

Exploring the Gap: Creative Mathematical Reasoning of Pre-service Teachers in Solving Multiple Solution Analytical Geometry Tasks

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Abstract: With the rapid changes in the times, the emphasis in education has also shifted, from equipping students with highly codified knowledge to developing routine skills to empower them to face and overcome complex and non-routine cognitive challenges. Students must be able to think flexibly and creatively when asked to solve problems for which they do not yet have strategies. The purpose of this study was to describe the creative mathematical reasoning (CMR) abilities of students in solving multiple solution tasks (MSTs) in analytical geometry problems. The CMR indicators in this study were novelty, plausibility, and mathematical foundation. The research used a qualitative descriptive design, describing how students' CMR abilities and their own abilities in solving MSTs questions. The subjects of this study were second-semester students in the mathematics education program at a university in Subang in the 2024/2025 academic year. The subjects were given MSTs questions, grouped into high, medium, and low groups based on their scores. Then, two participants were selected from each group. The criteria for selecting research subjects were: fulfilling the CMR ability aspect; being able to solve several MSTs questions to assess novelty, plausibility, and mathematical foundation, including the stages of initiating reasoning, developing reasoning (incubation, illumination), verifying reasoning, and justifying reasoning; and being able to communicate well. The research subjects were divided into three groups based on the MSTs' question scores: high, medium, and low. The results of the study illustrate that students' mathematical creative reasoning abilities in solving MSTs questions are not optimal, as reflected in the solutions they provide. Therefore, to optimize students' CMR abilities, assignments should combine question types and use interactive, practical, relevant, comfortable, and connected learning models. For example, project-based or technology-based learning models.

Keywords: analytical geometry, creative reasoning, multiple solution tasks.

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■ INTRODUCTION

Interest in creativity in mathematics education research is growing, and the research field is expanding (Joklitschke et al., 2022). In fact, by 2024, seven countries had joined the CTEG (Creative Thinking Expert Group), a group of experts specifically researching creative thinking (OECD, 2024). Mathematical creativity is viewed as a specific domain of creativity (Pitta-Pantazi, 2017). Researchers in mathematics education on creativity have begun to sort and

map existing research (Joklitschke & Schindler, 2022). Looking at research on creativity in mathematics education, there are many different aspects to study, as well as many theoretical assumptions underlying creativity, particularly the phenomena of creativity and giftedness in mathematics (Leikin & Sriraman, 2017), including the gender gap in education (OECD, 2014). Furthermore, students' creative processes in mathematics are increasingly important in mathematics education research (Schindler &

Lilienthal, 2020). Students' creative processes in mathematics are increasingly important in mathematics education research, and researchers often use Multiple Solution Tasks (MSTs) to foster and evaluate students' mathematical creativity (Schindler & Lilienthal, 2020). MSTs are tools for solving mathematical problems in multiple ways, using different representations, properties, or theorems. By providing students with rich mathematical tasks and encouraging them to find multiple solutions or proofs (Osakwe et al., 2023), MSTs can be used for mathematics learning and teaching. They can shift teachers' conceptions from a student-directed orientation to an action-directed orientation (Leikin, 2011). MSTs have a solution space that can be used to examine various aspects of problem-solving performance (Leikin & Lev, 2013).

The decline in creativity is evident in the lack of interaction among individuals and in the lack of stimulation in a creative school environment, despite the importance of fostering students' mathematical creativity (El Turkey et al., 2024). Educators should provide opportunities for students to see mathematics as a creative field (Cilli-Turner et al., 2023). Educators should integrate their knowledge of topics, mathematical structures, and practices to create a framework for mathematical creativity (Delgado-Rebolledo & Zakaryan, 2020). The use of effective learning strategies does not guarantee learning, as the approach's success depends on its implementation (Bahar et al., 2021). Some educators believe that children's creativity occurs naturally and cannot be taught or nurtured, and that creativity is better suited to the arts; such teachers are less motivated to foster creativity in the classroom (Bullard & Bahar, 2023). On the other hand, educators recognize the importance of creativity for student learning and the freedom to express new ideas (Katz-Buonincontro et al., 2020). Educators recognize the importance of creativity, but they often fail to foster a learning environment that

encourages students to develop their creative abilities (Bullard & Bahar, 2023). Mathematics educators need to utilize programs based on international studies such as TIMSS, PISA, PIRLS, and TAILS (Tashtoush et al., 2022). Students may have changed their reasoning methods due to support from work and increased experience with mathematical content and procedures (Jablonski & Ludwig, 2022). Mathematical reasoning is increasingly important in mathematics education (Hjelte et al., 2020). An educator must support mathematical reasoning when orchestrating classroom discussions on mathematics (Arnesen & Ro, 2024). Working memory capacity and mathematics anxiety affect the creative reasoning of prospective mathematics teachers in solving problems, where prospective mathematics teachers with high working memory capacity show fluency and smoothness in generating new ideas, can connect familiar mathematical concepts, can provide logical arguments to support the truth of the ideas generated, can cause loss of focus in solving complex problems and cannot provide arguments for the ideas generated due to high or low levels of mathematics anxiety (Hasan & Juniati, 2025).

Analytical geometry is a branch of mathematics that uses algebra and coordinate systems to study geometry. Analytical geometry's main characteristics are its methods and results. Analytical geometry bridges algebra and geometry. The ability to solve complex mathematical problems by translating between visual forms and their numerical formulas is essential for studying complex natural phenomena. School grades do not guarantee the ability to solve complex mathematical problems and therefore do not reflect their abilities in mathematical reasoning and mental computation (Singh et al., 2020). Although reasoning is a central concept in mathematics education research, the discipline still lacks a coherent theoretical framework for

mathematical reasoning (Kollosche, 2021), and the use of structural and procedural aspects remains too limited (Nhiry et al., 2023). Creative mathematical reasoning (CMR) is a critical domain of general views of mathematical reasoning (Hjelte et al., 2020). The CMR capability criteria that must be met are; Novelty, in the sense that a new sequence of reasoning is created for the thinker, or a forgotten sequence of reasoning is recreated; Plausibility, where there are arguments supporting the choice of strategy and/or implementation of the strategy that motivate why the conclusion is true or reasonable; Mathematical Foundation, in the sense that the argument is based on the intrinsic mathematical properties of the components involved in the reasoning (Lithner, 2008).

Other experts state that CMR encompasses creativity, plausibility, and anchoring (Jonsson et al., 2020; Dwirahayu et al., 2021). Other experts further clarify that CMR encompasses three aspects: novelty, plausibility, and mathematical foundation (McJames et al., 2023). Furthermore, they elaborate that the main principle of CMR is that students need to create and express independent arguments when learning mathematics. This process is guided by four principles: initiating, developing, verifying, and justifying reasoning. This can support the choice of strategies and/or implementation strategies that motivate why the conclusions are correct or reasonable (Olsson & Granberg, 2024). Many experts have conducted research on analytical geometry, including studies on mathematical communication skills (Darto et al., 2024), mathematical thinking (Fernanda & Kholid, 2023), learning interest (Listiani et al., 2024), and virtual platforms (Cardona-Reyes et al., 2025). Research on creative mathematical reasoning (CMR) is urgent and relevant to understanding the low level of creative mathematical reasoning (CMR), especially in the context of analytical geometry. This finding aligns with research

indicating that prospective mathematics teachers have limited content knowledge and skills for teaching analytical geometry in high school (Zulu & Brijlall, 2024). This high school-level material is essential for prospective teachers, and these findings can make a unique contribution to the literature on higher education mathematics by strengthening indicators of mathematical foundation and optimizing prospective teachers' CMR skills. The impact will be clearly visible when the prospective teacher students teach at school, where novelty and plausibility, from preparing learning tools to teaching and learning activities, are highly expected by the future generation.

