



Mapping Newman's Error Analysis to Mathematical Creative Thinking: A Diagnostic Tool for Identifying Cognitive Disruptions

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Abstract: This study examines the relationship between Newman's Error Analysis (NEA) stages and dimensions of Mathematical Creative Thinking (MCT) in solving contextual problems on relations and functions. Using a descriptive qualitative approach, 25 eighth-grade students were analyzed through two open-ended contextual essay items and semi-structured interviews. Errors identified at each NEA stage (reading, comprehension, transformation, process skills, encoding) were mapped to corresponding MCT dimensions to investigate correlations between error patterns and limitations in creative thinking. Findings indicate that students' primary difficulties emerged at higher-order cognitive stages. Most students succeeded in the reading (23 students on item 1) and comprehension stages (19 students), yet substantial errors occurred during transformation (14 errors), process skills (17 errors), and encoding (20 errors), a pattern similarly observed in Item 2. The narrowing of the Sankey diagram flow suggests that the core difficulties lie not in basic literacy skills but rather in increasing representational and procedural complexity, particularly at the transition from transformation to process skills. Case analyses revealed distinct profiles: high-ability students demonstrated strong fluency and flexibility but experienced a "cognitive transparency illusion" that constrained their elaboration; medium-ability students showed inconsistency in strategic execution due to strategic breakdowns and affective instability; and low-ability students encountered cascading failures beginning from the earliest stages. The study positions the NEA–MCT mapping as an interpretive diagnostic helpful framework for identifying cognitive–affective barriers to mathematical creativity. This framework supports differentiated interventions, including metacommunicative scaffolding for high-ability students, integrated cognitive–strategic–affective support for medium-ability students, and foundational representational instruction with affective scaffolding for low-ability students. Limitations include the small sample size and the narrow task context. Future studies should involve larger and more diverse participants, incorporate real-time think-aloud data, explore additional mathematical domains, and evaluate the framework's potential in digital learning environments.

Keywords: mathematical creative thinking, Newman's Error Analysis, problem solving, relations and functions.

▪ INTRODUCTION

One of the high-order thinking skills necessary for teaching mathematics in the twenty-first century is mathematical creative thinking (MCT). MCT is defined as students' ability to solve mathematical problems through innovation, flexibility, and originality (Bicer et al., 2024). MCT has four primary dimensions, according to the frameworks put forth by Guilford (1967) and Torrance (1974): fluency (the capacity to generate multiple ideas), flexibility (the capacity to employ a variety of strategies), originality (the capacity to produce original solutions), and elaboration (the capacity to explain concepts in detail). Students with high mathematical creativity are able to design non-routine solution strategies, explore alternative approaches, and construct varied representations of a problem (Arifianti & Baidawi, 2024; Asikin et al., 2019).

However, previous studies have consistently reported that students' mathematical creative thinking is still low. Students struggle to generate multiple ideas (fluency), develop alternative strategies (flexibility), propose unique solutions (originality), and articulate their reasoning in detail (elaboration) (Nufus et al., 2024). These difficulties hinder their ability to solve non-routine tasks, including open-ended problems that require creative thinking (Bicer, 2021). The low level of students' mathematical creative thinking is influenced by both internal and external factors. Internal factors include limited conceptual understanding, cognitive barriers to problem-solving, and low self-confidence to explore alternative solutions (Wahyuni et al., 2024). External factors include instructional approaches that emphasize routine procedures, limited use of open-ended problem-solving tasks, and learning environments that do not promote exploration and creativity (Bicer et al., 2024).

To address these issues, a diagnostic approach is needed to identify students' specific difficulties. Newman's Error Analysis (NEA) has been proven effective in identifying students' difficulties in mathematical problem-solving through its five hierarchical stages: reading, comprehension, transformation, process skills, and encoding (Kurniati et al., 2021; Newman, 1977). Although initially developed to trace the sources of procedural errors, cognitive studies indicate that each NEA stage also reflects more complex mental processes, such as interpreting information, selecting representations, developing procedures, and articulating reasoning (Schoevers et al., 2018; Elgrably & Leikin, 2021; Suherman & Vidákovich, 2023).

The empirical literature offers preliminary support for mapping the NEA stages onto the dimensions of MCT, although the relationship is complex and nonlinear. Errors in the reading and comprehension stages are often associated with limitations in fluency, as students fail to identify or interpret essential elements of a problem (Nufus et al., 2024; Schoevers et al., 2018), but may also stem from linguistic obstacles (Kania et al., 2024) or weak conceptual knowledge that constrains all creative dimensions (Schoevers et al., 2018). The transformation stage shows theoretical alignment with flexibility, as demonstrated by findings that students who are unable to reformulate contextual situations exhibit low strategic flexibility (Elgrably & Leikin, 2021; Lu et al., 2025) or by their inability to generate alternative representations, which relates to fluency and originality (Schoevers et al., 2018). The process-skills stage is associated with originality because non-routine procedures require divergent thinking (DeVink et al., 2022; Hartati et al., 2025), although errors at this stage may also arise from technical procedural weaknesses that do not reflect limitations in creativity (Hoth et al., 2022; Singer, 2018), or from failures in the preceding transformation stage, suggesting hierarchical interrelations among NEA stages (Mathaba et al., 2024). The encoding stage corresponds to elaboration, which demands the ability to organize reasoning coherently (Kozłowski et al., 2019; Suherman & Vidákovich, 2022), yet encoding errors do not always indicate weak elaboration, as they may also result from limited mathematical communication skills (Nguyen et al., 2025) or even technical carelessness despite otherwise accurate thinking. Thus, this study conceptualizes the NEA–MCT mapping as a flexible, interpretive diagnostic lens rather than a deterministic categorization, acknowledging that a single type of error may reflect multiple limitations simultaneously and that creative thinking is multidimensional, with interconnected cognitive constraints.

Although both domains have advanced, research on NEA and MCT continues to develop in isolation (Lubis et al., 2021) and exhibits notable methodological limitations. MCT research focusing on instructional interventions has been shown to enhance students' flexibility and originality (Leikin & Elgrably, 2020; Lu et al., 2025), yet it has three significant limitations: post-hoc assessments based on aggregate scores that fail to capture the real-time dynamics of students' thinking processes; the inability to identify the specific stages at which creative barriers occur, resulting in generic interventions; and a tendency to treat creativity as a unidimensional construct that overlooks the complexity of cognitive processes (Ibrahim et al., 2024; Säfström et al., 2024). Conversely, NEA research is effective in identifying the hierarchical location of errors (Lubis et al., 2021; Thomas & Mahmud, 2021). However, it operates within a convergent paradigm that interprets deviations as "errors" without acknowledging creative exploratory attempts (Buelban & Tan, 2024), treats errors merely as technical failures without linking them to dimensions of creative thinking, and adopts a static, retrospective perspective that fails to capture strategies that were attempted but later abandoned (Säfström et al., 2024; Zhang et al., 2025). As a result, MCT research provides diagnoses of creativity without localizing the underlying processes. In contrast, NEA research focuses on error localization without explaining the creative barriers that give rise to them.

This study applies the NEA framework to analyze students' mathematical creative thinking abilities in solving contextual problems in relations and functions. Specifically, this study aims to: (1) examine the structure and mechanisms underlying errors at each NEA stage, rather than merely identifying their typologies, and (2) explain how and why particular error patterns indicate limitations in the dimensions of fluency, flexibility, originality, and elaboration, as well as how these limitations influence students' reasoning processes. Through the integration of written tests and in-depth interviews, this study not only describes the types of errors that emerge but also interprets them as indicators of cognitive barriers related to creative thinking processes. This analytical focus provides a theoretical contribution by conceptualizing errors as cognitive indicators rather than technical symptoms, and a practical implication by offering educators guidance for designing pedagogical interventions that target specific dimensions of students' mathematical creativity in a nuanced manner (Bicer et al., 2024).

▪ METHOD

Research Design

A descriptive qualitative method was used in this investigation. This method was selected because it enables the researcher to thoroughly examine how pupils solve mathematical problems rather than focusing solely on outcomes. The study's primary goal was to examine students' error patterns using Newman's Error Analysis (NEA) stages and connect them to the markers of mathematical creativity—fluency, flexibility, originality, and elaboration.

