacles in the Pythagorean Theorem have implications for the theoretical and practical realms. Theoretically, the results of this study contribute to the TSD by providing empirical evidence that didactical obstacles are caused by the failure of the didactic system in breaking the didactic contract of the passive, flawed milieu, thus requiring TSD to further emphasize the analysis of the inadequacy of the problem design as a trigger for obstacles. Practically, these findings concretely recommend to junior high school mathematics teachers to re-engineer the learning milieu by providing variations in the visual orientation of right-wing triangles and non-standard problem contexts (creating effective a-didactic situations), as well as shifting the focus from memorization of formulaic techniques to an in-depth understanding of technology (conceptual justification) through self-validation activities by students, thereby breaking the causal chain of obstacle formation. This study revealed that there are didactic obstacles in solving the Pythagorean theorem problem, but these obstacles do not affect all students who take the written test. In the next study, a deeper exploration of the didactic obstacles to solving the Pythagorean theorem problem for all students who take the written test, even those with poor communication skills.

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