The use of the CMR capability framework must be supported by the learning environment, including: learning, task completion, textbooks, and assessments (Lithner, 2008), for example, to represent lines, curves, and shapes with equations and to solve geometric problems algebraically, to understand concepts such as distance, slope, midpoint, and transformations. Where the role of teachers in supporting productive interactions is very necessary to pay attention to, interpret, and shape further productive collaborative dynamics (Hansen & Naalsund, 2025). For example, collaborative technologies such as GeoGebra software support collaboration and mathematical creative reasoning by providing students with a shared workspace and responsive feedback (Granberg & Olsson, 2015). The results of other studies indicate that applying the CMR framework in early elementary schools regarding the concept of novelty warrants further study, as does the need to reconsider its plausibility criteria (Sjaastad, 2025). In creative mathematical reasoning (CMR), students with low memory capacity have difficulty providing proofs and rationales for solutions grounded in the intrinsic nature of mathematics (Palengka et al., 2022). The concepts of the Cartesian coordinate system, straight-line equations, circle

equations, parabola equations, ellipse equations, and hyperbola equations are part of the discussion of analytical geometry, which is generally taught based on procedures already presented in books. Compared to a procedure-based teaching model commonly found in schools, called Algorithmic Reasoning (AR), CMR was found to outperform AR (Jonsson et al., 2014). Students who perform poorly on creative tasks tend to resort to ineffective imitation strategies (Norqvista et al., 2019). Things to look at in completing a task, namely; A problematic situation if it is not clear how to proceed; Choosing a strategy to complete the task supported by predictive arguments; Applying the chosen strategy to complete the task; Obtaining conclusions (Lithner, 2008).

Research on learning designs that combine imitation and creative reasoning to examine whether, how, and why tasks and instructions enhance creative mathematical reasoning has long been conducted (Lithner, 2017). For example, research on collaborative processes to enhance understanding and creative mathematical reasoning of the mathematical properties of linear functions (Hansen, 2022), but unlike analytical geometry material, no research has yet been conducted. Regarding students' creative mathematical reasoning in solving mathematical problems, the need for research linked to mathematics learning (Marsitin et al., 2024). Based on the background provided, the researcher considers it necessary to conduct a study analyzing students' creative mathematical reasoning in analytical geometry problems, with a focus on their level of creative mathematical reasoning. This study analyzes the strategies, approaches, and explorations students use to overcome obstacles during the problem-solving process and find various solutions. It is hoped that the results of this study will contribute to understanding the root cause of low creative mathematical reasoning (CMR). In addition, this study is expected to identify differences between

groups in creative mathematical reasoning (CMR) abilities in the context of analytical geometry, as reflected in students' solutions.

■ **METHOD**

Participants

The researcher conducted research in the Mathematics Education Study Program, semester 2, at one of the universities in Subang in the 2024-2025 academic year. The research subjects were six students from high, medium, and low groups. The researcher named the first participant (P1) for the first high-ability participant, the second participant (P2) for the second high-ability participant, the third participant (P3) for the first medium-ability participant, the fourth participant (P4) for the second medium-ability participant, the fifth participant (P5) for the first low-ability participant, and the sixth participant (P6) for the second low-ability participant. Where each group consists of two participants. The criteria when selecting research subjects are: 1) Fulfilling aspects according to the level of creative mathematical reasoning ability (novelty, plausibility, mathematical foundation). 2) Can solve several MSTs questions to see creative mathematical reasoning, namely novelty, which includes the preparation stage (initiation), incubation, illumination, and verification. In addition, plausibility includes the stages: initiating reasoning, developing reasoning, verifying reasoning, and justifying reasoning. 3) can communicate well.

Research Design and Procedures

The research design used a qualitative descriptive approach, namely, to describe students' creative mathematical reasoning in solving multiple solution tasks. The researcher examined the stages of creative mathematical reasoning, both in the process of Plausibility and in the process of forming novelty, with different levels of creative mathematical reasoning in solving

multiple solution tasks. The subjects were grouped into three categories: high, medium, and low. Then, two samples were selected from each group to conduct an in-depth study of creative mathematical reasoning. The researcher gave six samples (subjects studied) questions to be used as instruments to assess the creative mathematical reasoning process, and the subjects studied worked on multiple solution tasks. After that, the research subjects were observed again by being interviewed using a semi-structured personal interview format, where the researcher had a list of questions but had flexibility depending on the respondents being interviewed, in order to obtain more detailed or more detailed information regarding creative mathematical reasoning in solving multiple solution tasks, along with the reasons why the research subjects carried out the steps used to solve these problems.

Instruments

In this study, the researcher used two test instruments and one interview guide. The Multiple Solution Tasks instrument will be used to assess the creative mathematical reasoning process. Then the interview guide instrument. Furthermore, the researcher created the interview guide to further explore students' creative mathematical reasoning in completing MSTs. In addition, to obtain more detailed information about the reasons for taking steps to resolve, an explanation of the data from the student resolution results. Before being used in the field, the instrument is evaluated for suitability of language, research objectives, and construction.

Data Analysis

Qualitative data analysis techniques with the Phenomenology approach were used in this study. The characteristics of qualitative data analysis techniques with the Phenomenology approach include: Focus, Type of Problem Best Suited for Design, Discipline Background, Unit of Analysis, Data Collection Forms, Data

Analysis Strategies, and Written Report (Creswell, 2007). This study focuses on understanding the essence of creative mathematical reasoning abilities. Therefore, at the Type of Problem Best Suited for Design stage, the researcher describes the essence of what happens to creative mathematical reasoning abilities within the Discipline Background of analytical geometry and the Unit of Analysis of students currently teaching analytical geometry courses. At the Data Collection Forms stage, the researcher gives essay questions to group students based on their level of creative mathematical reasoning, then selects two subjects from each group to describe the students' creative mathematical reasoning thinking process. Interviews are then conducted to deepen the creative mathematical reasoning process at each level. In the Data Analysis Strategies stage, the researcher used triangulation, comparing test results with interview results to provide a basis for categorizing and interpreting creative mathematical reasoning data using indicators of novelty, plausibility, and mathematical foundation. The coding approach used was inductive, where the researcher grouped data for deeper analysis. The operational process of the research, how the test results and interview transcripts were connected, began with students being given an analytical geometry test. The test results were further examined to determine the characteristics of students' creative mathematical reasoning, using indicators of novelty, plausibility, and mathematical foundation. Then, subjects were purposively selected, followed by in-depth interviews to determine students' difficulties and the ways in which they used their creative mathematical reasoning processes to solve analytical geometry tests. The next step was a discussion, and a description of all data obtained was presented in accordance with the research problem formulation. In the written report stage, the researcher interpreted the data obtained and what could be observed from it.

RESULT AND DISCUSSION

Before determining the type of question, the researcher reviews several reputable international journals on creative mathematical reasoning and Multiple solution Tasks. The researcher found 32 articles related to the researcher’s research objectives. After being studied, the researcher determined the type of creative mathematical reasoning question. The researcher has prepared questions related to analytical geometry material, which has been studied by all participants at a lower level. The researcher’s participants were given the Questions according to the relevant course schedule, under the researcher’s direct supervision. 16 Participants were given 60 minutes to work on the questions. Then the selected participants were six students, divided into high, medium, and low groups, with each group consisting of two participants, who were then studied in more depth by the researcher for further research through observation and direct interviews. Quantitative data is used to determine high, medium, and low groups.

Many opportunities for learning mathematical reasoning in schools encourage imitative learning procedures or algorithmic reasoning (AR) rather than engaging in more constructive reasoning processes, for example, creative mathematical reasoning (CMR), where encouraging students to engage in constructive processes when learning mathematical reasoning may have beneficial and long-lasting effects (Wirebring et al., 2021). Emotional traits will influence students, making their thoughts difficult to understand and control (Wallas, 1926). As for the following questions: The points (0, 0), (10, 0), and (2, 5) are the corner points of a parallelogram. Determine the fourth point of the parallelogram (there are three answers).

Table 1 is the result of mapping each participant (P1-P6) with each CMR component (Novelty, Plausibility, Mathematical Foundation), where in Table 1, there is concrete evidence from the results of the participants’ work (P1-P6) and relevant interview quotes to justify the author’s assessment regarding students’ CMR abilities.