Participants

Twenty-five eighth-graders from a public junior high school in Majalengka Regency, West Java, participated in this study. Purposive sampling was used to choose the participants based on their prior knowledge of Relations and Functions. The topic of Relations and Functions was selected strategically because it possesses three unique

characteristics that align well with the NEA–MCT framework: (1) its hierarchical conceptual structure (domain–codomain–mapping rule) allows for step-by-step tracing of errors in accordance with the stages of NEA; (2) its high representational flexibility (verbal descriptions, arrow diagrams, ordered pairs, graphs) naturally requires creativity in selecting and transforming representations—an essential aspect of flexibility and originality; and (3) its rich applicative context enables the design of open-ended tasks without compromising the mathematical structure necessary for systematic NEA analysis. These characteristics render the topic highly suitable for examining how conceptual misconceptions manifest as procedural errors and limitations in students’ creative mathematical thinking. The pupils were divided into three ability categories (high, medium, and low) based on the initial test results. Six students were selected as case-study participants: two from each ability category (high, medium, and low) to enable within-group comparison and ensure representativeness of reasoning patterns.

Instruments

The primary research instrument of this study consisted of two open-ended contextual essay questions designed to stimulate creative ideas and allow for variation in solution strategies. The questions were constructed by integrating indicators of mathematical creative thinking skills with the stages of Newman’s Error Analysis (NEA). In this study, originality is contextually defined at three levels: (1) procedural originality, referring to the use of solution methods that were not explicitly taught in prior instruction; (2) representational originality, referring to the use of mathematical representations (diagrams, tables, graphs, or symbolic expressions) that differ from those modeled by the teacher; and (3) strategic originality, referring to problem-solving approaches that are not employed by the majority of students within the same class. The mapping of mathematical creative thinking indicators to each test item is presented in Table 1.

Table 1. The test's indicators of mathematical creativity

Indicator of Creative Thinking Skill	Item Number
Generating multiple solution ideas (fluency)	1
Applying various strategies to solve the problem (flexibility)	1
Producing unique and different solutions or strategies (originality)	2
Presenting answers in a detailed, structured, and logical manner (elaboration)	2

The decision to distribute the four MCT indicators across two items was a deliberate methodological choice aimed at enhancing diagnostic precision rather than fragmenting the holistic nature of mathematical creative thinking. Separating the indicators enables clearer identification of stage-specific cognitive mechanisms within the NEA framework, allowing pinpointing whether creativity-related difficulties arise during comprehension, transformation, process skills, or encoding. This design also minimizes unnecessary cognitive load. When a single item requires students to demonstrate fluency, flexibility, originality, and elaboration simultaneously, junior high school learners risk becoming overwhelmed as they must coordinate multiple cognitive processes at once. Such combined demands can easily lead to cognitive overload, resulting in performance that reflects the task's complexity rather than students’ actual creative abilities. By distributing the indicators across two items, each task becomes more focused and mentally

manageable, allowing students to display their creative potential more authentically without being hindered by excessive task complexity. Furthermore, the separation provides structured data for mapping NEA stages to corresponding MCT dimensions, thereby supporting fine-grained analysis of where creativity barriers emerge.

To ensure the reliability of the instruments, content and construct validation were conducted by a senior mathematics education lecturer and an experienced mathematics teacher, followed by a pilot test with a group of students. This process ensured that the items effectively stimulated creative thinking while clearly revealing the stages of NEA. In addition to the written test, students' thought processes were explored through a semi-structured interview guide. The interviews focused on four aspects: (1) understanding the problem, (2) transforming information and selecting solution strategies, (3) generating creative solutions, and (4) reflecting on errors. All interviews were conducted in person, audio-recorded with participants' permission, and transcribed verbatim. To support the analysis, an NEA–MCT mapping sheet was developed as an analytical tool to connect identified errors at each NEA stage with the corresponding dimensions of mathematical creative thinking.

Research Procedures

The study was conducted from April to May 2024. It began with the development of two open-ended contextual items designed to integrate the five stages of Newman's Error Analysis (reading, comprehension, transformation, process skills, and encoding) and the four indicators of mathematical creative thinking—fluency, flexibility, originality, and elaboration. The instrument then underwent content and construct validation conducted by two validators: a senior mathematics education lecturer and an experienced mathematics teacher, both with expertise in assessment development and curriculum design. The validators evaluated the alignment of each item with the creative-thinking indicators, the clarity of language and instructions, and the appropriateness of operational verbs according to Bloom's Taxonomy. Their feedback highlighted several refinements, including: 1) shortening overly lengthy contextual statements, 2) clarifying instructions to stimulate a broader range of strategies, and 3) adding a justification component to items targeting originality. Based on these suggestions, revisions were made to simplify the wording, strengthen instructional clarity, and incorporate an explicit justification prompt. Experts subsequently re-evaluated the revised items to ensure theoretical consistency and operational clarity.

The validated and revised instrument was then pilot-tested with 25 eighth-grade students under regular classroom conditions with an allotted time of 90 minutes. The selection of the 25 students employed purposive sampling based on conceptual readiness criteria. Students were required to have studied Relations and Functions during the current semester, a condition verified through direct confirmation with the mathematics teacher and examination of instructional documents such as lesson plans and formative assessment records. The teacher confirmed that all students had achieved the minimum mastery criterion ($KKM \geq 70$) and demonstrated foundational understanding of relational representations, domain–codomain identification, and functional property evaluation. This verification ensured that the pilot test measured students' creative mathematical thinking rather than deficits in prerequisite conceptual knowledge. The pilot test aimed to evaluate the practical effectiveness of the instrument, including the clarity of the instructions, the ability of the items to differentiate student ability levels, and their

capacity to elicit authentic creative thinking. The results informed a final round of refinements concerning item presentation, instructional phrasing, and internal coherence, thereby enhancing the instrument's credibility for use in the main study.

Pilot-test scores were then used to classify the 25 students into three ability groups (high, medium, and low) using the 33rd and 66th percentiles as cut-off points. This tertile-based categorization is a common practice in educational research because it yields proportional and stable groupings, particularly for small samples. Students scoring above the 66th percentile were categorized as high-ability, those between the 33rd and 66th percentiles as medium-ability, and those below the 33rd percentile as low-ability. From these groups, six students were selected as case study participants using a maximum variation sampling strategy, with two students from each ability category. Selection criteria included completeness of written responses, distinctive error patterns characteristic of each ability level, and willingness to participate in follow-up interviews. This strategy ensured that the selected participants represented contrasting reasoning profiles and enabled an in-depth exploration of mathematical errors and creativity through Newman's Error Analysis (NEA).

To deepen the findings, semi-structured interviews were conducted with six students: two students from each ability category (high, medium, and low). Each interview lasted 10–15 minutes and followed a structured protocol designed to elicit authentic reasoning processes. The protocol consisted of four core components: (1) clarifying students' interpretations of the test items, (2) reconstructing their written solutions step by step, (3) exploring alternative strategies they considered or abandoned, and (4) prompting reflection on the creative aspects of their responses. To obtain richer data, the interviewer employed probing techniques such as prompting ("Can you explain how you decided to use this strategy?"), counterfactual probing ("If you had to solve this using a different approach, what might you try?"), and error-focused probing ("At which step did you feel uncertain or change your approach?") to elicit authentic reasoning processes. To minimize retrospective reconstruction, students were asked to verbalize their thinking while referring directly to their written work, and the interviewer followed the chronological sequence of their solution steps rather than requesting explanations at the end. Follow-up questions were also linked to specific segments of the students' written responses, ensuring that their explanations reflected their actual reasoning during the test rather than after-the-fact narratives. These procedures enhanced the reliability and depth of the interview data. Data from the written test, interviews, and the NEA–MCT mapping sheets were triangulated to obtain a comprehensive depiction of students' mathematical creative thinking abilities.

Data Analysis

To provide a thorough picture of students' mathematical creative thinking abilities and the kinds of mistakes they made, data were analyzed using a descriptive qualitative approach. Each step of Newman's Error Analysis was used to uncover faults in written test data, which were then connected to the pertinent signs of creative thinking.

Interview data were analyzed using thematic analysis (Braun & Clarke, 2006), employing both inductive and deductive approaches. The coding process began with repeated readings of the interview transcripts to achieve data familiarization, followed by initial coding in which meaningful units were identified and labeled according to both emergent patterns (inductive) and pre-established categories derived from the NEA–MCT

framework (deductive). Two coders independently generated the initial codes, and intercoder reliability was ensured through peer debriefing sessions in which discrepancies were discussed until full consensus was reached.

The primary analytical instrument in this study was the NEA–MCT Mapping Sheet, which links the five stages of Newman’s Error Analysis (reading, comprehension, transformation, process skills, and encoding) with the four dimensions of mathematical creative thinking (fluency, flexibility, originality, and elaboration). The sheet contains stage-specific indicators, behavioral markers for each MCT dimension, examples of correct and incorrect responses, and criteria connecting particular error patterns to corresponding limitations in creativity. During analysis, each coded interview segment was matched to the relevant NEA stage and then examined to determine whether the student’s reasoning reflected weaknesses in specific MCT dimensions. This procedure enabled systematic triangulation across written work, interview explanations, and the theoretical framework guiding the analysis.

The resulting thematic categories encompassed patterns in the emergence of indicators of creative thinking across error stages, students’ cognitive strategies for addressing errors, and their metacognitive reflections on their own thinking processes. Triangulation between written test responses and interview transcripts was conducted through an integrative comparison. First, errors in the written test, already mapped to NEA stages, were compared with the students’ explanations in the interviews. When discrepancies emerged (for example, a response coded as a transformation error on the written test but supported by a logically sound explanation in the interview), the research team revisited the reasoning through line-by-line transcript analysis to identify whether the error stemmed from a misconception, a procedural lapse, or an unclear written explanation. Interviews were also used to uncover conceptual weaknesses not visible in correctly written responses (such as incomplete understanding despite producing the correct answer), and these findings were incorporated into the final coding. Inconsistent cases were resolved through peer discussion to ensure coding consistency. This triangulated approach produced a more robust interpretation of how NEA-based errors align with students’ profiles of mathematical creative thinking.

Table 2. NEA–MCT mapping sheet

NEA Stage	Error Indicators	MCT Dimension	Behavioral Markers	Evidence Examples	Diagnostic Interpretation
Reading	Item 1: Misreading the sets and the rule “greater by one.” Item 2: Misreading the domain, the function, and the color codes.	Fluency	Missing essential information, failing to list all elements, and misinterpreting symbols.	Item 1: Wrote $A=\{1,2\}$ instead of $\{1,2,3\}$. Item 2: Did not note that $1=A$, $2=B$, etc.	Failure to grasp basic information prevents the student from initiating the problem-solving process appropriately.
Comprehension	Item 1: Misunderstanding “ $b = a + 1$.” Item 2: Not recognizing that each input yields a single output; misinterpreting number–letter conversion.	Fluency	Unable to restate the problem; confused about relational context; misunderstanding the task demand.	Item 1: Interpreted “greater by one” as $a + a$. Item 2: Assumed the function output was only a number.	Weak conceptual understanding hinders the student’s ability to construct an accurate interpretation of the problem.
Transformation	Item 1: Unable to write ordered pairs; relied solely on an arrow diagram. Item 2: Did not express $f(x)=2x+1$;	Flexibility	Using only one representation, not attempting alternative strategies, and rigid thinking.	Item 1: Only drew arrows without listing (a,b). Item 2: Calculated outputs directly without	Low strategic flexibility; the student struggles to shift between representations or explore alternative solution paths.

	failed to convert numbers into letter codes.			explicitly writing the function rule.	
Processes Skills	Item 1: Substitution errors; unsystematic work; stopping mid-process. Item 2: Computational errors; incorrect number-letter conversion; incomplete evaluation of all domain elements.	Originality	Following memorized procedures, not attempting more efficient methods, and relying on conventional textbook strategies.	Item 1: Computed values manually despite the clear pattern $b = a + 1$. Item 2: Did not construct a table or diagram, although it would simplify the process.	Limited originality; reliance on routine procedures restricts divergent thinking and creativity in problem solving.
Encoding	Item 1: Did not conclude whether the relation is a function; justification incomplete. Item 2: Did not state the final rule; produced no complete diagram; failed to relate results to the context.	Elaboration	Inability to justify steps; incomplete explanations; lack of verification; weak connection between results and context.	Item 1: Listed the ordered pairs but did not conclude about the function property. Item 2: Wrote $f(x)$ results but without explanation or a complete mapping diagram.	Weak elaboration; difficulty articulating and organizing reasoning coherently indicates limited metacognitive reflection and mathematical communication.

To support the analytical process, NEA–MCT mapping sheets were used to systematically link each identified error at the five stages of NEA with the corresponding dimensions of mathematical creative thinking. For each student, errors and reasoning indicators extracted from the written test and interview transcripts were coded onto the mapping sheet, allowing the researcher to trace how specific breakdowns in problem-solving aligned with limitations in fluency, flexibility, originality, or elaboration. These mapping sheets served as an integrative analytical tool, facilitating cross-case comparisons and enabling the development of interpretive themes connecting students' error patterns with their creative thinking profiles.

▪ RESULT AND DISCUSSION

In this section, the research findings are presented following the established qualitative analytical procedures. Data from the written work and interview excerpts were integrated to depict the dynamics of students' errors at each stage of Newman's Error Analysis (NEA) framework and their implications for mathematical creative thinking abilities.

Examining Students' Mistakes Using Newman's Error Analysis (NEA)

Table 3 shows the distribution of errors across the five Newman stages.

Table 3. Student errors based on NEA

NEA Stage	Q1 Achieved (students)	Q1 Error (students)	Q2 Achieved (students)	Q2 Error (students)
Reading	23	2	19	6
Comprehension	19	6	14	11
Transformation	11	14	5	20
Process skills	8	17	4	21
Encoding	5	20	0	25

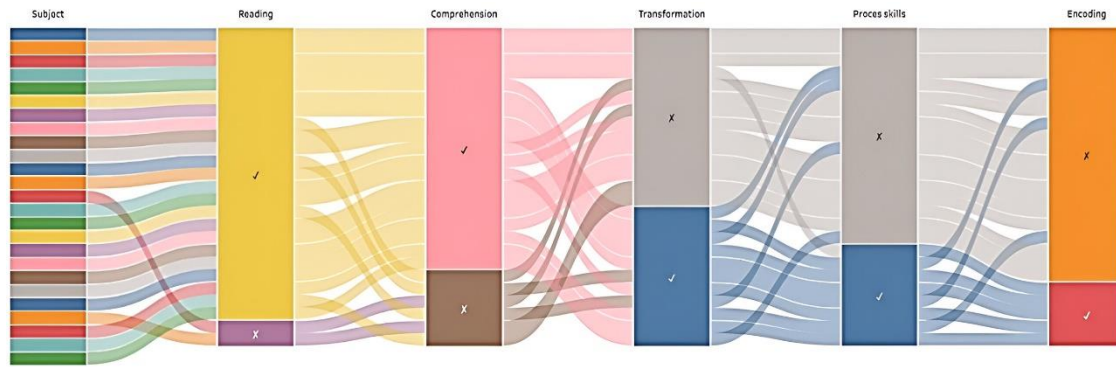


Figure 1. Flowchart of students’ Q1 solution path

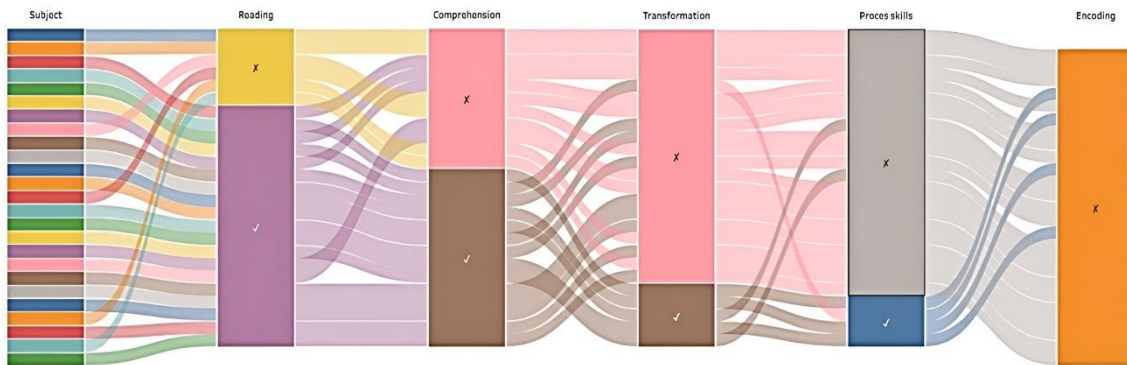


Figure 2. Flowchart of students’ Q2 solution path

The Sankey diagram illustrates the progressive reduction in the number of students who successfully passed each NEA stage without errors. In Item 1, the initially broad flow at the reading stage (23 students) narrows sharply at the transformation stage (11 students) and continues to constrict toward the encoding stage (5 students). In Item 2, the narrowing pattern is even more pronounced: from 19 students at the reading stage, only 5 passed the transformation stage without errors, and none reached the encoding stage correctly. This pattern reflects the cumulative nature of errors —disruptions at earlier stages substantially increase the likelihood of failure in subsequent stages. The transition point from transformation to process skills emerges as the most vulnerable segment, indicating weaknesses in representational flexibility, a crucial skill for fluent and creative problem-solving.