Table 1. Structured analysis table

Partici pants	Students’ Answer	CMR components			Interview Results
		Novelty	Plausibility	Mathematica l Foundation	
P1	<p>Jawaban</p> <p>A. menggunakan Vektor AC</p> $\text{Vektor AC} = C - A = (2,5) - (0,0) = (2,5)$ $D = B + AC = (10,0) + (2,5) = (12,5)$ <p>jadi titik keempatnya (12,5)</p>	X	V	V	<p>Researcher: Do you understand the problem and instructions in the question, and are you aware of what you should do?</p> <p>Participants P1, P2: Understand. You were asked to determine point D on a parallelogram using more than one answer.</p> <p>Participants P3, P4, P5, P6: Understand. You were asked to determine point D on a parallelogram, but you've forgotten how to solve it.</p>
P2	<p>Jawaban</p> <p>1a. dik: titik A (0,0), B (10,0), C (2,5)</p> <p>dit : D ?</p> <p>Jawab: Rumus vektor</p> $D = C + AB = (2 + 10, 5 - 0, 0) = (12,5)$ <p>→ Jadi titik keempat Jajargenjang adalah : D (12,5)</p>	X	V	V	<p>Researcher: What strategy have you developed to solve the problem?</p> <p>Participants P1, P2: Using the concept of vectors and Cartesian diagrams.</p> <p>Participants P3, P4, P5: Using Cartesian diagrams.</p> <p>Participant P6: Using the concept of vectors.</p> <p>Researcher: Are the properties of parallelograms the only ones you mentioned that can be used to solve the</p>

P3	<p>jadi titik ke 4 nya (12,5),</p>	X	V	V	<p>problem? What about other properties of parallelograms, such as those related to congruence, diagonals, and angles?</p> <p>Participants P1, P2, P3, P4, P5, and P6: The properties of parallelograms can be used to solve this</p>
P4		X	V	V	<p>problem. I only know that parallelograms have parallel sides.</p> <p>Researcher: Did you copy a previously established answer or solution procedure?</p> <p>Participants P1, P2, P3, P4, P5, and P6: Yes, I followed an established solution procedure.</p> <p>Researcher: When searching for a solution, do you base it on what you memorized or on your previous understanding?</p> <p>Researcher: Do you think it's more efficient to do it yourself, recalling previously learned material, or to search directly on Google?</p>
P5		X	V	V	<p>Participants P1, P2, P3, P4, P5, and P6: It's more efficient to search directly on Google.</p>
P6	$\rightarrow AC = C - A = (1,5) - (0,0) = (1,5)$ $DB = B - D = (10,0) - (4,7) = (10 - 4, 0 - 7)$ $= 2 - 10 \times \text{dikun 5} \quad 2 - 7$ $= 2 - 10 - 2 = 8 \quad \text{dikun 4} \quad 2 - 5$ $= D = (8, -5)$	X	V	V	

Visualization of several problem-solving strategies to obtain the correct solution to the parallelogram problem, as well as participant mapping of the solutions they found. Novelty is related to the participant's abilities, namely; Able to compile, develop several strategies or steps to be used in solving the problem; Able to avoid copying answers or copying solution procedures; Able to avoid memorized or algorithmic fixations when searching for solutions; Able to recreate forgotten solution methods; Able to do it yourself which leads to memory consolidation to be more

efficient than the learned solution method; Able to complete the task without the available solution method. Regarding mathematical foundations, participants must understand the concept of a parallelogram, including the following properties: two pairs of parallel sides, two pairs of equal sides; opposite angles are equal; the sum of adjacent angles is supplementary (180°); the sum of all angles is 360° ; two diagonals intersect at their midpoint; each diagonal bisects the other; each diagonal forms two congruent triangles (the same shape and size); parallelograms do not have

an axis of symmetry but have second-order rotational symmetry (they can occupy their frame in two ways). Plausibility relates to the process of determining the fourth point.

Strategy 1 is based on the concept of a parallelogram with two pairs of parallel sides, two pairs of equal lengths, and is created using vectors in a two-dimensional plane. Given a parallelogram ABCD with coordinates A (0, 0), B (10, 0), and C (2, 5). In a plane vector, or two-dimensional vector, there is a term called a position vector,

which originates at the origin (0, 0) and ends at a point (x, y). In the problem, the center of coordinates is A (0, 0), so vector $\overrightarrow{AB} = \vec{b} =$

$$\begin{pmatrix} 10 \\ 0 \end{pmatrix} \text{ and vector } \overrightarrow{AC} = \vec{c} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

Two pairs of parallel sides, namely: $AB \parallel CD$, $AC \parallel BD$ so that vector $\overrightarrow{AD} = \vec{d} = \overrightarrow{AB} + \overrightarrow{AC}$

$$\begin{pmatrix} 10 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$$

So the coordinates of point D(12,5)

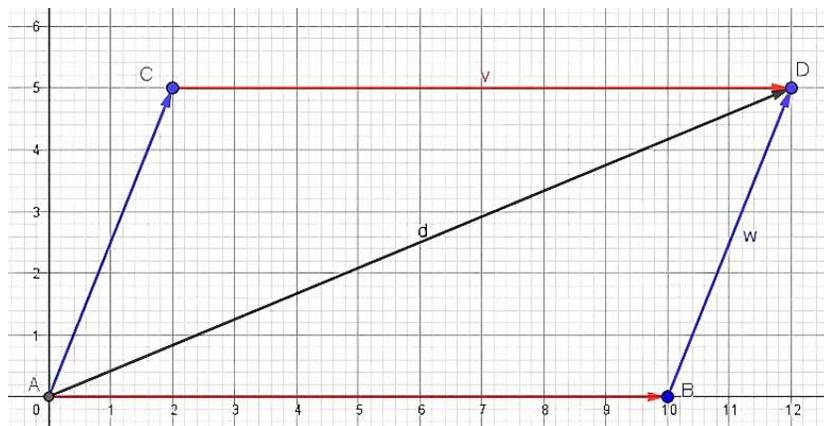


Figure 1. Two pairs of parallel sides $AB \parallel CD$, $AC \parallel BD$ coordinates of point D (12,5)

Two parallel sides, namely: $AB \parallel CD$

so that the vector $\overrightarrow{AD} = \vec{d} = \overrightarrow{AC} + \overrightarrow{CD} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} -10 \\ 0 \end{pmatrix} = \begin{pmatrix} -8 \\ 5 \end{pmatrix}$

So the coordinates of point D(-8,5)

Two parallel sides, namely:

$AC \parallel BD$, so that the vector $\overrightarrow{AD} = \vec{d} = \overrightarrow{AB} + \overrightarrow{BD} = \begin{pmatrix} 10 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \end{pmatrix}$

So the coordinates of point D(8,5)

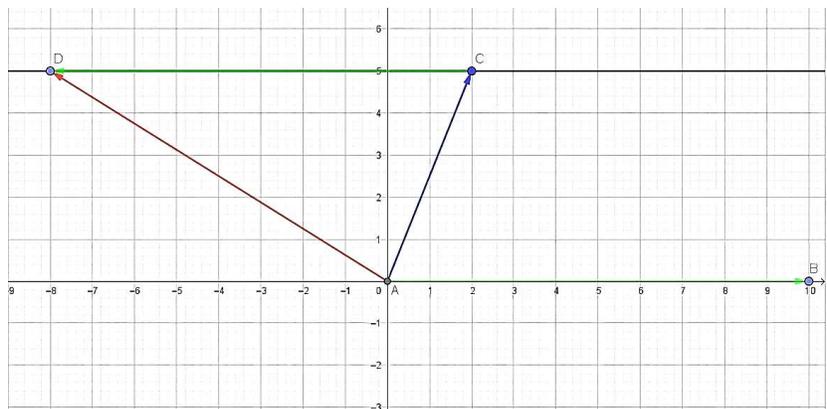


Figure 2. Two parallel sides $AB \parallel CD$ coordinates of point D(-8,5)

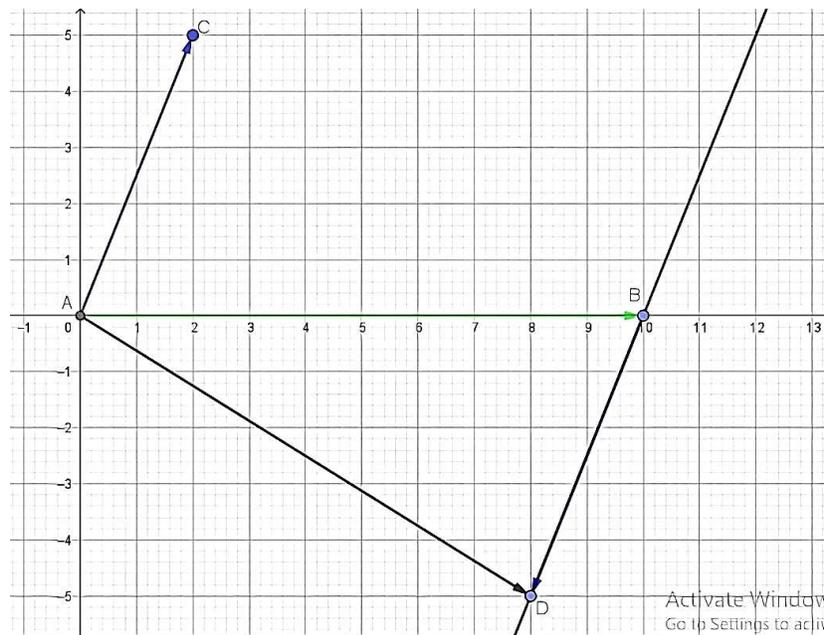


Figure 3. Two parallel sides $AC \parallel BD$ coordinates of point $D(8,-5)$

Strategy 2 is based on the concept of a parallelogram with two pairs of parallel sides and two pairs of equal-length sides, and is created in

a Cartesian coordinate system using GeoGebra. Two pairs of parallel sides, namely: $AB \parallel CD$, $AC \parallel BD$, the coordinates of point D are $(12,5)$.