The Sankey visualization further indicates that, for medium-ability students, failures at the transformation stage are systematically linked to errors that emerge in the process-skills stage. The flow patterns show that when students are unable to form complete and stable intermediate representations during transformation, they tend to enter the process-skills stage without a sufficiently coherent problem-solving structure. This is reflected in unsystematic computations, discontinuous application of the strategy, and fragmented procedural execution, as evidenced by the pronounced narrowing of the flow at this stage. Thus, the errors observed in the process-skills stage are not isolated procedural mistakes but downstream consequences of representational incompleteness at the transformation stage, forming a consistent cross-stage error pathway within the overall problem-solving process.

A comparison of the two items shows that Item 2 generated more reading and comprehension errors than Item 1, suggesting that the functional context required more complex symbolic processing. However, both items produced consistently high error rates at the transformation stage and beyond. This indicates that students' primary difficulties do not stem from contextual differences between items but rather from limitations in transforming conceptual understanding into procedural steps in a creative and flexible manner. These error patterns closely correspond to the dimensions of mathematical creative thinking. The relatively low frequency of reading errors suggests that initial fluency was adequate, yet it declined when students failed to identify essential elements during comprehension. The surge of errors at the transformation stage reflects low flexibility, as students tended to rely on a single form of representation. Errors in process skills illustrate weak originality, while the dominance of encoding errors—particularly in Item 2 indicates underdeveloped elaboration, namely the ability to articulate reasoning coherently and reflectively.

Case Analysis Across Three Ability Categories

Six students representing high (S1a, S1b), medium (S2a, S2b), and low (S3a, S3b) ability levels had their written responses and interview findings thoroughly examined to gain a deeper understanding of students' creative thinking profiles. Table 3 summarizes each student's achievement across the four indicators of mathematical creative thinking.

Table 4. Student performance according to the Newman stage

Newman Stage	S1 (High)				S2 (Medium)				S3 (Low)				
	1	2	3	4	1	2	3	4	1	2	3	4	
Reading	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	X
Comprehension	✓	✓	✓	✓	✓	✓	✓	✓	X	X	X	X	X
Transformation	✓	✓	✓	✓	✓	✓	X	X	X	X	X	X	X
Proces Skills	✓	✓	✓	✓	✓	✓	X	X	X	X	X	X	X
Encoding	✓	✓	✓	X	✓	X	X	X	X	X	X	X	X

S1: High- Ability Student

High-achieving students (S1a and S1b) demonstrated strong conceptual understanding and a systematic approach to solving problems on relations and functions. Although both exhibit high performance profiles, nuanced differences emerge in their reasoning processes, representational strategies, and affective dispositions, as revealed through the integrated analysis of written test responses and interview data. The study is presented sequentially according to the NEA stages, linked to the four indicators of mathematical creative thinking, and accompanied by interpretations of affective dimensions. For comprehensive context, excerpts from interviews with S1a and S1b are provided to illustrate the full scope of their cognitive and affective processes, followed by an in-depth interpretive analysis of each Newman stage.

P : “How did you understand the information in the problem and decide on your solution steps?”

S1a : “I immediately understood what was being asked and wrote down the key information. Then I converted it into a function and checked the patterns in

my mind. Since the steps were already clear to me, I did not feel the need to write everything in detail.”

- S1b** : *“The information was quite clear, so the functional pattern formed directly in my head. I completed most of the steps mentally and only wrote the final part because I felt the process was already clear and did not need to be written out fully.”*
- P** : *“What did you feel while working on the problem, especially regarding time pressure or the possibility of making mistakes?”*
- S1a** : *“I felt calm because I am used to this type of problem. I did not feel pressured by time and was not too worried about making mistakes because I understand the concepts.”*
- S1b** : *“I felt confident, so there was no panic or fear of being wrong. I just wanted to complete the problem as efficiently as possible, so I wrote the answer concisely.”*

The interview excerpts reveal a shared affective pattern between S1a and S1b: high confidence, low anxiety, and a tendency to conduct cognitive processing internally. The key difference lies in the motivation behind the brevity of their written elaboration: S1a's stems from perceived clarity (“the steps were already clear”), whereas S1b's is driven by an orientation toward efficiency (“as efficiently as possible”). These contrasting patterns carry different implications for the development of elaboration skills, which will be further discussed in the subsequent analysis.

Reading and Comprehension Stages

S1a and S1b demonstrated strong fluency in reading and comprehension, successfully identifying key information and understanding the problem structure quickly and without confusion. Responses such as “I immediately understood what was being asked” (S1a) and “the functional pattern directly formed in my head” (S1b) reflect automatic pattern recognition supported by mature conceptual schemas. Both students exhibit what Schoevers et al. (2018) and Bicer et al. (2024) describe as “domain-specific knowledge accessibility,” in which prior conceptual understanding enables direct pattern recognition without excessive cognitive load. Affective dimensions reinforce this fluency; both students reported feeling calm, confident, and unpressured by time. This sense of cognitive-emotional security reduces the cognitive load commonly induced by anxiety, thereby facilitating smoother retrieval of information from long-term memory (Lu et al., 2025; Wahyuni et al., 2024). Their fluency thus emerges from a combination of well-developed cognitive readiness and positive self-efficacy.

Transformation Stage

At the transformation stage, both students displayed flexibility by converting verbal descriptions into accurate mathematical representations, though through different cognitive styles. S1a used explicit intermediate steps during substitution, a deliberate, externally documented strategy. In contrast, S1b worked more concisely through internal mental processing (“only writing the final part”), reflecting cognitive compression and a high degree of procedural automatization. In several of S1b's responses (Q1 and Q2), the transformation stage was not visibly written, yet the process-skills and encoding outcomes remained accurate. Triangulation with interview data confirms that the

transformation occurred mentally without externalization. This phenomenon reflects the cognitive compression often present in high-achieving students, who process multiple NEA stages simultaneously in working memory without explicit documentation (Elgrably & Leikin, 2021).

These differences indicate that flexibility manifests in two forms: explicit (S1a's detailed documentation) and implicit (S1b's mental efficiency). Elgrably & Leikin (2021) emphasize that representational flexibility, as an indicator of mathematical creativity, can manifest in either form, depending on cognitive style and the degree of automatization. Turan et al. (2025) further assert that flexibility does not require explicit documentation to be considered creative, provided the underlying conceptual understanding is sound.

$A = \{1,2,3\}, B = \{2,3,4,5,6\}$
Based on the rule "a number that is one greater than a", the relationship can be expressed as $b = a + 1$, meaning that for

The relation is: $R = \{(a,b) \in A \times B | b = a + 1\}$

Substituo

Result: $R =$

Since every element of set A corresponds to exactly one element of set B, the relation satisfies the definition

1. Dik: $A = \{1,2,3\}, B = \{2,3,4,5,6\}$
Aturan: "Bilangan lebih besar satu dari a" maka $b = a + 1$
Urutan: Untuk setiap $a \in A$, perlu mencari satu atau lebih $b \in B$ yg memenuhi $b = a + 1$
Ditanyakan: $R = \{(a,b) \in A \times B | b = a + 1\}$

Jawab:
substitusi
 $a = 1 \rightarrow b = 1 + 1 = 2$
 $a = 2 \rightarrow b = 2 + 1 = 3$
 $a = 3 \rightarrow b = 3 + 1 = 4$
Hasil: $R = \{(1,2), (2,3), (3,4)\}$

A	B
1	2
2	3
3	4

Setiap anggota A berpasangan tepat satu anggota B. Maka relasi tersebut merupakan fungsi.

Reading
Comprehension
Transformation
Process skill
Encoding

Figure 3. S1a's learning outcome on Q1

Dik: $A = \{1,2,3\}$
 $B = \{2,3,4,5,6\}$
Aturan: "Bilangan lebih besar satu dari a"
Ditanyakan: $R = \{(a,b) \in A \times B | b = a + 1\}$

Jawab:
 $b = a + 1$
 $\rightarrow b = a + 1 \rightarrow a = 1$
 $b = 1 + 1$
 $b = 2$
 $\rightarrow b = a + 1 \rightarrow a = 2$
 $b = 2 + 1$
 $b = 3$
 $\rightarrow b = a + 1 \rightarrow a = 3$
 $b = 3 + 1$
 $b = 4$
 $R = \{(1,2), (2,3), (3,4)\}$

\rightarrow Setiap anggota A berpasangan tepat satu di B maka merupakan fungsi.

Reading
Comprehension
Process skill
Encoding

Figure 4. S1b's learning outcome on Q1

Figure 1 illustrates that S1 comprehends the problem thoroughly, accurately writes the relational rule $b = a + 1$, and transforms verbal information into appropriate mathematical representations and an arrow diagram. The ordered pairs (1,2), (2,3), and (3,4) were generated through correct calculations, accompanied by the conclusion that the relation constitutes a function because each element in set A is paired with exactly one element in set B. These findings reinforce that S1 has mastered all stages of information

processing, from reading and comprehension to transformation and encoding—systematically and consistently.

Process Skills Stage

At the process-skills stage, S1a and S1b demonstrated adaptive originality—not through rote enumeration, but by recognizing functional patterns (e.g., $b = a + 1$ in Q1) and independently constructing numerical–alphabetical mappings in Q2. However, neither student employed unconventional or divergent strategies; both adhered to standard procedures that they personalized in execution. This suggests that for high-achieving students, originality tends to emerge as an adaptive application of standard methods to achieve deeper understanding, rather than through radical or atypical strategies (DeVink et al., 2022). Differences in originality are evident: S1a engaged in explicit verification of each correspondence, whereas S1b relied on cognitive efficiency by conducting verification mentally without documentation (“checking all the patterns in my mind”).

Affective dimensions further illuminate this profile. S1a emphasized confidence in conceptual understanding (“I understand the concept”), whereas S1b exhibited an efficiency-oriented disposition (“wanting to solve the problem as efficiently as possible”). This high self-efficacy supports fluency and flexibility but constrains the exploration of alternatives, resulting in originality that remains adaptive rather than divergent. These findings align with Nufus et al. (2024) and Leikin & Elgrably (2020), who note that high-achieving students with strong conceptual confidence tend to select strategies that are proven effective and low-risk, meaning their creativity manifests in the optimization of procedures rather than in the search for non-conventional solutions.

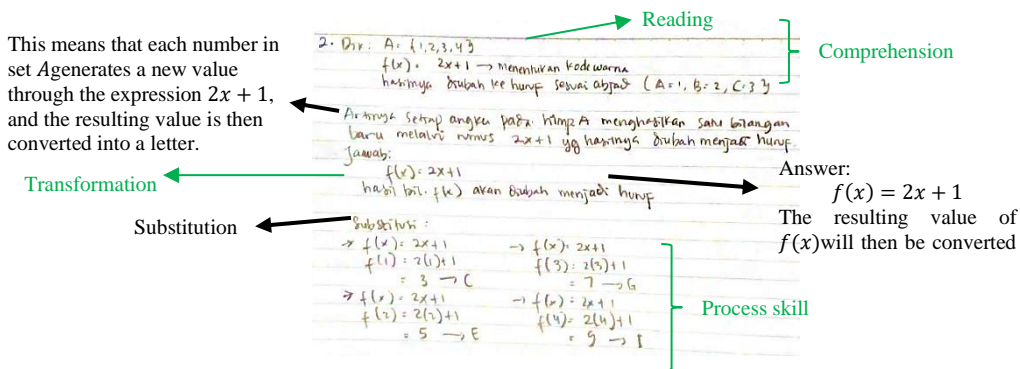


Figure 5. S1a’s learning outcome on Q2

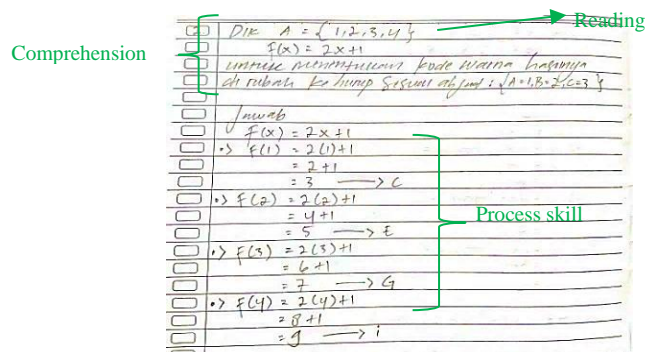


Figure 6. S1b’s learning outcome on Q2

Figure 2 shows that S1 solved Q2 systematically: reading the problem, understanding the function rule $f(x) = 2x + 1$, substituting each element of $A = \{1, 2, 3, 4\}$, obtaining the results $f(1)=3$, $f(2)=5$, $f(3)=7$, and $f(4)=9$, and subsequently converting them into letters (C, E, G, I). This process demonstrates mastery of functional concepts and the ability to correctly transform numerical representations into symbolic forms. However, at the encoding stage, S1's explanation lacks depth because it fails to connect the computational results to the task's contextual meaning. Even so, the solution remains logically structured and coherent.

Encoding Stage

At the encoding stage, S1a and S1b exhibit distinct elaboration patterns shaped by their cognitive mechanisms and metacommunicative dispositions. In Q1, both students demonstrated complete encoding performance, correctly applying the functional criterion. They explicitly stated that "each element in set A is paired with exactly one element in set B," indicating strong conceptual understanding and adequate mathematical communication skills (Kozłowski et al., 2019). However, the encoding limitation emerges in Q2, not in Q1. Although both students correctly derived the ordered pairs and the corresponding letter codes (C, E, G, I), they did not connect these results to the contextual meaning of the coloring task nor articulate whether the mapping satisfied the functional criterion. This absence of synthesis reflects a cognitive transparency illusion, the belief that their internal understanding is sufficiently clear and therefore does not require explicit articulation.

Interview data reveal that S1a tends to elaborate only when perceiving the concept as complex (metacommunicative selectivity), whereas S1b minimizes written explanation for efficiency (efficiency optimization). For both students, the cognitive transparency illusion reinforces the decision not to articulate their reasoning in Q2. Thus, the encoding limitation does not stem from conceptual weakness but from communicative judgments about when elaboration is deemed necessary. These findings align with previous research indicating that high-achieving students often demonstrate sufficient internal reasoning but inconsistent external communication (Suherman & Vidákovich, 2023), and the ability to explicate reasoning is a metacommunicative skill that requires intentional cultivation (Nguyen et al., 2025; Genç et al., 2025).

S2: Moderate-Ability Student

Students with medium ability, represented by S2a and S2b, demonstrate adequate conceptual understanding but lack consistency in applying it. Although both exhibit relatively similar achievement profiles, nuanced differences emerge in their affective dispositions, strategic decision-making patterns, and cognitive resilience when confronted with procedural obstacles. To provide a comprehensive context, the interview excerpts with S2a and S2b are presented first to illustrate their cognitive and affective processes, followed by an in-depth interpretive analysis of each NEA stage.

P : *"How did you understand the problem and decide on the steps you used?"*

S2a : *"I started by making an arrow diagram because it was the strategy I was familiar with, but I became confused about which values to test. I then shifted*

to substitution because it felt safer, but when the results did not match, I began to doubt myself and eventually stopped.”