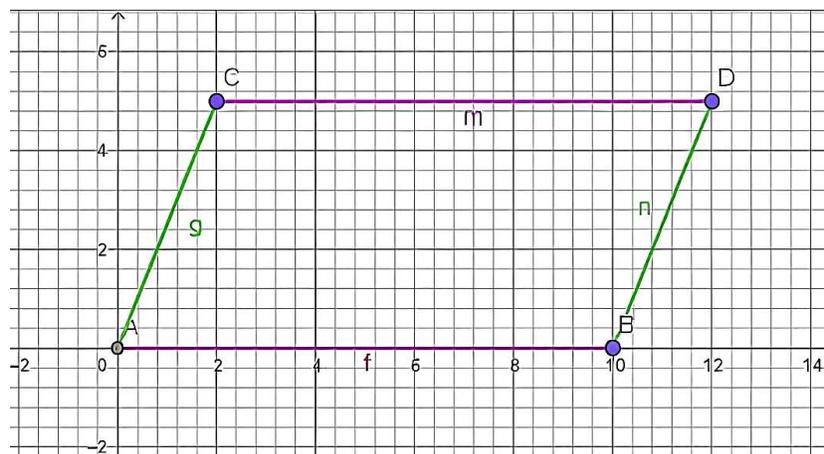


Figure 4. Two pairs of parallel sides $AB \parallel CD$ & $AC \parallel BD$

Two pairs of parallel sides, namely: $AB \parallel DC$, $BC \parallel AD$, so that the point coordinates are $D(-8,5)$

Two pairs of parallel sides, namely: $AC \parallel BD$, $AD \parallel BC$, so that the coordinates of the point $D(8,-5)$

Strategy 3 is based on the idea that each diagonal of a parallelogram forms two congruent triangles (the same shape and size). Point $D(x,y)$ is the fourth unknown point and is then found by drawing a triangle from points $A(0,0)$, $B(10,0)$, and $C(2,5)$. With the help of GeoGebra media,

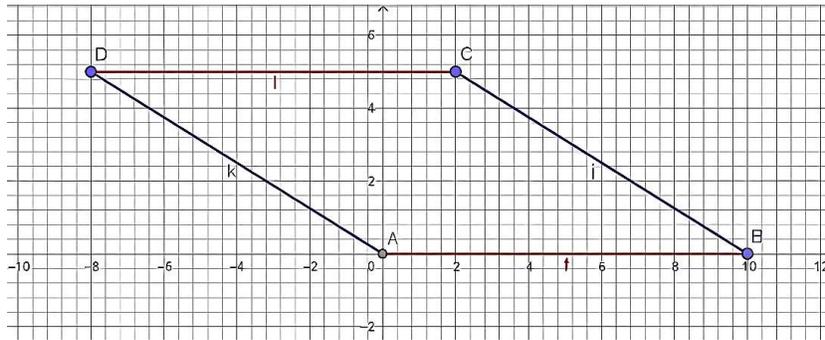


Figure 5. Two pairs of parallel sides $AB \parallel CD$ and $BC \parallel AD$

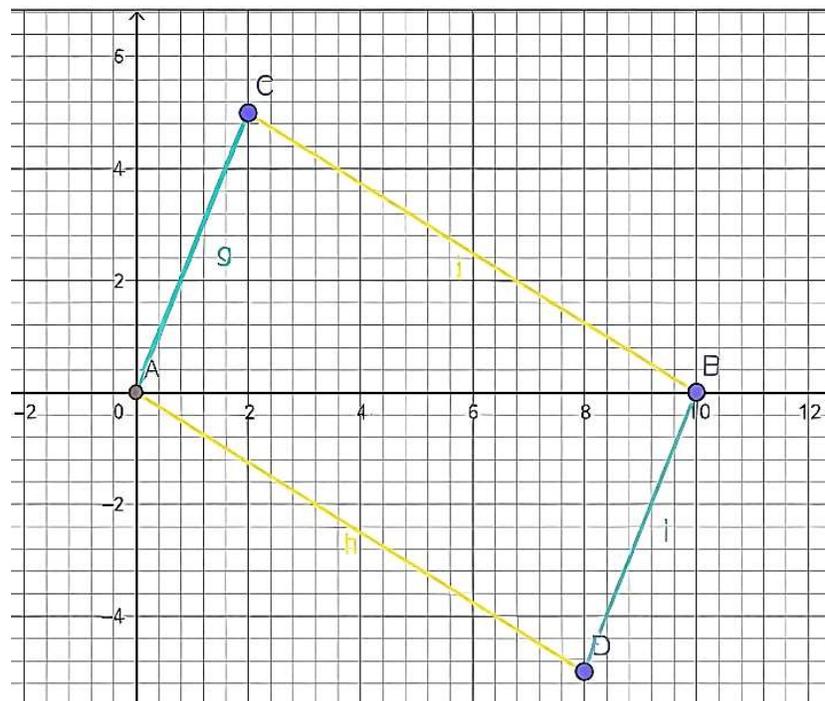


Figure 6. Two pairs of parallel sides $AC \parallel DB$ & $AD \parallel CB$

two congruent triangles (the same shape and size) are created, namely; Triangle ABC is congruent to triangle BCD, so that point D(12,5); Triangle ABC is congruent to triangle ACD, so that point D(-8,5); Triangle ABC is congruent to triangle ABD, so that point D(8,-5).

Strategy 4 is a strategy based on the concept of opposite parallelogram angles being equal; The sum of adjacent or neighboring angles is a supplementary angle (180°); The sum of all angles

is 360° . For example, points A(0, 0), B(10, 0), and C(2, 5) are the corner points of a parallelogram; point D(x, y) is the fourth unknown point, which is then found by calculation or created in GeoGebra. Make an angle with points A(0, 0), B(10, 2) and point C(2, 5), then determine the size $\angle A$, $\angle B$, $\angle C$. Point D(12, 5) is viewed from $\angle A$ which is opposite $\angle D$, where the size $\angle A = \text{the size } \angle D = 68.2^\circ$

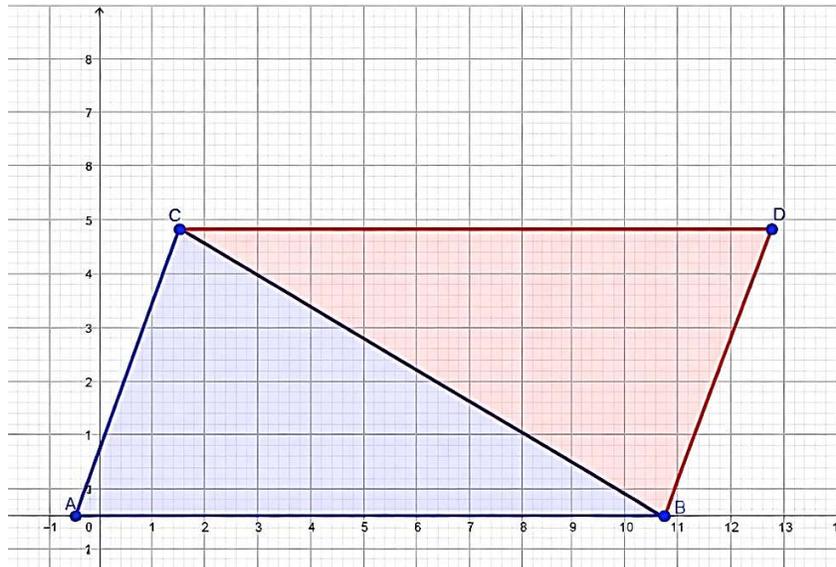


Figure 7. Triangle ABC is congruent to triangle BCD, point D (12,5)