S2b : *“I used substitution right away because it is the method I remember best. However, when the numbers did not fit, I began doubting my steps. I wanted to try another method, but I was not confident enough to do it.”*

P : *“What did you feel during the process, especially when doubts or confusion appeared?”*

S2a : *“I started feeling anxious when the table did not work. The uncertainty kept growing until I did not dare continue.”*

S2b : *“I was nervous from the beginning. Once a step felt wrong, my confidence immediately dropped, and I became afraid of making the next mistake.”*

The interview transcript reveals a similar affective pattern between S2a and S2b, characterized by unstable self-confidence, increasing anxiety when encountering obstacles, and a tendency to withdraw from problem-solving efforts when difficulties arise. The key difference lies in the timing of doubt: S2a experiences a breakdown in confidence after an initial unsuccessful strategy (post-attempt anxiety), whereas S2b experiences anxiety from the outset (anticipatory anxiety). These two patterns have distinct implications for the development of flexibility and originality, which will be discussed in the following analysis.

Reading and Comprehension Stages

S2a and S2b successfully identified key information and accurately interpreted the given functions, demonstrating initial fluency in extracting mathematical information. Statements such as “start by making an arrow diagram” (S2a) and “directly use substitution” (S2b) indicate the activation of procedural schemas stored in long-term memory. However, unlike S1, who displayed automatic pattern recognition, S2's fluency is procedural-rote rather than conceptual-relational. This aligns with Schoevers et al. (2018), who distinguish between superficial comprehension (grasping isolated elements) and relational understanding (recognizing structural relationships among elements). In Q1, S2b exhibited an anomalous pattern in which transformation and process-skills steps were attempted even though comprehension had not been fully achieved. This represents a form of fragile comprehension, adequate for routine application but insufficient for non-routine problem-solving that requires strategic flexibility. S2b relied on memorized patterns derived from similar problems to carry out technical steps without understanding the underlying relational meaning (Schoevers et al., 2018). He was able to “recognize a pattern” and follow standard procedures, yet weak conceptual grounding led to incomplete solutions.

Affectively, S2b experienced anticipatory anxiety (“nervous from the beginning”), which imposed additional load on working memory, whereas S2a displayed conditional confidence, self-assurance contingent upon the success of the subsequent step. In contrast to S1's stable self-efficacy, S2's confidence is fragile and context-dependent, easily disrupted when confronted with procedural obstacles. Their foundation is procedurally strong but vulnerable when required to perform more complex representational transformations.

Transformation Stage

At the transformation stage, S2a and S2b demonstrated attempted flexibility. In Q1, both were able to initiate the transformation process by constructing mathematical representations from the verbal description. S2a attempted to draw an arrow diagram, while S2b proceeded directly with substitution. These initial attempts indicate a foundational level of representational awareness, namely the understanding that a problem can be approached through multiple forms of representation (Elgrably & Leikin, 2021). However, when their initial strategies failed to produce accurate solutions, both encountered a strategic breakdown. S2a stated that she was “confused about which value to try” and subsequently “stopped” when the substitution did not work. S2b hesitated when the result did not match expectations, revealing weak metacognitive monitoring. They were unable to determine whether the inaccuracy stemmed from a conceptual misunderstanding, a procedural error, or an inappropriate strategy choice, consistent with the characteristics of poor metacognitive control described by Schindler & Lilienthal (2020). Their strategic knowledge appears fragmented: they know several methods but lack understanding of when and why particular strategies are effective, reflecting insufficient strategic conditional knowledge (Lu et al., 2025). Both also exhibit a low persistence threshold, giving up quickly when encountering obstacles (Nufus et al., 2024).

Affective dimensions further exacerbated this issue. S2a reported that “my uncertainty kept growing, so I did not dare to continue,” whereas S2b stated that “my confidence immediately dropped and I was afraid of making another mistake.” Procedural failure triggered a negative affective spiral (strategic difficulty → decreased confidence → increased anxiety → task avoidance), which hindered flexibility because students became unwilling to take cognitive risks required for strategic exploration (Wahyuni et al., 2024). The difference between S2a and S2b lies in the point at which doubt emerged: S2a experienced doubt after attempting a strategy (reactive doubt), whereas S2b was already uncertain from the outset (proactive doubt). These findings emphasize that flexibility is not merely about knowing alternative strategies, but about sustaining effort amid strategic uncertainty and evaluating and adapting one’s approach. Among moderately achieving students, flexibility tends to remain surface-level—knowing alternatives but being unable to apply them adaptively due to the interaction between weak metacognitive control and unstable affective regulation.

- Given $A = \{1,2,3\}, B = \{2,3,4,5,6\}$
- The rule “a number greater by one than a” means that $b = a + 1$.
- This means that for every $a \in A$, it is necessary to find one or more $b \in B$ that satisfy $b = a + 1$.

Relation : $R = \{(a,b) \in A \times B \mid b = a + 1\}$

Result: $R = \{(1,2), (2,3), (3,4)\}$

Penjelasan

Informasi diketahui: $A = \{1, 2, 3\}$. $B = \{2, 3, 4, 5, 6\}$.

aturan bilangan lebih besar satu dari A berarti $b = a + 1$

- Ati aturan untuk setiap $a \in A$. Perlu mencari satu atau lebih $b \in B$ yang memenuhi $b = a + 1$

$R = \{(a,b) \in A \times B \mid b = a + 1\}$

$a = 1 \rightarrow b = 2$

$a = 2 \rightarrow b = 3$

$a = 3 \rightarrow b = 4$

Hasil $R = \{(1,2), (2,3), (3,4)\}$

Reading

Comprehension

Transformation

Process skill

Figure 7. S2a’s learning outcome on Q1

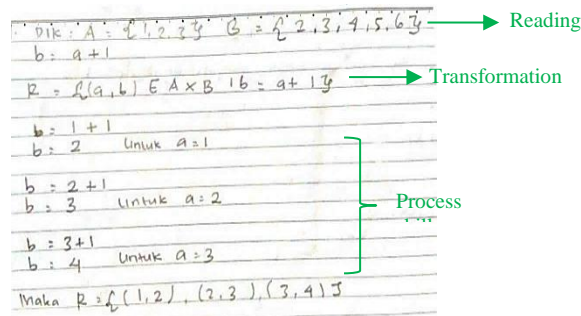


Figure 8. S2b’s learning outcome on Q1

Process Skills Stage

At the process-skills stage, S2a and S2b exhibited significant limitations in originality, characterized by strong reliance on memorized procedures and limited capacity for creative adaptation. In Q1, although both recognized the pattern $b=a+1$, they failed to apply it systematically: S2a began with correct manual calculations and successfully generated several ordered pairs: (1,2), (2,3), (3,4), but discontinued the process prematurely due to uncertainty about whether all domain elements had been checked or whether the relation qualified as a function. S2b performed partial substitutions and obtained some correct results, but did not verify completeness. In Q2, S2a did not construct an organizational structure to ensure that all domain elements were evaluated, whereas S2b produced numerical values without consistently converting them into letter codes, indicating that their procedures were fragmentary and not goal-directed. S2b also demonstrated premature procedural activation, immediately attempting computations without first forming a mathematical representation (Mathaba et al., 2024). These behaviors reflect that earlier difficulties in forming stable representations during the transformation stage left both students without a coherent plan to guide their procedural work, leading to process-skills errors that emerged as direct downstream consequences of transformation breakdowns. This pattern illustrates a hierarchical interrelation among NEA stages: insufficient transformation directly constrains the quality and coherence of subsequent procedural execution.

These originality constraints stem primarily from procedural anchoring, the tendency to cling to familiar strategies (DeVink et al., 2022), the absence of pattern-based reasoning that leaves them trapped in element-by-element calculations without structural abstraction (Leikin, 2013), and cognitive risk aversion linked to fragile self-efficacy (Hartati et al., 2025). The lack of metacognitive reflection prevented both students from evaluating the efficiency or quality of their solutions, leaving them unaware that their procedures were suboptimal (Schindler & Lilienthal, 2020). Their manifestations differ: S2a displayed attempted flexibility without follow-through, recognizing her strategic limitations and attempting to switch approaches but lacking the metacognitive capacity to execute the shift effectively. S2b showed early commitment with premature abandonment, choosing a single method from the outset and giving up upon encountering difficulty without attempting adaptation, reflecting lower metacognitive monitoring. Overall, these findings indicate that among moderately achieving students, limited originality is not merely a lack of creative ideas but the inability to sustain or execute non-

standard approaches due to weak metacognitive regulation, fragmented procedural knowledge, and unstable affective regulation.