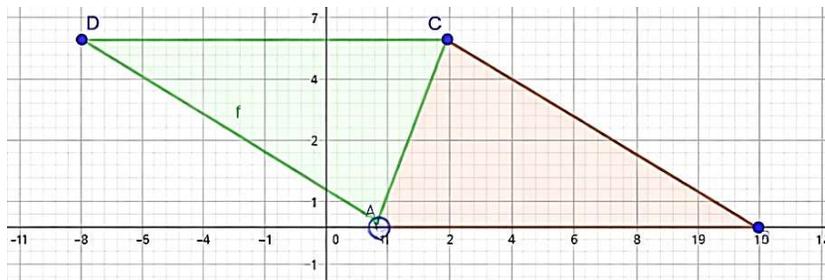


Figure 8. Triangle ABC is congruent to triangle ACD, point D (-8,5)

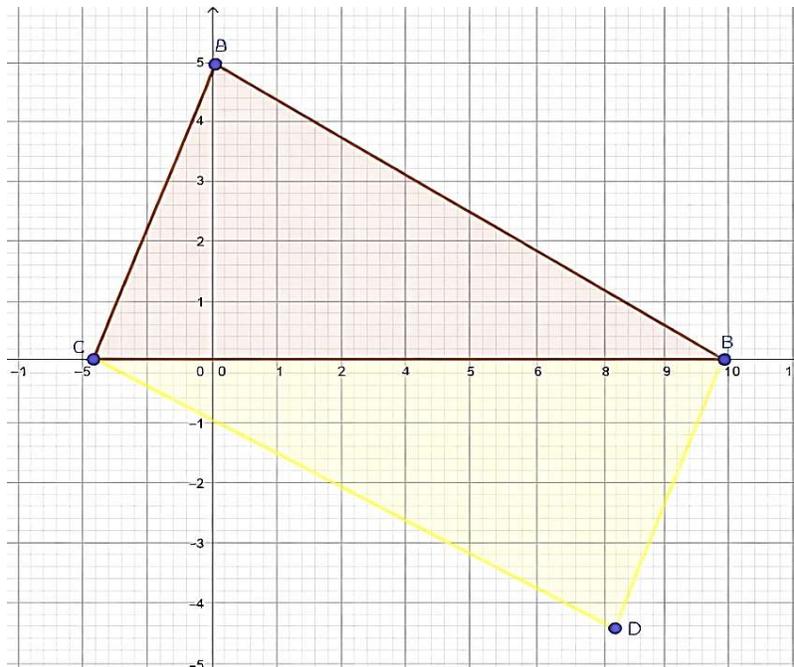


Figure 9. Triangle ABC is congruent to triangle ABD, point D (8,-5)

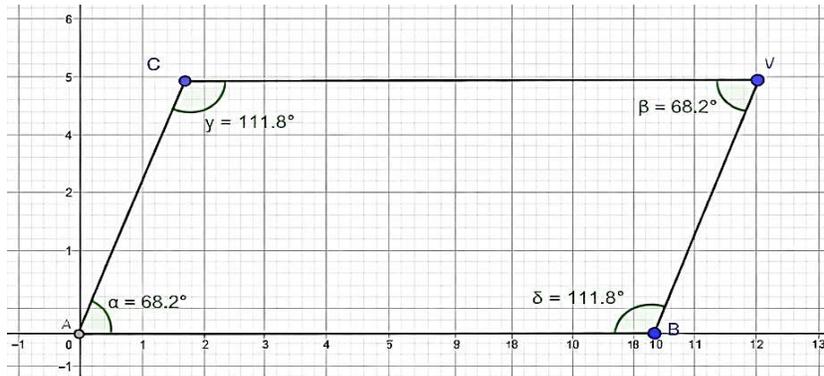


Figure 10. Point D(12,5) viewed from $\angle A$ opposite $\angle D$

Point D(-8,5) is viewed from $\angle B$ which is opposite $\angle D$, where the magnitude of $\angle D =$ the magnitude of $\angle B = 32^\circ$

Point D(8,-5) is viewed from $\angle C$ which is opposite $\angle D$, where the magnitude of $\angle C =$ the magnitude of $\angle D = 79.8^\circ$

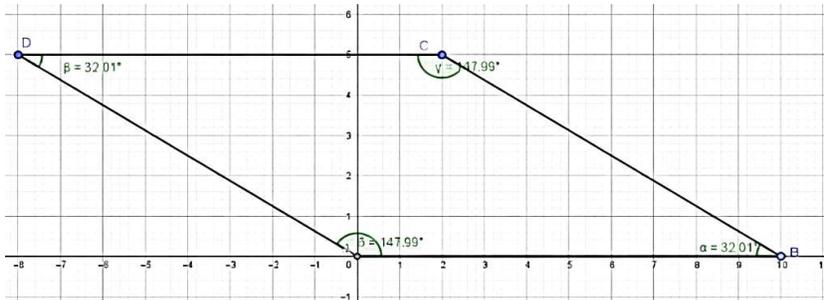


Figure 11. Point D(-8,5) viewed from $\angle B$ which is opposite $\angle D$

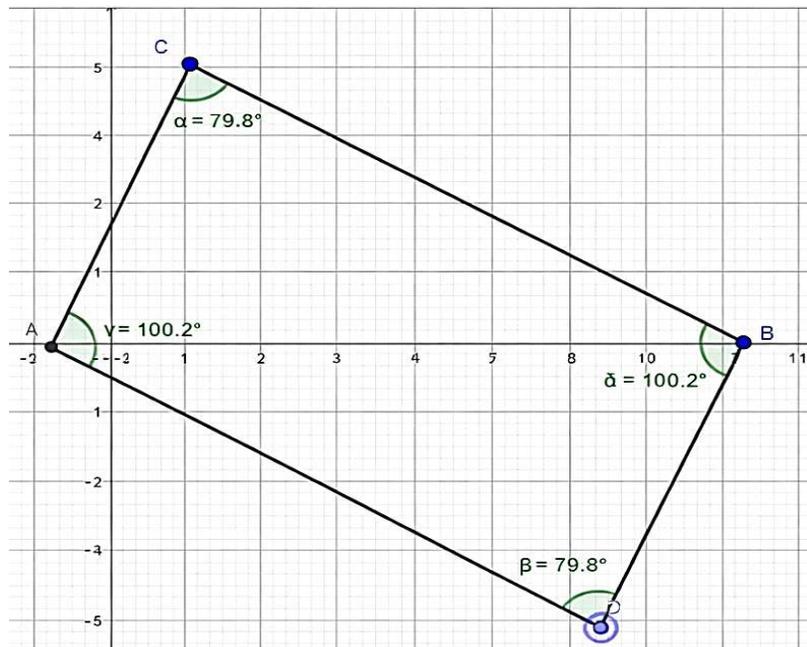


Figure 12. Point D(8,-5) viewed from $\angle C$ which is opposite $\angle D$

Strategy 5 is based on the concept of two diagonals intersecting at their midpoints, with each diagonal bisecting the other. There are three ways to determine which pairs of points form the intersecting diagonals. For example, points A(0, 0), B(10, 0), and C(2, 5) are the vertices of a parallelogram, and point D(x, y) is the unknown fourth point. Possibility 1:

Determining the midpoints of diagonals AC and BD determines point D.

$$\begin{aligned} \text{Midpoint of the diagonal AC} &= \left(\frac{0+2}{2}, \frac{0+5}{2}\right) \\ &= \left(1, \frac{5}{2}\right) \end{aligned}$$

$$\text{Midpoint of the diagonal BD} = \left(\frac{10+x}{2}, \frac{0+y}{2}\right)$$

Determine the point D(x,y)

Midpoint of the diagonal AC = Midpoint of the diagonal BD

$$\begin{aligned} \left(1, \frac{5}{2}\right) &= \left(\frac{10+x}{2}, \frac{0+y}{2}\right) \\ 1 &= \frac{10+x}{2} \quad \frac{5}{2} = \frac{0+y}{2} \\ 2 &= 10+x \quad \frac{10}{2} = y \end{aligned}$$

$$x = -8 \quad y = 5$$

So, point D(-8,5)

Possibility 2:

Determine the midpoint of the diagonal AB & CD determine the point D

$$\text{Midpoint of the diagonal AB} = \left(\frac{0+10}{2}, \frac{0+0}{2}\right) = (5,0)$$

$$\text{Midpoint of the diagonal CD} = \left(\frac{2+x}{2}, \frac{5+y}{2}\right)$$

Determine the point (x,y)

Midpoint of the diagonal AB = Midpoint of the diagonal CD

$$(5, 0) = \left(\frac{2+x}{2}, \frac{5+y}{2}\right)$$

$$5 = \frac{2+x}{2} \quad 0 = \frac{5+y}{2}$$

$$10 = 2+x \quad 0 = 5+y$$

$$x = 8 \quad y = -5$$

So, point D(8,-5)

Possibility 3:

Determine the midpoint of the diagonal BC & AD Determine the point D

$$\text{Midpoint of the diagonal BC} = \left(\frac{10+2}{2}, \frac{0+5}{2}\right) = \left(6, \frac{5}{2}\right)$$

$$\text{Midpoint of the diagonal AD} = \left(\frac{0+x}{2}, \frac{0+y}{2}\right)$$

Determine the point D(x,y)

Midpoint of the diagonal BC = Midpoint of the diagonal AD

$$\left(6, \frac{5}{2}\right) = \left(\frac{0+x}{2}, \frac{0+y}{2}\right)$$

$$6 = \frac{0+x}{2} \quad \frac{5}{2} = \frac{0+y}{2}$$

$$12 = 0+x \quad \frac{10}{2} = 0+y$$

$$x = 12 \quad y = 5$$

So, point D(12,5)

The author visualized the correct solution, then created Table 2, which shows which participants found which solution and what strategies they used.

Table 2. Solution strategy for each participant

Participant	Solution Strategy				
	Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5
P1	v	v	x	x	x
P2	v	v	x	x	x
P3	x	v	x	x	x
P4	x	v	x	x	x
P5	x	v	x	x	x
P6	v	x	x	x	x

Figure 13 and figure 14 show the results of the work of the first participant (P1) and the second participant (P2) have not yet emerged

anything new, this is clearly seen P1 & P2 have not been able to compile, develop several strategies or steps that will be used in solving the

problem and have not been able to avoid copying answers or still copying existing solution procedures this is reflected in the results of in-depth interviews. P1 & P2's plausibility is only as an initial step to understand the problem, namely by stating what is known in the problem and by interpreting known data with unknown data. In addition, although they have been able to find the relationship between the information in the problem and have been able to plan a solution strategy for solving the problem that will be used, namely strategy 1 using the vector concept and strategy 2 using the Cartesian coordinate system concept, as seen in figures 13 and 14. In strategy 1, P1 & P2 are less precise in writing vectors and vector operations. P1 & P2 in strategy 1 have not been able to solve the problem by applying a strategy based on the concept of a parallelogram with two pairs of parallel sides and two pairs of equal-length sides, using vectors on a two-dimensional plane. In strategy 2, P1 has not correctly drawn the parallelogram's parallel sides. In contrast, P2, in addition to not correctly drawing the parallel sides

of the parallelogram, plots the coordinate points on the Cartesian diagram incorrectly. This shows that participants P1 & P2 are still not able to apply the strategies or steps developed to solve the problem. In addition, in terms of plausibility, P1 & P2 are limited to describing the simplest forms of questions and to understanding the types of appropriate answers; in terms of providing arguments, they have not been able to provide a conclusion that explains the relationship between the results and the mathematical concept. Meanwhile, regarding the Mathematical Foundations, it is not yet optimal. However, P1 & P2 can identify and explain the information needed to solve problems and can identify the initial needs and strategies for solving them. However, they have not been able to identify or explain the mathematical operations derived from mathematical concepts. This is because the understanding of mathematical concepts related to vectorization is still lacking, as evidenced by how vectors are written and by the Cartesian coordinate system being made less precise.

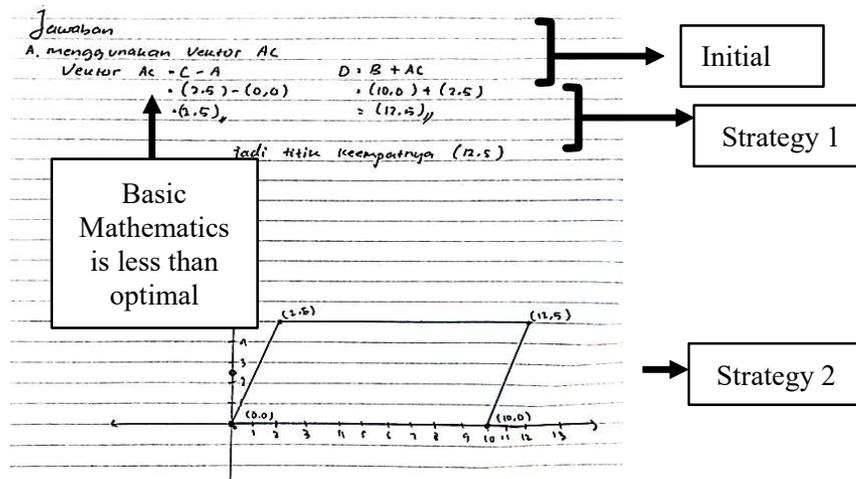


Figure 13. Results of participant one's (P1) question work

In Figure 15, Figure 16 and Figure 17, it is clearly seen that Participant 3 (P3), Participant 4 (P4) and Participant 5 (P5) in terms of novelty, have not yet emerged anything new, this is clearly

seen P3, P4 & P5 have not been able to compile, develop several strategies or steps that will be used in solving the problem and have not been able to avoid copying answers or still copying

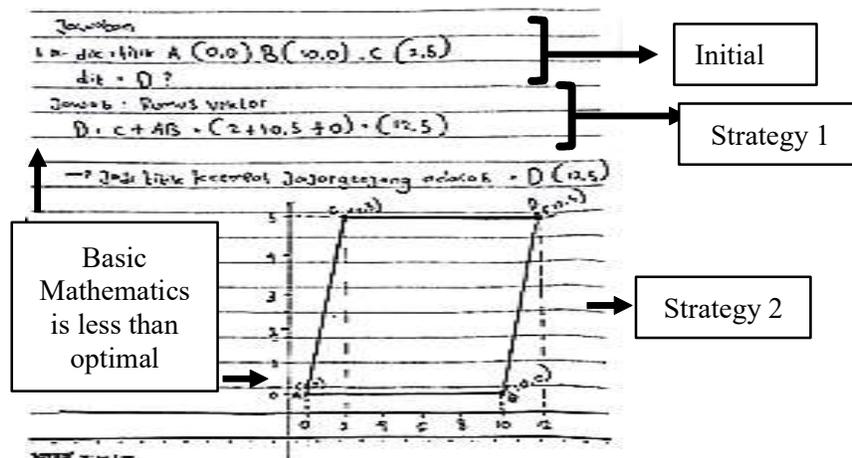


Figure 14. Results of participant one's (P2) question work

existing solution procedures this is reflected in the results of in-depth interviews. Plausibility P3, P4, & P5 are still less than optimal because they do not understand the problem or the instructions in the question and have not realized what should be done. Only have one strategy compiled to solve the problem. Confidence in the truth of the solution, accompanied by arguments for the steps applied to prove the truth, is not clear. The strategy is based on the concept of a parallelogram with two pairs of parallel sides and two pairs of equal sides, which is then created using a Cartesian diagram. Furthermore, although they identified the interconnectedness of the information in the problem and planned a solution strategy, they used only one strategy: the concept of a parallelogram with two pairs of parallel sides and two pairs of equal sides, which was then represented in a Cartesian diagram. In strategy 1, P3, P4, and P5 still did not correctly draw the parallelogram's parallel sides, either in terms of unit scale or in their parallelism, and they did not correctly plot the coordinate points on the Cartesian diagram. This indicates that participants P3, P4, and P5 were still unable to apply the strategies or steps developed to solve the problem. Furthermore, in terms of plausibility, P3, P4, and P5 only outlined the simplest forms of questions and understood the appropriate types

of answers. In terms of argumentation, they were not yet able to provide a conclusion that explains the interconnectedness between the results and the mathematical concept. Meanwhile, regarding the Mathematical Foundations, it is not optimal, as P3, P4, & P5 have not been able to identify and explain the information required to solve problems, nor explain the mathematical operations derived from mathematical concepts. This is because the understanding of mathematical concepts related to parallelogram properties is still lacking, as evidenced by the inappropriate Cartesian coordinate system created.