This means that each number in set A generates a new value through the formula $2x + 1$, and the result is then converted into a letter to determine the color or code of the lamp post.

function rule

Reading

Comprehension

Transformation

Process skill

Figure 9. S2a's learning outcome on Q2

Reading

Comprehension

Figure 10. S2b's learning outcome on Q2

Encoding Stage

At the encoding stage, S2a and S2b demonstrated minimal elaboration, the inability to articulate their thinking processes, connect results to the problem context, or provide coherent justification. In Q1, S2a generated several ordered pairs without concluding whether the relation constituted a function, while S2b produced only partial results lacking structure or explanation. In Q2, S2a computed several function values but did not complete the conversion to letter codes, whereas S2b wrote down the formula without applying it systematically across the domain. Their limited elaboration was driven by cognitive overload that had already drained mental resources during the transformation and process-skills phases; cognitive capacity became consumed by unsuccessful procedures, leaving insufficient working memory for metacognitive articulation (Suherman & Vidákovich, 2023). Additionally, the students lacked a communicative disposition, an awareness that mathematical explanation is an essential component of problem-solving, resulting in a product-rather-than-process orientation (Nguyen et al., 2025). Incomplete processing produced a recursive cycle: unfinished reasoning prevented elaboration, and the absence of elaboration, in turn, prevented recognition of the incomplete reasoning. This limitation was exacerbated by affective shutdown as anxiety intensified. S2b's remark, "I got stressed and eventually gave up," reflected full cognitive disengagement (Wahyuni et al., 2024).

The patterns differed between students: S2a produced incomplete but structured responses. Some organizational attempt was present but abandoned midway due to cognitive overload and premature disengagement. S2b exhibited fragmentary, unstructured responses, with neither organizational pattern nor communicative awareness; the mathematical explanation was almost absent. Their failure in elaboration was not merely a communication issue but the downstream consequence of a chain of cognitive and affective disruptions, unstable conceptual understanding, insufficient strategic monitoring, and mental-emotional fatigue that ultimately prevented them from reflecting on their thinking processes.

S3: Low-Ability Student

Low-achieving students, represented by S3a and S3b, had fundamental difficulties understanding the concepts of relations and functions, which affected the entire process of mathematical problem-solving. Unlike S1 and S2, who were able to navigate the initial stages with relatively little difficulty, S3 encountered cognitive obstacles from the reading and comprehension stage onward, leading to a cascading failure across subsequent NEA stages. Before presenting the analysis of each NEA stage, the complete interview transcript with both subjects is provided below.

P : *“How did you understand the problem, and what made it difficult for you to solve it?”*

S3a : *“When I read the problem, I forgot the rules and conditions I needed. I tried to recall the previous lessons, but I was not sure, so I did not know where to start. The more confused I became, the more I decided to stop.”*

S3b : *“I panicked as soon as I saw the function problem. I tried sketching something to help myself understand, but it only made me more confused. I did not know how to connect the numbers with the outputs of the function, so in the end I just guessed.”*

P : *“What did you feel while working on the problem, especially when you started to feel confused or uncertain?”*

S3a : *“I immediately felt afraid of making mistakes because I was not sure whether I remembered the material correctly. The more I tried to make sense of it, the less confident I became, so I stopped.”*

S3b : *“I felt anxious and confused from the beginning. As I understood less and less, I became stressed and eventually gave up and just guessed.”*

The interview transcript reveals an affective–cognitive pattern in S3 that is qualitatively distinct from that of S1 and S2: a critical combination of fundamental conceptual deficits, debilitating mathematical anxiety, and the absence of compensatory strategies. The key difference lies in their dominant failure mechanisms: S3a experienced retrieval failure the inability to access prerequisite knowledge from long-term memory which triggered cognitive paralysis, resulting in the complete cessation of problem-solving effort; whereas S3b exhibited an immediate panic response, leading to random guessing as a maladaptive coping mechanism. These patterns carry different implications for remediation and for supporting the development of mathematical creative thinking.

Reading and Comprehension Stages

S3a and S3b experienced a fundamental fluency breakdown during reading. Although they could read the text literally, they failed to translate the phrase “a number one greater than a” into an appropriate mathematical structure. They merely copied the information without understanding the relations between sets, demonstrating mathematical reading difficulties resulting from the absence of relational schemas in long-term memory (Kania et al., 2024). Consistent with Schoevers et al. (2018), students with weak conceptual understanding lack the semantic networks needed to map linguistic expressions onto mathematical structures. This inability to read meaningfully immediately propagated to the comprehension stage. S3a reported “not knowing where to start,” indicating a retrieval failure of prerequisite knowledge. At the same time, S3b attempted to draw something but only became more confused because it did not know what needed to be represented, an indication of absent representational metacognition (Elgrably & Leikin, 2021). Consequently, fluency never emerged, as fluency depends on the ability to extract and integrate information (Bicer et al., 2024).

Affective dimensions further exacerbated these cognitive vulnerabilities: S3a experienced a fear of making mistakes, which reduced working-memory capacity and impaired comprehension, whereas S3b showed panic from the outset, triggering a premature abandonment of deeper processing (Wahyuni et al., 2024). Anxiety in S3 was both causal and consequential; weak knowledge triggered anxiety, and anxiety further weakened cognitive processes, creating a self-reinforcing cycle that prevented the development of even basic fluency.

Transformation Stage

At the transformation stage, both S3a and S3b failed to convert verbal information into mathematical representations, rendering flexibility structurally impossible. Unlike S2, who had strategies but failed to execute them, S3 did not even know what needed to be represented or how to represent it, demonstrating that flexibility can arise only when a sufficient conceptual foundation exists. S3a merely copied parts of the problem in Q1 without constructing a relational representation; the statement “forgot the rules and conditions” reflects representational paralysis. Meanwhile, S3b attempted to draw “something.” However, the representation was arbitrary and non-mathematical, indicating the absence of representational competence, an understanding of what must be represented and in what mathematical form (Elgrably & Leikin, 2021). The claim of “not knowing how to connect numbers with function outputs” reveals a complete absence of functional thinking. Failure at the transformation stage eliminates any basis for flexibility. Flexibility requires a strategy repertoire, an understanding of when strategies are appropriate, and the procedural fluency to execute them (Lu et al., 2025).

Affective and metacognitive responses further intensified the breakdown: S3a chose to stop entirely (avoidance coping), while S3b resorted to guessing (a low-effort heuristic), both indicating the absence of productive persistence. Misattributions in the interviews, S3a's belief that the issue was merely “forgetting rules,” and S3b's “not knowing how to connect numbers, reflect a lack of metacognitive awareness; they identified symptoms but failed to recognize the conceptual roots of their difficulties, thereby precluding effective self-regulation of learning.

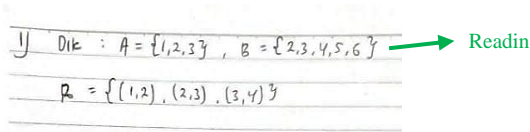


Figure 12. S3a’s learning outcome on Q1

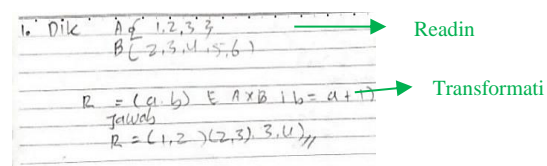


Figure 11. S3b’s learning outcome on Q1

Process Skills Stage

In the process skills stage, S3a and S3b exhibited a consequential absence of originality they did not merely fail to innovate, but lacked the prerequisites for doing so. Mathematical originality requires mastery of standard procedures, an understanding of the principles that allow variation, and the metacognitive willingness to explore non-standard approaches (DeVink et al., 2022; Hartati et al., 2025). S3 possessed none of the basic procedural knowledge there was no computation, no application of rules, and the work traces that appeared (such as random numbers) reflected guessing rather than reasoning. Consequently, divergent thinking could not arise; one cannot depart productively from a norm they do not understand (Leikin, 2013). The “different” behavior shown by S3 is not meaningful creativity but random deviation without mathematical purpose; true originality entails purposeful and productive divergence, not accidental confusion.

Affective dimensions further eliminated the possibility of originality. High anxiety, fear of error, and panic responses inhibited the cognitive exploration that requires tolerance for uncertainty (Bicer et al., 2024). The absence of originality in S3 reflects deeper mathematical difficulties: weak conceptual–procedural foundations compounded by severe anxiety. Effective intervention must therefore prioritize strengthening these foundational skills and addressing affective barriers before creativity can be meaningfully developed.