In Figure 18, it is clear that participant 6 (P6) in terms of novelty, has not yet emerged anything new, this is clearly seen P6 has not been able to compile, develop several strategies or steps to be used in solving problems and has not been able to avoid copying answers or still copying existing solution procedures, this is reflected in the results of in-depth interviews. P6's plausibility is still less than optimal, because he is still unable to identify the problem and understand the instructions in the question and realize what should be done; has not been able to develop a strategy in solving the problem; has not been able to believe in the truth of the solution accompanied by arguments for the steps applied to prove the truth; has not been able to conclude the

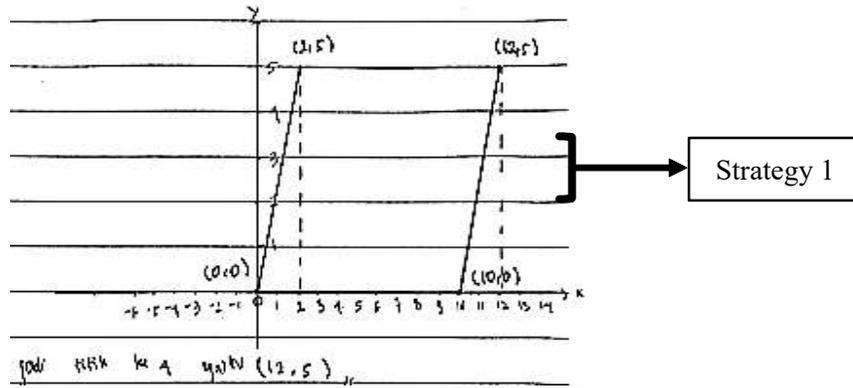


Figure 15. Results of participant three's work (P3)

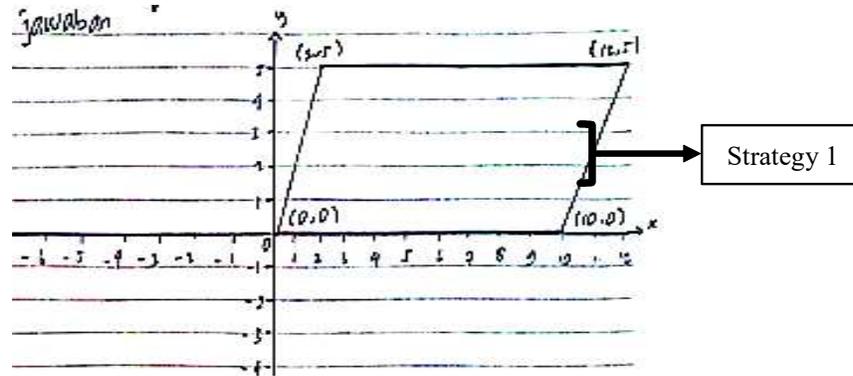


Figure 16. Results of participant four's (P4) work

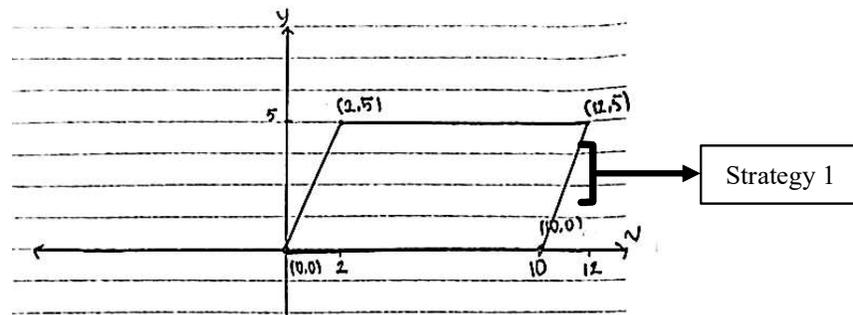


Figure 17. Results of participant five's (P5) question work

application of correct or reasonable methods. Meanwhile, regarding the Mathematical Foundations, it is not optimal: P6 has not identified or explained the information required to solve the problem, nor explained the mathematical operations derived from the concepts. This is due

to a lack of understanding of the mathematical concept of a parallelogram with two pairs of parallel sides and two pairs of equal-length sides, as applied to vectors on a two-dimensional plane. In addition, P6 does not yet understand vectors and vector operations.

Basic Mathematics is less than optimal

$$\rightarrow AC: C.A : (1,5) - (0,0) : (1,5)$$

$$DB: B.D : (0,0) - (-1,4) : (10-x, 0-y)$$

$$: 2: 10-x \text{ dan } 5: -y$$

$$: x : 10 - 2 : 8 \text{ dan } y : -5$$

$$: D : (8, -5)$$

Strategy 1

Figure 18. Results of participant five's work (P6)

Creative mathematical reasoning ability with Multiple Solution Tasks (MSTs) is a type of mathematical task to identify differences between groups related to creative mathematical reasoning (CMR). When conducting field research, researchers identified several root causes of low creative mathematical reasoning among students, based on their work and in-depth interviews with selected individuals. The strategy is unclear because the mathematical foundation indicator is still low in analytical geometry. The mathematical foundation is less than optimal, even though analytical geometry (or coordinate geometry) is a branch of mathematics that uses algebraic equations and coordinate systems (such as the Cartesian plane) to study geometric figures, allowing for the representation of lines, curves, and shapes with equations and solving geometric problems algebraically. This branch bridges algebra and geometry, translating between visual forms and their numerical formulas, which is important for understanding concepts such as distance, slope, midpoint, and transformation. The results of students' work in the upper, middle, and lower creative mathematical reasoning groups remain low and do not meet the level of creative mathematical reasoning ability (Novelty, Plausibility, and mathematical foundation). Geometric images often provide an unclear understanding of the meaning of algebraic results.

In addition, Plausibility, which includes the stages of initiating, developing, verifying, and justifying reasoning, remains unclear. The role of creativity in personal understanding, according to Beghetto & Schreiber (2017), is vital; the more inappropriate learning stimuli (that is, something that is different from students' previous understanding and expectations, the more the stimulus will tend to be ignored by students or included in what students have known before. Furthermore, creative mathematical reasoning abilities with Multiple Solution Tasks (MSTs) will increase if mathematics lecturers more often give assignments with MSTs types of questions. A mathematical assignment that identifies differences between groups related to creative mathematical reasoning (CMR) can be combined with a learning model that is comfortable and connected to students. The researcher validated the findings' accuracy by conducting direct interviews with selected research subjects. The interview results are as follows:

Researcher: Have you ever worked on a problem like this?

Participants P3, P4, P5, P6: Yes, when I was in junior high or high school.

Participants P1, P2: Yes, if I'm not mistaken, back in 10th or 11th grade.

Researcher: Do you understand the meaning of

the problem?

Participants P1, P2, P3, P4, P5, and P6: I understand, but I've forgotten how to solve it.

Researcher: Do you understand the problem and instructions in the question, and are you aware of what you should do?

Participants P1, P2: Understand. You were asked to determine point D on a parallelogram using more than one answer.

Participants P3, P4, P5, P6: Understand. You were asked to determine point D on a parallelogram, but you've forgotten how to solve it.

Researcher: What strategy have you developed to solve the problem?

Participants P1, P2: Using the concept of vectors and Cartesian diagrams.

Participants P3, P4, P5: Using Cartesian diagrams.

Participant P6: Using the concept of vectors.

Researcher: Did you use any mathematical concepts to solve the problem?

Participants P1, P2, and P6: I used the concept of vectors and Cartesian diagrams.

Participants P3, P4, P5: I used Cartesian diagrams.

Researcher: Do you know the mathematical concepts, namely the properties of parallelograms, that can be used to solve the problem?

Participants P1, P2, P3, P4, P5, and P6: I know the property of a parallelogram is that its sides are parallel.

Researcher: Are the properties of parallelograms the only ones you mentioned that can be used to solve this problem? What about other properties of parallelograms, such as those related to congruence, diagonals, and angles?

Participants P1, P2, P3, P4, P5, and P6: The properties of parallelograms can be used to solve this problem. I only

know that parallelograms have parallel sides.

Researcher: Do you have any other ways to solve this problem?

Participants P1, P2, P3, P4, P5, and P6: No, other than what was mentioned earlier.

Researcher: Did you copy an existing answer or solution procedure?

Participants P1, P2, P3, P4, P5, and P6: Based on what I remember about the material.

Researcher: Do you reinvent the solution method to find the solution based on your previous understanding, or do you follow an established solution procedure?

Participants P1, P2, P3, P4, P5, and P6: I follow an established solution procedure.

Researcher: Are you able to complete the task without referring to an established solution procedure?

Participants P1, P2, P3, P4, P5, and P6: I complete the task by referring to an established solution procedure.

Researcher: What have you tried to test your idea/solution?

Participants P3, P4, P5: I will draw a parallelogram by plotting the known points on a Cartesian diagram.

Participants P1, P2: First, I will work with vectors, then plot them on a Cartesian diagram.

Researcher: Why did you write the vector $\overrightarrow{AC} = \overrightarrow{C} - \overrightarrow{A}$ and $\overrightarrow{D} = \overrightarrow{B} + \overrightarrow{AC}$? What makes you sure this step is correct?

Participant P1: I'm not sure about my answer. D stands for AD, and B stands for BD. Then why are they added to AC? Because BD is a line parallel to AC.

Researcher: Are you sure you're writing the vectors that way?

Participant P1: As far as I know, vector $\overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A}$, $\overrightarrow{BC} = \overrightarrow{C} - \overrightarrow{B}$ in a parallelogram. $\overrightarrow{AD} = \overrightarrow{BC}$, so $\overrightarrow{D} =$

- $B + \overline{AC}$ can be written as $D = A +$
- Researcher: Why did you write $D = C + AB$?
What makes you sure this step is correct?
- Participant P2: I'm not sure about my answer. D stands for AD, and C stands for AC. Then why are they added to AB? Because CD is a line parallel to AB.
- Researcher: Why doesn't the graph start at the coordinate center (0,0)? It should be in a plane vector or a two-dimensional vector. There's a term called a position vector, which is a vector that starts at the coordinate center (0,0) and ends at a point (x, y).
- Participants P1, P2, and P6: I don't understand that.
- Researcher: What do you do to more effectively recall previously taught material to solve this problem?
- Participants P1, P2, P3, P4, P5, and P6: Searching for answers, searching online, and using AI.
- Researcher: Do you think it's more efficient to do it yourself by recalling previously learned material while doing the assignment or to search for answers directly on Google?
- Participants P1, P2, P3, P4, P5, and P6: It's more efficient to search for answers directly on Google.
- Researcher: Think back to when you were in middle school or high school. Did you understand when your math teacher explained things?
- Participants P1 & P2: All of your math teachers were good. When they explained things, you understood, but if you didn't understand, you'd often ask friends who already understood.
- Researcher: When you were in middle school or high school, did your math teacher often give you homework?
- Participants P1, P2, P3, P4, P5, P6: Your math teacher rarely gave you homework.
- Researcher: Why did it take so long to do it, even though you'd already studied the material?
- Participants P1 & P2: Because you forgot how to do it, and you also looked up the answer online.
- Researcher: Did you find the answer to that question online?
- Participants P1, P2, P3, P4, P5, P6: If you found something similar, you didn't understand the process.
- Researcher: Did you ask the AI?
- Participant P1, P2: AI's answer is not as expected.
- Researcher: Have you tried using a math app to solve the problem?
- Participants P1, P2, P3, P4, P5, P6: If you don't understand the app on your phone, you don't understand it.
- Researcher: Do you have a math app installed on your phone?
- Participants P1, P2, P3, P4, P5, P6: You've had the app installed on your phone for a long time, but you rarely use it because you don't understand it.
- Researcher: What app is installed on your phone?
- Participants P1, P2, P3, P4, P5, P6: The GeoGebra app is installed on your phone.
- Researcher: Do you know the function of the GeoGebra app?
- Participants P1, P2, P3, P4, P5, P6: To learn geometry

Based on more in-depth interviews, it was discovered that although participants had worked on similar problems in junior high or high school and understood the meaning, they had forgotten how to solve them, even after trying to remember. However, when their math teacher explained the solution, they were able to understand it. Meanwhile, those who didn't understand, many asked their friends who did, even though their

math teacher was good. These interview results are highly urgent and relevant to this study, which aims to determine the description of creative mathematical reasoning (CMR) abilities in the indicator of novelty that has not yet emerged, the indicator of plausibility, and the indicator of mathematical foundation that is still low in the context of analytical geometry, as reflected in the results of their solutions. In the indicator of plausibility, students still have a low understanding of the problem and instructions in the problem, and are not yet aware of what should be done. Furthermore, there is a lack of strategies developed to solve the problem. The student teachers, with their solutions and the arguments for the steps applied, still have many errors, for example, how to determine vectors, how to write symbols and operate vectors, and a lack of understanding of the properties of parallelograms, resulting in suboptimal conclusions.

The interview results revealed that the novelty indicator had not yet appeared in all participants. In participants P1 & P2, the novelty indicator still showed more than one strategy for solving the problem. Although the solution procedure still imitated the existing one, when searching for a solution, participants based it on what they had memorized or on their previous understanding of parallelograms. In other words, participants were able to recreate the solution method using their previous understanding. Participants felt that recalling previously learned material was less efficient than directly searching for answers on Google when working to find a solution. All participants knew the parallelogram material in the problem, but were not yet able to use the mathematical concept to solve it in the solution. For example, the problem related to parallelograms is taught at the high school level. Regarding mathematical foundations, participants must understand the concept of a parallelogram, including the following properties: two pairs of parallel sides, two pairs of equal sides; opposite

angles are equal; the sum of adjacent angles is supplementary (180°); the sum of all angles is 360° ; two diagonals intersect at their midpoint; each diagonal bisects the other; Each diagonal forms two congruent triangles (the same shape and size); parallelograms do not have an axis of symmetry but have second-order rotational symmetry (they can occupy their frame in two ways). This high school-level material is essential for student teachers, and this finding can make a unique contribution to the literature on higher education mathematics by strengthening mathematical foundation indicators and optimizing student teachers' CMR abilities. The impact will be clearly visible when these student teachers teach in schools, where novelty and plausibility, from preparing learning materials to teaching and learning activities, are highly anticipated by the next generation.

The questions asked by the researchers explored past experiences ("Have you ever worked on a problem like this?"), study habits ("did the math teacher often give homework?"), Moreover, technology use was studied because the researchers wanted to understand the causes of students' low CMR. Interview results showed that participants took a long time to work on problems because they searched the internet and AI for answers, even though the answers did not match their expectations. Participants also tried math applications on their mobile phones, but they did not really understand them. These interview results show that learning mathematics requires frequent practice solving problems, but this is not supported by teachers at school, where homework is rarely given. In fact, CMR skills will continue to develop if supported by frequent problem-solving practice. In addition, CMR skills are needed to solve complex mathematical problems by translating between visual forms and their numerical formulas to explore new domains in geometry and to create new applications of mathematics study complex natural phenomena.

■ CONCLUSION

Researchers can conclude that analyzing students' creative mathematical reasoning with multiple solution tasks on analytical geometry material is very useful for developing students' and lecturers' knowledge and experience. By knowing the root of the problem of low creative mathematical reasoning (CMR) of students, lecturers will try their best to find ways to improve students' creative mathematical reasoning (CMR) abilities. One way is that mathematics lecturers more often give assignments with MSTs types of questions that can be combined with learning models that are comfortable and connected to students. Although it provides valuable insights into the factors that influence creative mathematical reasoning in analytical geometry, it has the limitation that participants are not fully involved in the learning. Participants' intentions towards technology integration may be limited to seeking help because they have difficulty when asked to solve problems from questions in the form of multiple solution tasks. Future research should explore teachers' perspectives in the office to understand how teaching experience influences technology adoption and to explain the challenges and opportunities experienced educators face when integrating technology into their pedagogical practices.

The qualitative nature of this study means its limitations include the inability to generalize the findings broadly across contexts. Future research should use mixed methods or large-scale quantitative approaches to support its findings. In-depth studies that focus on the characteristics of memory processes, ethics, other mathematical abilities, and pedagogical implications of technology use in the classroom are also needed. Recommendations for future research include investigating the effectiveness of different technology-enabled pedagogical approaches and developing policies to ensure equitable access to technology across educational settings. Finally, studies with the same variables and the same

groups or individuals, repeatedly and over a long period of time, will help explain the long-term effects of technology integration on students' mathematical abilities.

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