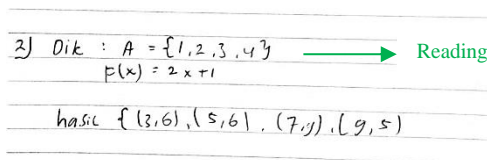


Figure 13. S3a’s learning outcome on Q2

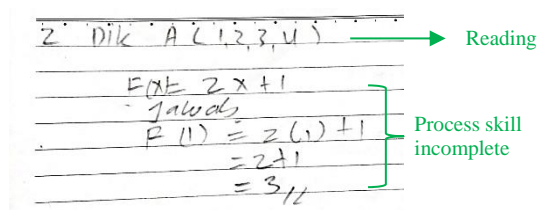


Figure 14. S3b’s learning outcome on Q2

Encoding Stage

At the encoding stage, the failure of elaboration in S3a and S3b is a direct consequence of the cumulative breakdowns from the reading phase through the process-skills phase. Because no coherent solution was ever produced, elaboration becomes structurally impossible there is no reasoning, result, or pattern available to explain. In both Q1 and Q2, S3a merely copied or wrote symbols without meaning, while S3b generated random numbers or arbitrary representations. This reflects non-elaboration, rather than minimal elaboration. Elaboration in mathematical creativity requires complete understanding, metacognitive engagement, mathematical communication skills, and motivation to explain (Kozlowski et al., 2019). S3 lacks all of these foundations:

understanding never formed (“forgot the rules,” “did not know how to connect the numbers”); metacognition is weak because reasoning never occurred; and limited mathematical language prevents the articulation of ideas (Nguyen et al., 2025).

Affective dimensions (anxiety, fear of error, and affective shutdown) further eliminate the possibility of elaboration. Consequently, S3 becomes trapped in a cycle: weak understanding → inability to elaborate → no reflection → understanding remains weak. Although their outward patterns differ (S3a through silent abandonment and S3b through meaningless production), both converge toward the same outcome: encoding failure as the final manifestation of accumulated cognitive and affective breakdowns.

The analysis of S1, S2, and S3 yields a theoretical contribution showing that the relationship between Newman’s Error Analysis (NEA) stages and the dimensions of Mathematical Creative Thinking (MCT) is neither linear nor one-to-one. Instead, it is strongly shaped by students’ internal representations, the stability of their metacognition, and their affective states. High-ability students (S1a, S1b) demonstrated strong fluency, flexibility, and adequate basic justification skills. However, they experienced a “cognitive transparency illusion, the belief that internal understanding is self-evident and therefore requires minimal external articulation, which constrained the depth of their elaboration and contextual-reflective communication. Importantly, strong fluency and flexibility did not guarantee optimal elaboration; the findings reveal that elaboration is not merely the recording of steps but requires a sustained disposition for mathematical communication and reflective metacognition. This extends theoretical understanding by showing that limitations in elaboration can emerge even when conceptual understanding is mature, indicating that encoding errors do not necessarily reflect cognitive weakness but may stem from expressive barriers and self-regulatory constraints. Medium-ability students (S2a, S2b) demonstrated fragile comprehension, procedurally adequate but conceptually shallow, leading to inconsistent strategic execution characterized by premature discontinuation of solution processes, strategic breakdowns under cognitive load, and affective instability when encountering obstacles. The analysis clarifies that their limitations in flexibility and originality arose not from a lack of ideas but from the inability to sustain, monitor, or shift to alternative strategies, illustrating how cognitive overload and weakened self-monitoring interact to destabilize performance. Low-ability students (S3a, S3b) demonstrated heterogeneous performance in literal reading, succeeding on simpler items but experiencing consistent cascading failures from the comprehension stage onward across all items. These failures were triggered by an inability to transform literal information into mathematical meaning, the absence of representational schemas, and debilitating mathematical anxiety that prevented sustained cognitive engagement. For S3, early-stage breakdowns left no cognitive space for creativity to emerge. The theoretical contribution here is a new interpretation: errors within NEA function as multidimensional indicators of creative ability, with each MCT dimension manifesting differently across ability profiles. Thus, the resulting NEA–MCT model does not merely locate errors but explains the cognitive–affective mechanisms underlying variations in mathematical creativity.

This study implies that NEA-based error analysis should be viewed as a diagnostic tool for creativity processes rather than simply an indicator of procedural weaknesses. Errors at each NEA stage provide essential information about the quality of internal representations, metacognitive stability, and students’ affective states. Consequently,

mathematics assessment and instruction should shift from emphasizing final answers to evaluating thinking processes, including how students construct, transform, and communicate their ideas. In practical terms, high-achieving students require support in elaboration and mathematical communication; moderate groups need scaffolding for strategic regulation to prevent cognitive overload; and lower-achieving students require representational and affective scaffolding to build the foundations of creative thinking. Overall, the NEA–MCT model offers a foundation for more differentiated, diagnostic, and cognitively and affectively responsive designs of assessment and instructional interventions.

This study has several limitations. The limited number of participants renders the identified patterns exploratory. In addition, the NEA–MCT mapping may not fully capture the multidimensional complexity of creative thinking. Finally, the focus on relations and functions may constrain the generalizability of the findings to other mathematical topics. Future research may employ a more diverse sample, incorporate real-time think-aloud protocols, and explore additional mathematical domains.

▪ CONCLUSION

The integration of Newman’s Error Analysis with the dimensions of Mathematical Creative Thinking in this study produced an NEA–MCT mapping model with three key findings. First, the pattern of student errors reveals a hierarchical structure: most students completed the reading and comprehension stages but encountered substantial difficulties in the transformation, process skills, and encoding stages. The narrowing flow illustrated in the Sankey diagram indicates that the primary challenges are not related to basic literacy skills but arise from increasing representational and procedural complexity, particularly at the transition between the transformation and process-skills stages. Second, the mechanisms underlying these errors involve complex interactions among cognitive factors (conceptual understanding, representational competence, procedural fluency), metacognitive factors (strategic monitoring, metacognitive awareness), and affective factors (anxiety, self-efficacy, cognitive confidence). In-depth analysis of six students across three ability categories revealed distinct profiles: high-ability students (S1a, S1b) demonstrated strong fluency and flexibility but experienced a “cognitive transparency illusion” that constrained their elaboration; medium-ability students (S2a, S2b) exhibited inconsistent strategic execution due to strategic breakdowns, cognitive overload, and affective instability; whereas low-ability students (S3a, S3b) experienced cascading failures beginning from the early stages, triggered by retrieval failure and representational paralysis. Third, the relationship between NEA stages and MCT dimensions is nonlinear and contextual, indicating that errors at different stages reflect specific limitations in creative thinking shaped by varying cognitive affective mechanisms. The resulting NEA–MCT mapping model contributes theoretically by conceptualizing errors not as technical failures but as cognitive affective indicators of barriers to creative thinking, thereby extending the understanding that error analysis can serve as a diagnostic tool for mathematical creativity processes.

These findings have significant implications for mathematics education, particularly in the design of assessment and instructional systems that are more diagnostic, differentiated, and sensitive to students’ cognitive–affective processes. The NEA–MCT model enables teachers to identify not only where students make errors but

also which dimensions of creativity are hindered and which cognitive–affective mechanisms underlie those difficulties, allowing pedagogical interventions to be more precisely targeted. High-ability students require metacommunicative scaffolding to externalize their internal reasoning; medium-ability students need integrated cognitive–strategic–affective support to prevent strategic breakdowns and regulate anxiety; whereas low-ability students require foundational representational instruction combined with affective scaffolding to mitigate anxiety that inhibits cognitive exploration. This study emphasizes the need for mathematics learning to shift from a focus on final answers toward an emphasis on students’ thinking processes. Nevertheless, this study has several limitations. The small number of participants makes the identified patterns exploratory, and the focus on two problems on relations and functions limits the generalizability of the findings. Moreover, the current NEA–MCT mapping may not yet capture the full complexity of students’ creative thinking processes, indicating the need for further refinement. Future studies may involve more diverse samples, expand the range of problem contexts, and examine the model across different mathematical domains or educational levels, including its potential integration with richer affective variables or technology-based assessments.

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